Persistence without too much price stickiness: the role of variable factor utilization

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Abstract

We study the propagation of monetary shocks in a sticky price model with capital utilization and labor effort. Variable factor utilization enriches the propagation mechanism of monetary shocks by reducing the sensitivity of marginal costs to changes in aggregate output. Variable labor effort is relatively more important for generating persistence than variable capital utilization, except when depreciation is fairly unresponsive to changes in utilization. In addition to reinforcing the propagation mechanism of monetary shocks, volatilities and comovements of output, capacity utilization and hours produced by the model are close to those observed in the UK.

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1. Introduction

The relationship between monetary shocks and real activity has been a central topic of debates for at least forty years. What kinds of models are consistent with the empirical features of the propagation mechanism of monetary shocks, documented for example in Christiano et al. (1999)? In particular, what kind of models can account for the magnitude of the impact response of output to monetary policy shocks and for its persistence over time? After a decade of studies employing flexible price, cash-in-advance type models, the literature has turned to a sticky price, monopolistic competitive framework. The ‘New Keynesian’ Phillips curve, a relationship between prices and real marginal costs, has become a distinctive feature of these models and represents the mechanism through which monetary disturbances are propagated to prices and the real economy. Following a monetary shock, demand increases and—given the monopolistic competitive assumption—employment and output also increase. The expansion in economic activity raises real marginal costs, which, in turn, exerts a cost-push effect on inflation.

Chari et al. (2000) (henceforth CKM) have shown that the only way to produce sizeable real effects in response to monetary policy disturbances in a basic sticky price model is to assume a high degree of price stickiness. This is a rather unsatisfactory mechanism for at least two reasons. First, there is substantial controversy in the literature surrounding the existence and the magnitude of price adjustment costs. Bils and Klenow (2002) estimate that the degree of price stickiness for US goods is rather low, 4.33 months, compared to the 12 months that Taylor (1999) supports as summary evidence in the Handbook of Macroeconomics. Second, even if there were an agreement on the existence of price adjustment costs, there is no accepted framework for modeling the costs that firms face for changing their price (see for example Mankiw and Reis, 2002). One way to address these criticisms is to reduce the model’s reliance on nominal rigidities as a mechanism for propagating shocks. However, if the degree of nominal rigidity is low, monetary policy shocks have essentially no real effects.

The findings of CKM have engineered a growing literature aimed at producing alternative mechanisms for generating persistence without the need of extreme price stickiness. Most of these works use mechanisms that induce a steeper individual marginal cost and revenue curves, and/or flat aggregate marginal cost of output. The most popular approach is sticky nominal wages (see for example, Erceg, 1997 and Huang and Liu, 2002). Edge (2002) and Ascari (2000) question the ability of nominal wage contracts to generate persistence and Edge (2002) highlights the importance of firm specific factor inputs in generating persistent real responses to monetary shocks. Along the same lines, Bergin and Feenstra (1998) develop a model in which money shocks induce persistent output responses via an input–output structure of production.

As an alternative to nominal wage rigidity the literature has also used real rigidities for generating persistence. Ball and Romer (1990) indicate that real rigidities may play a crucial role in making nominal shocks non-neutral and in amplifying (small) nominal rigidities—an argument echoed in Farmer (2000). Kimball (1995) shows how real rigidities can generate persistence for a general class of models. Similarly, Gertler and Gilchrist (2000) emphasize the role of investment delays in generating hump-shaped output dynamics. Assuming highly elastic factors of production may also generate persistence.
This increases persistence by flattening the marginal cost curve for all firms equally. However, within a standard labor market-clearing framework this can only be achieved by assuming infinite labor supply elasticity (see Kiley, 1997 and Dotsey et al., 1997). This paper adds to this growing literature by exploring the role of variable factor utilization in generating flat marginal costs. Variable factor utilization appears to be useful for making responses sizeable and more persistence, since it increases the elasticity of effective factor inputs without the need of assuming unrealistic factor elasticities.

The mechanism we employ is based on two empirically relevant features:

(a) variable capital utilization, and
(b) variable labor effort (see e.g. Burnside and Eichenbaum, 1996 and Cook, 1999).

Factor utilization is a key factor of the supply side of the economy and policymakers frequently regard it as an indicator of the state of the real activity. For example, the Federal Reserve Board uses capacity utilization as a measure of the maximum sustainable level of output. As a result, capacity utilization is often used for providing information about the build-up of inflationary pressures in the economy (see, e.g., Larsen et al., 2002). Besides its relevance from a policy perspective, Burnside and Eichenbaum (1996), Basu and Kimball (1997) and Basu and Fernald (2000) have shown that variable capital and labor utilization is an important factor for explaining the cyclicality of productivity.

The mechanism by which time-varying factor utilization contributes to persistence is as follows. Time-varying factor utilization makes output highly sensitive and responsive to demand shocks without proportionally altering real marginal costs.

This is achieved by making labor more elastic, through movements in effort, and by increasing the responsiveness of investment to changes in demand, through movements in capital utilization. The increased sensitivity of output to changes in demand allows for a reduction in the degree of nominal rigidity needed to produce sizeable and persistent real effects. The flattening of marginal costs enhances also inflation persistence through the Phillips curve. We are able to disentangle the contribution of variable labor effort and variable capital utilization to persistence. We find that the presence of variable labor effort stretches over time the effects of monetary shocks and is relatively more important for generating persistence, except for set-ups in which the elasticity of depreciation to changes in utilization is low.

While our model with factor utilization can generate investment and output volatilities and co-movements in output, hours and measures of capital and labor utilization which replicate those found in the UK, we still fail to quantitatively replicate the observed persistence of output and inflation. Many current contributions have pointed out that it is unlikely for a single friction model, like ours, to account for all the effects of monetary shocks in the economy. Christiano et al. (2001) (CEE) and Dotsey and King (2001) have used a combination of different frictions to achieve this. CEE use a model with variable capital utilization and wage stickiness which can account for output persistence and inflation inertia. Their model, however, includes a number of features that deviate from the basic setting used in the literature (for example, habit persistence in consumption). Therefore, it is hard to disentangle the contribution of each added feature in generating output and inflation sluggishness. Dotsey and King (2001) also introduce variable capital utilization in
a model with intermediate inputs in production. This additional feature dramatically alters the behavior of real marginal costs relative to our case.

Many real business cycles model have also used adjustment lags in labor inputs (see Bils and Cho, 1994; Burnside and Eichenbaum, 1996; and Wen, 1998) to generate persistence. We show that such feature combined with nominal rigidities could produce problematic outcomes. In fact, while labor supply rigidities enhance the propagation mechanism for output, they decrease inflation persistence and could induce a negative relation between persistence and the degree of price stickiness.

