Gains from International Monetary Policy Coordination:

Does It Pay to Be Different?∗

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Abstract

In a two country world where each country has a traded and a non-traded sector and each sector has sticky prices, optimal independent policy in general cannot replicate the natural-rate allocations. There are potential welfare gains from coordination since the planner under a cooperating regime internalizes a terms-of-trade externality that independent policymakers overlook. If the countries have symmetric trading structures, however, the gains from coordination are quantitatively small. With asymmetric trading structures, the gains can be sizable since, in addition to internalizing the terms-of-trade externality, the planner optimally engineers a terms-of-trade bias that favors the country with a larger traded sector.

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1 Introduction

As countries become more interdependent through international trade, should they conduct monetary policies independently or should they coordinate their policies? In other words, are there gains from international monetary policy coordination? This question lies at the heart of the intellectual discussions about optimal monetary policy in open economies.

The literature has produced a strong conclusion in favor of inward-looking policies and flexible exchange-rate regimes. This conclusion has been drawn not only in the traditional literature within the Mundell-Fleming framework that features ad hoc stabilizing policy goals, but also in the more recent New Open-Economy Macro (NOEM) literature that features optimizing individuals, monopolistic competition and nominal rigidities, with the representative household's utility function serving as a natural welfare metric for optimal policy. In the traditional literature, many have argued that the gains from coordination are likely to be small because a flexible exchange-rate system would effectively insulate impacts of foreign disturbances on domestic employment and output (e.g., Mundell, 1961; McKibbin, 1997). In the NOEM literature pioneered by Obstfeld and Rogoff (1995), it has been shown that, although gains from coordination are theoretically possible, they are quantitatively small (e.g., Obstfeld and Rogoff, 2002; Corsetti and Pesenti, 2001).

The remarkably strong conclusion about the lack of gains from coordination has stimulated a lively debate and a growing strand of literature in search of sources of coordination gains by enriching the simple framework built by Obstfeld and Rogoff (2002). Several potential sources have been identified. For instance, the gains from coordination can be related to the degree of exchange-rate pass-through (e.g., Devereux and Engel, 2003; Duarte, 2003; Corsetti and Pesenti, 2005). Even with perfect exchange-rate pass-through, inward-looking monetary policy can be suboptimal and be improved upon by coordination, depending on the values of the intertemporal elasticity and the elasticity of substitution between goods produced in different countries (e.g., Clarida, et al., 2002; Benigno and Benigno, 2003; Pappa, 2004). Policy coordination may also produce welfare gains if the international financial markets are incomplete (e.g., Benigno, 2001; Sutherland, 2002), policy makers have imperfect information (e.g., Della, 2006), or domestic shocks are imperfectly correlated across sectors (e.g., Canzoneri, et al., 2005).

Most of the studies on international policy coordination focus on countries with similar characteristics. The theoretical framework used in these studies typically features two countries that are identical except that they might be buffeted by different shocks. Such a framework is not suitable, and indeed, is not designed to address issues on policy coordination between countries at different

\footnote{Corsetti and Dedola (2005) show that, if the distribution of traded goods requires local inputs, then international markets would be endogenously segmented, rendering exchange-rate pass-through incomplete. This feature also provides a scope for international monetary policy cooperation.}
stages of development or countries with different institutional structures that render their production and trading structures asymmetric. Recent events such as the rise of China as an increasingly important player in the world economy and the accession of some Eastern European countries to the European Monetary Union render it important to understand the implications of international policy coordination between dissimilar countries.

The present paper takes a first step in this direction by emphasizing the role of cross-country differences in trading structures in generating gains from policy coordination. For this purpose, we build a two-country model in the spirit of the NOEM literature, with two production sectors within each country. One sector produces traded goods that enter the consumption baskets in both countries, and the other sector produces non-traded goods that enter the domestic consumption basket only. To allow for real effects of monetary policy, we assume staggered price setting in both sectors. A key point of departure from the NOEM literature is that we allow the share of traded goods in the consumption basket to be different across countries.\(^2\) One possible interpretation of the asymmetry here is that it may reflect differences in tastes. For instance, the Americans seem to enjoy spending a large share of their income on housing services or paying their local dealers at various stages of distribution, while the Chinese seem to love McDonald’s, Starbucks, and Dell computers. Another possible interpretation of the asymmetry assumed in our model is that it may capture cross-country differences in production structures as a consequence of different technology progress. For instance, if technology progress is faster in the manufacturing sector than in the service sector and goods and services are poor substitutes, then the long-run share of the service sector will rise and the manufacturing sector will decline (e.g., Ngai and Pissarides, 2007). Different patterns of technology progress can thus create differences in production structures across countries. A third possibility is that, in the presence of transportation cost or other trade barriers, some goods are traded but others are not (e.g., Eaton and Kortum, 2002; Melitz, 2003). Cross-country differences in trade policy or transportation technologies may thus create differences in the relative size of the traded sector. Finally, if the countries involved have different sizes of the government sector, then, to the extent that public services are not traded, the countries should also have different shares of expenditure on non-traded goods, as we assume in our model. There are many other possible interpretations of the structural asymmetry in our model. However, understanding the sources of such asymmetry is beyond the scope of our current paper. We focus on examining how the presence