The paper is organized as follows. Section 2 describes the model; Section 3 discusses the calibration and Section 4 compares the dynamic responses across a number of models. Section 5 examines the sensitivity of our conclusions to changes in some key features of the model. Finally, Section 6 concludes.

2. The model

The economy consists of infinitely lived agents, firms, and a government sector. Households and firms optimize intertemporally and have rational expectations. Monopolistic firms set their price to maximize profits, but cannot always adjust them instantaneously in response to changing economic conditions. Nominal price stickiness is modeled as in Calvo (1983). Firms produce a continuum of differentiated goods, which are aggregated to produce a single composite good that can be used for consumption and investment. Households derive utility from the transaction services provided by real balances, and the economy is subject to real and monetary shocks. We assume that the technology for producing differentiated goods depends on capital and labor services. The former is defined as capital utilization times the existing physical stock of capital. The latter as labor effort times total hours worked. Capital is predetermined and its accumulation is subject to adjustment costs. The rate at which capital depreciates depends on its utilization. As a result, firms may over or under-utilize (e.g. hoard) capital in equilibrium.

2.1. Households

Households consume a continuum of differentiated goods indexed by $i \in [0, 1]$. The composite consumption good ($C_t$), a Dixit–Stiglitz aggregate over a multiplicity of goods, and price index ($P_t$) are defined as:

$$C_t = \left[ \int_0^1 c_t(i)^{\rho/(\rho-1)} i^{1-\rho} \, di \right]^{1/(1-\rho)}$$

(1)

and

$$P_t = \left[ \int_0^1 p_t(i)^{1-\rho} i \, di \right]^{1/(1-\rho)}$$

(2)

where $\rho$ is the elasticity of substitution between differentiated goods and is assumed to be greater than one.
The economy is inhabited by a large number of households. Agents derive utility from consumption \((C_t)\) and real money balances \((M^d_{t+1}/P_{t+1})\), and disutility from working. The latter depends both on the hours spent at work, \(h_t\), and on the level of effort expended while at work, \(e_t\), which we assume is observable.

The representative household owns the capital stock and at each period \(t\) receives income from renting the effective capital stock to the firm at a rate \(r_t\), and working at a wage rate \(w_t\), interest payments from a riskless bond \(R_t-1\) and lump-sum firm profits and government transfers, \(V_t\) and \(\Gamma_t\), respectively. It then chooses a sequence of current consumption \((C_t)\), hours, \(h_t\), effort \(e_t\), and utilization \(U_t\), nominal money balances \((M_t)\), bonds \((B_t)\), and capital, \((K_t)\), to maximize her present value utility:

\[
E_t \sum_{j=0}^{\infty} \beta^j \left[ \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{1}{1+\theta_n} h^{1+\eta_n}_t - \frac{1}{1+\theta_e} e^{1+\eta_e}_t + \frac{1}{1-\epsilon} \left( \frac{M^d_{t+1}}{P_{t+1}} \right)^{1-\epsilon} \right]
\]

subject to a series of budget constraints:

\[
P_{t+j}(C_{t+j} + I_{t+j}) + M^d_{t+j} + B_{t+j+1} = P_{t+j}(w_t+j h_{t+j} e_{t+j} + r_t+j U_{t+j} K_{t+j}) + M^d_{t+j-1} + R_{t-1+j} B_{t+j} + V_{t+j} + \Gamma_{t+j}
\]

\(\forall j = 0, 1, \ldots, \infty\), where \(\sigma, \theta_n, \theta_e, \lambda_n, \lambda_e, \epsilon > 0\), and \(\beta \in (0, 1)\). The parameter \(\sigma\) is the risk aversion coefficient, while \(\theta_n, \theta_e\) determine the supply elasticity of hours and effort respectively. Notice that for high values of \(\theta_e\) agents supply a constant level of effort so that a model with no variable employment margin can be nested in (3).

Investment \((I_{t+j})\) is related to the capital stock by:

\[
I_{t+j} = K_{t+j+1} - (1 - \delta(U_{t+j})) K_{t+j} + v \left( \frac{K_{t+j+1}}{K_{t+j}} \right) K_{t+j}
\]

where \(v(\cdot)\) is a function of investment and regulates capital adjustment costs. We adopt a quadratic specification of the form:

\[
v \left( \frac{K_{t+j+1}}{K_{t+j}} \right) = b \left[ \frac{K_{t+j+1}}{K_{t+j}} \right] - 1 \right]^2
\]

In addition, the depreciation function is parameterized as:

\[
\delta(U) = \delta U^\phi
\]

where \(\delta\) and \(\phi\) are positive constants.

Following Greenwood et al. (1988) we assume that using capital more intensively increases the rate at which capital depreciates, where \(\delta'(U) > 0\). The elasticity of marginal depreciation costs, \(\phi\), is negatively related to the responsiveness of utilization to shocks. When \(\phi\) is large, the negative effects of utilization on depreciation dominate the positive effects of utilization on output and firms choose to keep utilization constant.\(^1\)

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\(^1\) Bils and Cho (1994) suggest a set-up for capital utilization that has greater intuitive appeal in that an increase in total hours automatically raises the degree to which the existing physical capital stock is utilized. Both the dynamics and the persistence properties of the model remain unchanged when we incorporate their specification.
Thus, the model nests the standard sticky price model studied in CKM for \( \phi, \theta \to \infty \).

### 2.2. Firms

There is a continuum of monopolistically competitive firms, indexed by \( i \in [0, 1] \). Each firm \( i \) chooses its factor inputs, labor services \((h_t, e_t)\) and capital services \((K_t, U_t)\), in order to minimize the costs of producing a given level of output \((Y_t)\):

\[
w_t h_t e_t + r_t K_t U_t
\]

subject to the technological constraint:

\[
Y_t = (K_t U_t)^{1-\alpha} (X_t e_t h_t) \alpha,
\]

where \( 0 < \alpha < 1 \) and the level of technology is assumed to be exogenous.

Firms choose labor and capital services such that:

\[
w_t = \alpha Y_t mct, \tag{10}
\]

\[
r_t = \frac{(1-\alpha) Y_t mct}{K_t U_t}, \tag{11}
\]

where \( mct \) denotes the unit cost function, or real marginal cost.

Each firm \( i \) is allowed to reset its price \((P_t^i)\) according to a stochastic (time-dependent) rule that depends on receiving a signal at a constant random rate \((1-\eta)\). The parameter \( \eta \) governs the degree of nominal price rigidity: as \( \eta \) approaches 0, prices become perfectly flexible; as \( \eta \) approaches 1, firms charge a fixed price. Producers face an idiosyncratic risk due to the uncertainty of price adjustment. The probability that the price set at time \( t \) still prevails at \( t+j \) is \( \eta^j \). When a firm has the opportunity of changing its price, it will choose its level so as to maximize profits, taking aggregate output \((Y_t)\), the aggregate price level \((P_t)\), and nominal marginal cost \((MC_t^i)\) as given, i.e.:

\[
\max_{\tilde{P}_t^i} E_t \sum_{j=0}^{\infty} A_{t+j} \eta^j [\tilde{P}_t^i - MC_{t+j}^i] Y_{t+j}^i \tag{12}
\]

subject to a downward sloping demand for its good \((Y_t^i)\):

\[
Y_{t+j}^i = \left[ \frac{P_t^i}{P_t} \right]^{-\rho} Y_{t+j}, \quad \forall j = 0, 1, \ldots, \infty.
\tag{13}
\]

The solution to this problem is given by:

\[
\tilde{P}_t^i = \frac{\rho}{\rho - 1} \frac{E_t \sum_{j=0}^{\infty} A_{t+j} \eta^j [mc_{t+j}^i Y_{t+j}^i]}{E_t \sum_{j=0}^{\infty} A_{t+j} \eta^j Y_{t+j}^i}
\tag{14}
\]

where \( \rho/(\rho - 1) \) is the steady state mark-up, or the inverse of the steady-state real marginal cost. Equation (14) illustrates that the optimal price depends on current and expected future demand and real marginal cost \((mc_t^i)\). Intuitively, firms know that the price they set today may also apply in future periods, so the expected state of the economy influences current prices.
Given the pricing decisions of each firm $i$, the aggregate price index evolves according to:

$$P_t = \left[ \eta P_{t-1}^{1-\rho} + (1 - \eta) \tilde{P}_{t-1}^{1-\rho} \right]^{1/(1-\rho)}.$$  \hspace{1cm} (15)

The price level is therefore a weighted average of the optimal period $t$ price and prices set at $t - 1$ since some firms cannot reset their price in the current period.

2.3. Monetary policy

The money supply process is taken to be exogenous and represented by

$$M_t = \mu_t M_{t-1}. \hspace{1cm} (16)$$

Although it is well known that a large portion of the movements in monetary variables represents endogenous changes to the state of the economy, we initially maintain the assumption of money supply exogeneity to have a benchmark for comparison to the existing literature. In later sections we consider endogenous monetary policy.

The government finances its lump-sum transfers to the representative household through seignorage. The budget constraint is given by:

$$\Gamma_t + j = M_t + j - M_{t-1} + j, \hspace{1cm} (17)$$

for all $j = 0, 1, \ldots, \infty$.

2.4. Resource constraint

The economy is subject to the following resource constraint:

$$Y_t + j = C_t + j + I_t + j. \hspace{1cm} (18)$$

2.5. Shocks

There are two types of disturbances in this model: a technology and a nominal money supply shock. Each shock is assumed to follow an AR(1) process:

$$\hat{x}_t = \rho_x \hat{x}_{t-1} + \varepsilon_{xt}, \hspace{1cm} (19)$$

$$\hat{\mu}_t = \rho_\mu \hat{\mu}_{t-1} + \varepsilon_{\mu t}, \hspace{1cm} (20)$$

where $\varepsilon_{xt}$ and $\varepsilon_{\mu t}$ are mutually independent white noise processes.

In order to investigate the dynamics of the model, we log-linearize the equilibrium conditions around the steady state. The system of log-linear equations is presented in Appendix B.

3. Calibration

We calibrate the parameters of the model so that simulated series match salient features of the UK economy for the period 1977:Q1 to 2000:Q4. A full list of our choices is given in Table 1.

The discount factor is calibrated so that the steady-state annualized net real interest rate is 4%. Using the Euler equation and UK data, the estimates of the coefficient of relative
Table 1
Parameter values

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>Discount factor</td>
<td>0.9901</td>
</tr>
<tr>
<td>$1/\sigma$</td>
<td>Intertemporal elasticity of substitution</td>
<td>1.0</td>
</tr>
<tr>
<td>$\sigma/\varepsilon$</td>
<td>Consumption elasticity of money demand</td>
<td>0.2924</td>
</tr>
<tr>
<td>$\omega$</td>
<td>Interest semi-elasticity of money demand</td>
<td>$-0.2895$</td>
</tr>
<tr>
<td>$1/\theta_n$</td>
<td>Labor supply elasticity</td>
<td>0.25</td>
</tr>
<tr>
<td>$1/\theta_e$</td>
<td>Effort supply elasticity</td>
<td>2.0</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Elasticity of effective labor</td>
<td>0.717</td>
</tr>
<tr>
<td>$\delta$</td>
<td>Steady state rate of depreciation</td>
<td>0.0207</td>
</tr>
<tr>
<td>$\psi$</td>
<td>Working hours as a % of total time endowment</td>
<td>0.3</td>
</tr>
<tr>
<td>$e, U$</td>
<td>Steady state level of effort and capital utilization</td>
<td>1.0, 1.0</td>
</tr>
<tr>
<td>$\phi$</td>
<td>Elasticity of depreciation with respect to utilization</td>
<td>1.483</td>
</tr>
<tr>
<td>$\rho/\rho - 1$</td>
<td>Gross steady state mark-up</td>
<td>1.168</td>
</tr>
<tr>
<td>$\eta$</td>
<td>Probability that a firm is unable to change its price</td>
<td>variable</td>
</tr>
<tr>
<td>$\psi$</td>
<td>Capital adjustment costs</td>
<td>12.94</td>
</tr>
<tr>
<td>$\rho_X$</td>
<td>AR(1) parameter on productivity shock</td>
<td>0.895</td>
</tr>
<tr>
<td>$\rho_{\mu}$</td>
<td>AR(1) parameter on money shock</td>
<td>0.679</td>
</tr>
<tr>
<td>$\sigma_X$</td>
<td>Standard deviation of technology innovations</td>
<td>0.0083</td>
</tr>
<tr>
<td>$\sigma_{\mu}$</td>
<td>Standard deviation of money innovations</td>
<td>0.0063</td>
</tr>
</tbody>
</table>

risk aversion, $\sigma$, vary from close to zero in Patterson and Pesaran (1992) to close to one in Attanasio and Weber (1993). We choose $\sigma = 1$, since log utility gives exactly offsetting income and substitution effects. The consumption elasticity of money demand is estimated using the regression: $\ln(M_t/P_t) = \gamma_0 + \gamma_1 t + \gamma_2 \ln c_t + \gamma_3 R_t + \varepsilon_t$, where $c_t$ denotes the level of real consumption, $R_t$ the treasury bill rate, $M_t$ nominal money balances (M0), $P_t$ CPI prices, $\varepsilon_t$ an error term, and where one lag of consumption and one lag of the interest rate are used as instruments. The estimated value $\gamma_2 = 0.2924$ implies a value for $\varepsilon$ of 3.42. This value then implies an interest semi-elasticity of money demand, $\omega = 1/\varepsilon R^{1.0} = -0.2895$, which is higher than the value estimated in the data, $\gamma_2 = -0.11$.