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\(^2\)Since a large component of services is non-traded, it is informative to examine the cross-country differences in value-added share of services in order to get a sense of the empirical relevance of the structural asymmetry assumed in our model. The data suggest substantial international differences in the share of services, particularly across countries at different stages of development. According to the OECD STAN Indicators database, the value-added share of services in developed countries lies between 65% and 75% during the 1990s, while the share of services in less developed countries ranges between 50% and 55% during the same period. See Liu and Pappa (2005) for similar evidence based on data from the World Bank.
of multiple sectors and sectoral asymmetries across countries affects macroeconomic stability and welfare under independent and cooperating policy regimes.

To isolate the role of asymmetric trading structures, we make two assumptions that simplify our analysis. First, we assume log-utility in aggregate consumption, a unitary elasticity of substitution between domestically-produced traded goods and imported goods, and a unitary elasticity of substitution between traded and non-traded goods in the consumption baskets. Second, we assume the existence of a complete international financial market so that country-specific risks are perfectly insured. Although these assumptions typically go against the need for international policy coordination (e.g., Clarida et al., 2002; Pappa, 2004; Benigno, 2001; and Sutherland, 2002), we still obtain sizable welfare gains from coordination in our model with structural asymmetry.

The gains from coordination arise through the following channels. First, optimal independent policy cannot replicate the natural-rate allocations when facing multiple sources of domestic nominal rigidities, creating a scope for coordination. Second, under a cooperating regime, the social planner internalizes a terms-of-trade externality that independent policymakers overlook. Third, with symmetric trading structures, welfare gains from coordination are quantitatively small; but with asymmetric structures, the gains can be sizable because, in addition to internalizing the terms-of-trade externality, the planner engineers a terms-of-trade bias in favor of the country with a larger traded sector. This bias originates from differences in the countries' initial wealth levels. In the optimal-policy problem, the planner needs to balance the terms-of-trade bias against the desire to stabilize fluctuations in the terms-of-trade gap, among other variables in the policy objective derived from the first principle. We further show that the welfare gain from policy coordination increases with the share of imported goods in the traded consumption baskets and with the durations of pricing contracts; but decreases with the correlations of domestic shocks.

Our work contributes to the literature in two aspects. First, by introducing asymmetric structures in an otherwise standard two-sector NOEM model, we have identified the terms-of-trade bias as a new channel of welfare gains from cooperation. The model enables us to go beyond the special results obtained in the literature (e.g., Obstfeld and Rogoff, 2002) and to find sizable welfare gains from international policy coordination. Second, we make a methodological contribution to the literature by deriving an explicit expression for the welfare objective both for the independent policy regime and for the cooperating regime. To our knowledge, we are the first to derive such a welfare criterion in an open economy with multiple sectors based on quadratic approximations of households’ utility functions.\(^3\)

\(^3\)For a comprehensive discussion of the general approach to deriving the welfare criterion for optimal policy based on quadratic approximations, see Woodford (2003, Chapter 6).
The Model

Consider a world economy with two countries, home and foreign, each populated by a continuum of identical, infinitely-lived households. The representative household in each country is endowed with one unit of time, and derives utility from consuming a basket of final goods. The consumption basket consists of traded goods, either domestically produced or imported (e.g., manufacturing goods), and of non-traded goods (e.g., services). Final consumption goods are composites of differentiated intermediate goods produced in two sectors, a traded good sector, and a non-traded good sector. Production of intermediate goods requires domestic labor as the only input, which is supplied by domestic households. Labor is mobile across sectors, but not across countries. The production structures in the two countries are symmetric but the trading structures may differ in that the share of traded goods in the final consumption basket may be different across countries.

Time is discrete. In each period of time \( t = 0, 1, \ldots \), a productivity shock is realized in each intermediate-good sector. Firms and households make their optimizing decisions after observing the shocks. All agents have access to an international financial market, where they can trade a state-contingent nominal bond. The government in each country conducts monetary policy and uses lump-sum transfers to finance production subsidies.