Based on a 16-hour maximum workday, the quarterly total time endowment $\tau$ is 1460 hours. The number of hours worked by an employed person in the UK is 436.6 hours per quarter. By normalizing total time endowment to one, this implies that the percentage of working hours in the steady state is 0.3. We set the parameter $\lambda_n$ to match this number and $\lambda_e$ so that the steady state effort level is one, where $\lambda_e = \lambda_n h^{1/(1+\theta_n)}$.

Although Ball et al. (1988) have highlighted that very elastic labor supply can generate large nominal rigidities, actual labor supply elasticities have been estimated to be fairly small. Microeconomic studies using the Labor Force Survey data for the UK have estimated hours supply elasticities to be in the interval [0.05, 0.5] (see, for example, Arellano and Meghir, 1992). We set $\theta_n = 4$, which implies a labor supply elasticity of 0.25.

In equilibrium the value of the effort supply elasticity affects the variability of output. In order to match the variability of our simulated series with its empirical counterpart, for plausible values of $\sigma$, we set $\theta_e = 0.5$, which implies an effort supply elasticity of 2. In Section 6 we evaluate how results change when these two parameters are changed within a reasonable range.
The steady state share of labor income is set equal to 0.717, which matches the UK average over the sample and $\delta = 0.0207$ is selected so that the steady state investment-to-output ratio is the same as its sample average ($I/Y = 0.19$). Following CKM we calibrate $b$, the parameter determining the size of capital adjustment costs, to match the ratio of investment to output standard deviations, $\sigma(I)/\sigma(Y) = 2.94$, for the sample period considered.

The elasticity of depreciation to changes in utilization depends on the depreciation rate and the real interest rate: i.e. $\phi = U^* \delta''/\delta' + 1 = r^*/\delta$. Given previous choices, the implied value for $\phi = 1.483$.

The chosen value for $\rho$ implies a steady-state mark-up of 16.8%, which is consistent with the estimated mark-up reported in Crafts and Mills (2001) for the 1976–1996 period.

In standard sticky-price models, $\eta$ is set to 0.75, indicating that firms change their price on average once a year. Estimates of this parameter vary from 0.75 in Gali and Gertler (1999), to 0.5 in Gali et al. (2001) and 0.25 in Bils and Klenow (2002). We experiment with all these values in our exercises.

The values of $\rho_X$ and $\sigma_X$, the persistence and standard deviation of productivity shocks correspond to the estimates obtained for the UK TFP adjusted for variable factor utilization. We have assumed a zero average rate of money growth, the rest of the parameters that govern the money process, $\rho_\mu$ and $\sigma_\mu$, are estimated from quarterly UK series of M0 money growth from 1977 to 2000.

4. Dynamics and persistence

4.1. The benchmark model

The responses of a standard sticky price model without variable factor utilization are presented in Fig. 1 for $\eta = 0.25$ and in Fig. 2 for $\eta = 0.75$. An increase in aggregate demand cannot be met by increases in capital in the impact period, thus, hours increase to cover excess demand and output instantaneously increases and smoothly converges back to its original steady state.

Persistence of both output and inflation is directly related to the degree of price stickiness. In fact, the half-life of output responses, $\xi_y$, is smaller than a quarter (0.81 quarters) when $\eta = 0.25$ and it reaches 1.57 quarters when $\eta = 0.75$ (see Table 2). The same occurs for inflation persistence: the half-life of inflation responses, $\xi_\pi$, increases with $\eta$ and reaches its maximum for ($\xi_\pi = 1.65$) for $\eta = 0.75$.

As in Kimball (1995), the presence of capital adjustment costs makes the real interest rate net of depreciation fall (moderately) after the monetary injection. When the degree of price rigidity is 0.75, the real interest rate falls more persistently because in this case a monetary expansion increases the relative price of capital more putting downward pressure on real interest rates via the arbitrage relationship.

---

2 When we estimate TFP without accounting for variable factor utilization $\rho_X = 0.90$, and $\sigma_X = 0.0088$. Thus, in accordance to previous studies, variable factor utilization decreases both persistence and variability of the Solow residual (see, for example, Bils and Cho, 1994 and Burnside and Eichenbaum, 1996). Yet, it does not do so substantially in our model, since we do not incorporate any rigidity in the labor supply.
The pattern of responses of the real interest rate is crucial to understand the persistent properties of the model, since its movements reflect changes in consumption and investment which, in turn, determine demand and output. In other words, the magnitude and persistence of the demand effect on output from changes in the monetary stance is not only affected by the degree of price rigidity but from the behavior of the real interest rate as well. In fact, if a monetary expansion persistently decreases the real interest rate, the effect of the initial shock is propagated through time. Moreover, since the shape of the real interest rate responses is affected by the presence of capital adjustment costs, the latter are very important for generating persistence. This is true for all the model variants we investigate in this paper, we return to this issue later on in Section 5.

Investment, consumption and hours move one-for-one with output over the adjustment path. The size of output, consumption, hours, and investment responses to monetary shocks are all very small for low $\eta$ and the low standard deviation of output and the low relative

\[3\] Standard deviations and autocorrelation functions are calculated for both productivity and policy shocks (the parameters of the productivity and policy shock processes are given in Table 1.).
volatility of investment reflect the limited response of real variables to shocks in an environment of low nominal rigidity (see Table 3). Output and investment variabilities increase with $\eta$, but they still remain relatively low even when $\eta = 0.75$. To match the volatility of
Table 3
Investment and output volatility

<table>
<thead>
<tr>
<th>Model setting</th>
<th>( \eta = 0.25 )</th>
<th></th>
<th>( \eta = 0.50 )</th>
<th></th>
<th>( \eta = 0.75 )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( \sigma(Y) )</td>
<td>( \Sigma(I) )</td>
<td>( \sigma(Y) )</td>
<td>( \sigma(I) )</td>
<td>( \sigma(Y) )</td>
</tr>
<tr>
<td>CKM model</td>
<td>0.79</td>
<td>1.76</td>
<td>0.77</td>
<td>1.75</td>
<td>0.85</td>
</tr>
<tr>
<td>Variable capital utilization</td>
<td>1.00</td>
<td>2.76</td>
<td>1.11</td>
<td>2.85</td>
<td>1.56</td>
</tr>
<tr>
<td>Variable labor utilization</td>
<td>0.91</td>
<td>2.03</td>
<td>0.93</td>
<td>2.12</td>
<td>1.60</td>
</tr>
<tr>
<td>Variable factor model</td>
<td>1.34</td>
<td>3.74</td>
<td>1.42</td>
<td>4.15</td>
<td>2.72</td>
</tr>
</tbody>
</table>

UK data: \( \sigma(Y) = 1.41, \sigma(I) = 4.15 \).

these variables it is necessary to have a high elasticity of hours supply and a higher degree of nominal rigidity. For example, setting \( \eta = 0.83 \) and \( 1/\theta_n = 0.5 \) would do the job.

4.2. Variable capital utilization

We examine each variable factor separately so as to gain some intuition for its role in generating persistence. We start by studying what variable capital utilization does.

As it is clear from Figs. 1 and 2 the introduction of variable capital utilization magnifies the responses of output, hours and investment relative to the benchmark case, while it does not change substantially consumption, real interest rate, marginal costs and the nominal variables. For a low degree of nominal rigidities, increases in demand are met with small increases in capital utilization; however, when \( \eta \) is high the excess demand generates a large increase in utilization and hours, due to production complementarities, and this magnifies output and investment responses in the impact period. This sizeable reaction is reflected in the high standard deviations of output and investment for \( \eta = 0.75 \) (see Table 3).

Table 2 and Fig. 3 indicate that the effect of utilization on persistence is marginal. Half-lives are similar in magnitude to those of the benchmark model for low \( \eta \) and they increase a bit for \( \eta = 0.75 \). The autocorrelation functions generated by the variable capital utilization model are almost identical to those of the benchmark model for \( \eta = 0.25 \). When the degree of nominal rigidity increases, the autocorrelation coefficient of inflation displays slightly more persistence (see Fig. 3).

Persistence is not altered considerably because capital utilization does not alter substantially the relative responses of marginal costs and output and, hence, the elasticity of marginal costs to changes in output. It is still true that in the case of variable capital utilization real wages increase less and the real interest rate falls more than in the standard case. Wages increase less since capital utilization decreases relative labor demand compared with the benchmark model. When capital utilization varies, the increase in utilization increases the rate of capital depreciation and, as a result, the real interest rate net of depreciation falls more relative to the CKM model. However, the effects of capital utilization on real wages and interest rates are not strong enough to enhance persistence.

Persistence is positively related to the degree of nominal rigidity. Higher values of \( \eta \) induce larger increases in utilization and depreciation and, consequently, larger and more persistent declines in the real rate net of depreciation and stronger propagation of monetary shocks in the economy.
One lesson one can draw from this first set of exercises is that in order to generate persistent real dynamics following nominal shocks, variable capital utilization must be combined with some other friction. Capital adjustment costs alone or combined with variable capital utilization cannot dramatically enhance persistence.

4.3. Variable labor utilization

For low $\eta$, hours increase by almost the same magnitude as in the benchmark model. However, variations in effort induce larger variations in all real variables relative to the benchmark case (see Fig. 1). Hence, output and investment variability increases for all degrees of nominal rigidity (see Table 3).

Along with sizeable effects variable effort brings also considerable persistence for both real and nominal variables. In Table 2 the half-life of output for $\eta = 0.25$ increases marginally and that of inflation somewhat more. The increase in the half-life is more striking for higher degrees of nominal rigidities. For $\eta = 0.75$, half-life of output responses is 3.44 quarters and for inflation responses is 2.51 quarters, while it takes two years for the output effect of the shock to die out. This is twice the time prices are assumed to be sticky. Similar interpretation can be obtained from the plot of the autocorrelation functions.

The mechanism through which variable effort increases persistence is well understood. Variable effort increases the elasticity of the labor supply. Since a high elasticity of the labor supply flattens real marginal costs relative to changes in output and relative to the benchmark case, the model generates more persistent output and inflation responses.
The elasticities of the effort and hours supply are important determinants of the size of these effects. We investigate the importance of these parameters below.

4.4. A variable factor model

In this subsection we let both margins vary in response to aggregate shocks. The dynamics of the model in response to money shocks are depicted as a continuous line in Figs. 1 and 2.

Firms would like to increase their inputs to meet the increase in demand induced by the monetary expansion. In the previous models, the demand effect on output is dampened. In the benchmark model, and in the model with variable effort this is due to the short-run rigidity of capital and in the variable capital utilization model due to the steepness of the effective labor supply function. When factor inputs vary, firms can increase capital and labor services instantaneously and this magnifies the initial effect of the money shock on the real variables.

This larger effect is reflected in the statistics of Table 3. The standard deviation of output replicates estimates obtained for the UK economy for $\eta = 0.5$ and remains high even when $\eta$ is reduced to 0.25. The increased variability of investment relative to the standard model is the result of the depreciation-through-use assumption and the desire of firms to invest to keep the productivity of capital in line with that of labor, whereas the increased variability of output is mainly due to the higher responsiveness of effort to changes in the aggregate demand. Since both margins increase the responsiveness of investment and output to aggregate demand and since higher nominal rigidity intensifies the increase in demand, their variability increases more than proportionally with $\eta$.

Allowing for both margins to vary does not only enhance the impact effect of the shock but also enriches its propagation mechanism. Interesting insights on how the two margins affect persistence can be obtained by examining the relationship between output and real marginal costs. Write the production function in log-linear terms as:

$$y_t = \alpha(h_t + e_t) + (1 - \alpha)(k_t + u_t)$$

where $\alpha$ is the effective labor’s cost share in output. Marginal costs are determined by factor prices:

$$mc_t = \alpha w_t + (1 - \alpha)r_t$$

where $w_t$ is the real wage and $r_t$ is the rental rate of capital. The rental rate of capital is given by $r_t = w_t + y_t/\alpha - (k_t + u_t)/\alpha$; the hours supply curve is: $\theta_n h_t = w_t - \sigma c_t + e_t$, while the effort supply curve is: $\theta_e e_t = w_t - \sigma c_t + h_t$. Since labor market equilibrium implies

$$w_t = \frac{(1 + \theta_n) + \alpha \sigma \kappa}{\alpha} y_t - \frac{(1 + \theta_n)(1 - \alpha)}{\alpha} (k_t + u_t),$$

where $\kappa$ is a parameter capturing the proportionality of output with consumption during the cycle, marginal costs are:

$$mc_t = \frac{(1 + \theta_n) + (1 - \alpha) + \alpha \sigma \kappa}{\alpha} y_t - \frac{(2 + \theta_n)(1 - \alpha)}{\alpha} (k_t + u_t) - e_t$$

$$= \xi y_t - \xi \lambda (k_t + u_t) - e_t.$$
As is clear from the expression above, both capital utilization and effort tend to decrease the impact response of marginal costs. That is, they flatten the marginal cost responses and via the Phillips curve the responses of inflation. Moreover, they also decrease the elasticity of marginal cost to changes in output, \( \zeta_y \) (it goes from 6.71 in the benchmark model to 2.11 in the model with variable capital utilization, to 1.22 in the model with variable effort and down to 0.58 when both margins are allowed to vary). There are two reasons for this. First, variable capital utilization increases the effective output elasticity of employment and second, variable labor effort increases the effective labor supply elasticity.

Thus, by reducing the sensitivity of marginal costs to changes in output, the utilization of both margins increases output persistence. Moreover, by flattening the marginal costs curve, inflation dynamics also become more persistent.

Despite the increase, the autocorrelation coefficients are still quite different from those in the data. In order to generate nominal responses that match the UK data additional frictions such as the ones suggested by CEE, Dotsey and King (2001) could be needed. Given that the purpose of our exercise is to assess the role of variable factor utilization in generating persistence, we do not pursue this issue further.

It is important to stress that in the model with variable factor utilization higher degrees of nominal rigidity do not increase only persistence, but also the variability of investment and output responses to monetary shocks. While with the parameterization employed unconditional variabilities are high, there exist plausible parameter configurations (for example, \( b = 25, \sigma = 6, \phi = 2 \)) such that the model with variable margins matches the volatility and dynamics of the UK data for \( \eta = 0.75 \) without loosing its relative advantage in generating persistence.

A model with variable factors produces reasonable comovements of output, hours, effort and capacity utilization (see Table 4). To calculate the correlations in real data we use the Confederation of the British Industry’s (CBI, 2001) Industrial Trends Survey measure of capital utilization and the series of percentage utilization of labor (PUL) produced by Bennett and Smith-Gavine (1987). This last measure is very noisy, covers a short period of data (1979:Q1–1992:Q3) and does not track down well changes in effort of UK workers, but it is the only one available. The model (for \( \eta = 0.5 \)) matches well hours and utilization correlations with output, but it produces a higher correlation of output and effort.

### 5. Robustness exercises

In this section we examine the robustness of our results to alterations in some of the primitives of the economy and their sensitivity to some crucial parameters.
5.1. Lags in labor supply

In Neiss and Pappa (2002), as well as in many real business cycle models, variable capital utilization is combined with rigidities in the dynamics of employment. For example, Burnside and Eichenbaum (1996) assume a lag in employment in heads, while Bils and Cho (1994) assume costs of adjustment for the capital to labor ratio. These assumptions increase output persistence in frictionless models. However, when nominal rigidities are present, the introduction of short-run labor supply rigidities is problematic because it generates three unappealing features:

(a) a negative (or no relation) between $\eta$ and $\xi_y$,
(b) unrealistic volatilities for investment and output for high degrees of nominal rigidity, and
(c) a reduction of inflation persistence.

To illustrate this point, we introduce labor supply rigidities via habit formation on leisure, as in Wen (1998). This implies a utility function of the form

$$
E_t \sum_{j=0}^{\infty} \beta^j \left[ \frac{C_{t+j}^{1-\sigma}}{1-\sigma} - \lambda_e \frac{1}{1+\theta_e} (h_{t+j} - \psi h_{t+j-1})^{1+\theta_e} 
- \lambda_e \frac{1}{1+\theta_e} \frac{1}{1-\epsilon} \left( \frac{M_{t+j}^d}{P_{t+j}^{1-\epsilon}} \right)^{1-\epsilon} \right]
$$

(22)

where $\psi$ determines the degree of habit persistence in hours worked. Wen (1998) estimates $\psi$ using quarterly data to be equal to 0.625, while Eichenbaum et al. (1988) estimate $\psi$ for monthly data in the interval (0.68, 0.83). We set $\psi = 0.65$.

When habit formation on working hours is added, persistence is substantially increased (see Table 2). Habit formation increases persistence both for the benchmark and the factor utilization model for low degrees of nominal rigidity. For example, in the benchmark model the half-life of output is almost seven times larger than in the standard model and in the variable factor utilization around three and a half times larger when $\eta = 0.25$.

Nonetheless, the persistence of hours responses decreases inflation persistence and in the benchmark model persistence and the degree of price stickiness are negatively related.

It is easy to understand why habit formation in hours increases output persistence. Temporal complementarities of leisure choices induce persistent movements in hours. Since output dynamics are directly related to hours, output also reacts sluggishly in response to

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4 Such an assumption is not difficult to rationalize. Though it is plausible for individuals to substitute current for future leisure so as to optimize their marginal productivity of work, it is very costly to change their habit of sleeping for extra hours of work. Surveys suggest that individuals prefer to increase the amount of effort at work contemporaneously rather than the time spent at work. For example, they prefer to get job done during the weekdays at the cost of extra effort than sacrificing their free time during the weekend in order to maintain a constant effort level during the week.

5 Wen considers longer lags on habit formation. 0.625 represents the resulting sum of the lag coefficients.
aggregate shocks. On the other hand, the sluggishness of hours coupled with rigidities in prices result in puzzling inflation dynamics.

First, notice that the assumption of rigid hours does not alter the behavior of marginal costs and of inflation relative to the benchmark case, since wages and real interest rate can adjust freely to changes in demand. Next, notice that, as a general rule, changes in the money supply induce increases in demand. However, rigidities in the hours supply imply that output and, hence, consumption cannot be augmented on impact. Consumers would be willing to postpone their consumption if only and only if they will receive a higher interest rate. As a result, the interest rate increases so as to bring the economy to equilibrium. However, an increase in the real rate reduces inflation persistence relative to the benchmark case. Furthermore, the higher is the degree of nominal rigidity, the higher is the increase in the interest rate needed for making consumers postpone their consumption and this reduces output persistence for higher degrees of nominal rigidity.

In the model with variable effort this effect is absent since firms can move effort instantaneously to satisfy the increased demand. In contrast, in the model with variable capital utilization most of the persistence in output disappears due to the immediate reaction of capital utilization and, thus, of marginal costs and inflation in the impact period of the shock. Consequently, when both factors vary, the combined movements of effort and utilization result in intermediate values for the half-life of output and inflation.

We conjecture that features, such as habit formation in consumption and/or sticky wages can lessen the puzzles generated by the model with rigid hours supply.

5.2. Endogenous monetary policy

The preceding discussion has focused on equilibria where monetary policy decisions are exogenous. Several papers (e.g. Gordon and Leeper, 1994) have argued that money supply movements are largely due to the endogenous reaction to the state of the economy. In this section, we analyze whether the persistence properties of our model change when the money supply rule (16) is replaced by a rule à la Taylor (1993), where the interest rate responds to changes in the state of the economy. We assume that such a rule takes the form

\[ R_t = \rho_R R_{t-1} + (1 - \rho_R) b_\pi \pi_t + (1 - \rho_R) b_y y_t + \mu_t \]  

(23)

where, for the sake of comparisons with the existing literature, we set \( \rho_R = 0.5 \), \( (1 - \rho_R) b_\pi = 1.5 \), and \( (1 - \rho_R) b_y = 0.5 \). As before, \( \mu_t \) represents a policy shock.

Endogenizing policy decreases the persistence properties of all models since the endogenous reactions of the interest rate to aggregate demand changes affect the dynamics of output and inflation and hence their persistence. Yet, the variable factor model still generates relatively more persistent responses (see the last two rows of Table 2).

5.3. Sensitivity analysis

In this section we analyze the sensitivity of the results to alterations of some of the parameters which may potentially affect persistence.
5.3.1. Capital adjustment costs

So far, we have set \( b \) to match the ratio of investment to output standard deviations. In this section we perform a sensitivity analysis over a plausible range for this parameter to investigate how the half-life of output and inflation responses changes with changes in \( b \), for the models we considered in Section 4. We report results in Fig. 4 for \( \eta = 0.75 \). Many interesting insights emerge from the figure.

First, the model where both factors are allowed to vary outperforms all the other model variants for all values of \( b \). Thus, capital adjustment costs do not give an edge to our favorite propagation mechanism. Second, the relation between persistence and capital adjustment costs is concave. In other words, persistence increases with \( b \), but with a decreasing rate. Therefore there is a limit to the degree that one can enhance the propagation mechanism by varying \( b \). Third, for low values of \( b \), persistence for the models with variable effort increases faster. This is because the effort margin affects both utility and production. In all models the transition from zero to positive adjustment costs puts downward pressures on real rates. In the models with variable effort this pressure is stronger since movements in effort require larger compensated increases in consumption when capital is costly to adjust. Finally, for zero capital adjustment costs all models are equally unable to generate persistence, since the positive effect of aggregate demand on the real interest rate renders the propagation mechanism of monetary shocks ineffectual (see also Casares and McCallum, 2000).

5.3.2. Hours and effort supply elasticities

Persistence in our favorite model is mainly generated by the introduction of variable effort. Effort increases the elasticity of effective labor to changes in demand and this flattens the marginal costs curve generating persistence in both inflation and output. In Fig. 5 we investigate how persistence changes when we let both labor and effort supply elasticity to vary. Both hours and effort supply elasticity are positively related with persistence. For given values of the hours supply elasticity, increases in the effort supply elasticity increase both \( \xi_y \) and \( \xi_\pi \) and thus persistence in our preferred model. Effort and hours supply elasticity have to assume very small values for the persistent effects of the variable utilization model on output and inflation to disappear.

5.3.3. Elasticity of depreciation

Finally, we investigate whether our results are specific to the depreciation function we assumed. There are two prevailing assumptions for the depreciation function \( \delta(U) \) used in the literature. Following Burnside and Eichenbaum (1996), we assume only ‘wear and tear’ costs, while Basu and Kimball (1997) additionally assume ‘rust and dust’ costs in depreciation. According to the specification of Basu and Kimball (1997): \( \delta(U) = \delta_0 + \delta_1 U^\phi \), where \( \delta_0 \) and \( \delta_1 \) are constants and \( \phi = 2 \). They suggest that this specification would induce capital to be less variable, since the higher is \( \phi \), the larger are the negative effects of utilization on depreciation and, as a result, firms would opt to smooth utilization as much as possible. This in turn may weaken the propagation mechanism of monetary shocks since smoother changes in utilization imply less elastic responses of output to changes in demand. In Fig. 6 we plot how the coefficients \( \xi_y \) and \( \xi_\pi \) change with changes in \( \phi \). We consider values for \( \phi \) in the interval \([0.45, 5.0]\).
Fig. 4. Capital adjustment costs.
Unsurprisingly, the half-lives for both output and inflation decrease with $\phi$. However, for plausible values of the elasticity of depreciation to changes in utilization, the value of $\xi_y$ and $\xi_\pi$ do not change dramatically and remain well above the corresponding values of the benchmark model. Therefore, if a negative effect of higher $\phi$ on persistence exists, it appears to be quantitatively minor.

Also notice that for very low values of $\phi$ persistence increases substantially. This is so because low $\phi$ implies that capital is costless to utilize and, as a consequence, firms in equilibrium utilize capital more intensively. The more intensive use of capital instead results in the complete flattening of marginal costs (the elasticity of marginal costs to changes in output equals 0.06 for $\phi = 0.45$). This occurs for two reasons: the more intensive use of
capital reduces (a) the relative need for labor in production and, therefore, wages and (b) the interest rate net of depreciation (higher utilization increases depreciation even if $\phi$ is low).

Thus, capital utilization becomes very important for persistence when the elasticity of depreciation is low. For example, for $\eta = 0.5$ and $\phi = 0.5$ in Fig. 6, $\xi_y$ and $\xi_\pi$ equal approximately three quarters and, contrary to our findings so far, capital utilization becomes more important for generating persistence relative to effort.

6. Conclusion

CKM have forcefully shown that standard sticky price models of the business cycles cannot generate persistent real responses of monetary shocks. This result has been the starting point for a new literature aimed at constructing models that increase the propagation mechanism of monetary shocks. In this paper we contribute to this literature by studying the role of variable factor utilization in shaping output and inflation persistence.

Variable factor utilization is able to generate relatively more persistence because it flattens the marginal costs curve. The mechanism we employ makes labor more elastic, through movements in variable effort, and increases the effective output elasticity of employment, through changes in capital utilization. We find that variable labor effort is more important for generating persistence than variable capital utilization. However, variable capital utilization can also be very important for persistence when capital utilization does not increase substantially the rate of capital depreciation. We also find that a model with
variable factor utilization allows us to match volatilities and comovements of output, hours, effort and capital utilization in the UK economy using moderate degrees of nominal rigidity.

However, variable factor utilization alone is not sufficient to quantitatively replicate the persistence of output and inflation responses to monetary shocks found in the UK data. In order to provide a mechanism that accounts for all the empirical regularities of the transmission mechanism of monetary shocks one has to include additional frictions. Our exercise indicates that labor supply rigidities are unlikely to produce the required changes without altering other properties of the model.

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Appendix A. Representative household’s first-order conditions

Here we present the FOC for the general case with habit formation on leisure. The FOC for the case of no habit formation can be achieved by setting $\psi = 0$. Allowing $\lambda_t$ to denote the Lagrange multiplier on the household’s budget constraint, the first-order conditions for the representative agent are given by:

Consumption:
\[
C_t^{\sigma} = P_t \lambda_t. \tag{A.1}
\]

Money demand:
\[
\left( \frac{M_t^d}{P_t} \right)^{-\varepsilon} = P_t \lambda_t - \frac{E_t \beta P_{t+1} \lambda_{t+1} P_t}{P_{t+1}}. \tag{A.2}
\]

Effort:
\[
\lambda_t \theta_t e_t = \lambda_t P_t w_t h_t. \tag{A.3}
\]

Hours:
\[
\lambda_t (h_t - \psi h_{t-1}) \theta_e - \beta \psi \lambda_t (h_{t+1} - \psi h_t) \theta_e = \lambda_t P_t w_t e_t. \tag{A.4}
\]

Utilization:
\[
r_t = \delta \phi U_t^{\phi-1}. \tag{A.5}
\]

Capital:
\[
P_t \lambda_t \left( 1 + b \left[ \frac{K_{t+1}}{K_t} - 1 \right] \right) = E_t \beta P_{t+1} \lambda_{t+1} \left[ r_{t+1} U_{t+1} + (1 - \delta U_t^{\phi}) + \frac{b}{2} \left( \frac{K_{t+1}}{K_t} \right)^2 - 1 \right]. \tag{A.6}
\]
Bonds:
\[ 0 = -P_t \lambda_t + E_t \beta P_{t+1} \lambda_{t+1} R_t P_t / P_{t+1}. \] (A.7)

Appendix B. Log-linearized conditions

The system of equations that solve the model is given by:

Money demand:
\[ \hat{m}_t = \frac{\sigma}{\varepsilon} \hat{c}_t - \frac{1}{R^3 \varepsilon} \hat{R}_t. \] (B.1)

Hours supply:
\[ \frac{1}{(1 - \psi)(1 - \beta \psi)} \left[ (1 + \beta \psi^2) \theta_n \hat{h}_t - \theta_n \psi \hat{h}_{t-1} - \beta \psi \theta_n \hat{h}_{t+1} \right] = -\sigma \hat{c}_t + \hat{w}_t + \hat{e}_t. \] (B.2)

Effort supply:
\[ (1 + \theta_n) \hat{h}_t = (1 + \theta_n) \hat{c}_t. \] (B.3)

Utilization:
\[ (\phi - 1) \hat{u}_t = \hat{r}_t. \] (B.4)

Capital motion:
\[ \hat{k}_{t+1} = \delta \hat{h}_t + (1 - \delta) \hat{k}_t - \delta \phi \hat{u}_t. \] (B.5)

Capital FOC:
\[ \frac{1}{\beta (\sigma \hat{c}_t - b \delta \hat{k}_t + b \delta \hat{u}_t) \phi} \left\{ \frac{1}{\sigma} \hat{c}_t + \frac{1}{\sigma} \hat{h}_t - \frac{1}{\sigma} \hat{e}_t \right\} = E_t \left\{ \frac{1}{\sigma} \hat{c}_t + \frac{1}{\sigma} \hat{h}_t - \frac{1}{\sigma} \hat{e}_t \right\}. \] (B.6)

Production:
\[ \hat{y}_t = (1 - \alpha) (\hat{k}_t + \hat{u}_t) + \alpha (\hat{h}_t + \hat{c}_t + \hat{x}_t). \] (B.7)

Euler equation:
\[ \hat{c}_t = E_t \hat{c}_{t+1} - \frac{1}{\sigma} (\hat{R}_t - E_t \pi_{t+1}). \] (B.8)

Real wages:
\[ \hat{w}_t = \hat{y}_t + \hat{m} \hat{c}_t - \hat{h}_t - \hat{e}_t. \] (B.9)

Real rate:
\[ \hat{r}_t = \hat{y}_t + \hat{m} \hat{c}_t - \hat{k}_t - \hat{u}_t. \] (B.10)

Calvo pricing:
\[ \pi_t = \beta E_t \pi_{t+1} + \frac{(1 - \eta)(1 - \beta \eta)}{\eta} \hat{m} \hat{c}_t. \] (B.11)
Resource constraint:
\[ \hat{y}_t = \frac{c}{\hat{y}_t} + \frac{i}{\hat{y}_t}, \]  
(B.12)

Money supply:
\[ \hat{m}_t = \hat{m}_{t-1} - \pi_t + \hat{\mu}_t. \]  
(B.13)

Real rate net of depreciation:
\[ z_t = \hat{R}_t - E_t \pi_{t+1}. \]  
(B.14)

Hated variables denote log-deviations from their steady state values. The 14 equations describe the path of 14 endogenous variables: output (\( \hat{y}_t \)), utilization (\( \hat{u}_t \)), capital (\( \hat{k}_t \)), effort (\( \hat{e}_t \)), hours (\( \hat{h}_t \)), consumption (\( \hat{c}_t \)), investment (\( \hat{i}_t \)), nominal money balances (\( \hat{m}_t \)), the nominal interest rate (\( R_t \)), the real interest rate (\( \hat{r}_t \)) and real interest rate net of depreciation (\( z_t \)), the real wage (\( \hat{w}_t \)), real marginal cost (\( \hat{m}_c_t \)), and inflation (\( \pi_t \)).

References