2.1 Representative Households

The preferences of households regarding final consumption goods are symmetric across countries, so we focus on the representative household in the home country. The utility function is given by

\[
E \sum_{t=0}^{\infty} \beta^t [\ln C_t - \Psi L_t],
\]

where \( 0 < \beta < 1 \) is a subjective discount factor, \( C_t > 0 \) denotes final consumption, \( L_t \in (0, 1) \) denotes hours worked, and \( E \) is an expectation operator.

The purchase of consumption goods is financed by labor income, profit income, and a lump-sum transfer from the government. In addition, the household has access to an international financial market, where state-contingent nominal bonds (denominated in home currency) can be traded. The period-budget constraint facing the household is given by

\[
P_t C_t + E_t D_{t,t+1} B_{t+1} \leq W_t L_t + B_t + \Pi_t + T_t, \quad t = 0, 1, \ldots,
\]

where \( P_t \) is the price level, \( B_{t+1} \) is the holdings of the state-contingent nominal bond that pays one unit of home currency in period \( t + 1 \) if a specified state is realized, \( D_{t,t+1} \) is the period-\( t \) price of such bonds, \( W_t \) is the nominal wage rate, \( \Pi_t \) is the profit income, and \( T_t \) is the lump-sum transfer from the government.
The household maximizes (2.1) subject to (2.2). The optimal labor supply decision implies
\[ \Psi C_t = W_t / P_t, \]  
which states that the marginal rate of substitution between leisure and consumption equals the real consumption wage. The optimal consumption-saving decision is described by
\[ D_{t,t+1} = \beta C_t / C_{t+1} P_t / P_{t+1}, \]  
so that the intertemporal marginal rate of substitution equals the price of the state contingent bond. Define the nominal interest rate on a risk-free bond as \( R_t = [E_t D_{t,t+1}]^{-1} \). Then (2.4) implies that
\[ 1 / C_t = \beta E_t \left[ 1 / C_{t+1} P_t / P_{t+1} R_t \right], \]  
which is the familiar intertemporal Euler equation.

The final consumption basket consists of traded goods (domestically produced and imported) and non-traded goods. Denote \( C_{Nt} \) the composite good that is non-traded, and \( C_{Tt} \) the composite of goods that are traded. Then we have
\[ C_t = \bar{\alpha} C_{Tt}^{\omega} C_{Nt}^{1-\omega}, \quad \bar{\alpha} = \alpha^{-\omega}(1 - \alpha)^{\omega-1}. \]  
The traded component \( C_{Tt} \) is itself an aggregate of domestically produced good \( C_{Ht} \) and imported good \( C_{Ft} \), that is,
\[ C_{Tt} = \bar{\omega} C_{Ht}^{\omega} C_{Ft}^{1-\omega}, \quad \bar{\omega} = \omega^{-\omega}(1 - \omega)^{\omega-1}. \]  
Solving the household’s expenditure-minimizing problem yields the following demand functions for non-traded and traded goods:
\[ C_{Nt} = (1 - \alpha) P_t C_t / \bar{P}_{Nt}, \quad C_{Tt} = \alpha P_t C_t / \bar{P}_{Tt}, \]  
where \( \bar{P}_{Nt} \) is the price of final non-traded goods, and \( \bar{P}_{Tt} \) is the price of final traded goods, which are related to the price level \( P_t \) by
\[ P_t = \bar{P}_{Tt}^{\omega} \bar{P}_{Nt}^{1-\omega}. \]  
The induced demand functions for domestically produced traded goods and for imported goods are respectively given by
\[ C_{Ht} = \omega \bar{P}_{Tt} C_{Tt} / \bar{P}_{Ht}, \quad C_{Ft} = (1 - \omega) \bar{P}_{Tt} C_{Tt} / [E_t \bar{P}_{Ft}], \]  
where \( \bar{P}_{Ht} \) is the price index of home-produced traded goods, \( \bar{P}_{Ft}^* \) is the price index of foreign-produced traded goods, and \( E_t \) is the nominal exchange rate. These prices are related to \( \bar{P}_{Tt} \) by
\[ \bar{P}_{Tt} = \bar{P}_{Ht}^* [E_t \bar{P}_{Ft}^*]^{1-\omega}. \]
Throughout our analysis, we assume that firms set prices in the sellers’ local currency and the law-of-one-price holds, so that the cost of imported goods in the home consumption basket is simply the price of traded goods charged by foreign exporting firms, adjusted by the nominal exchange rate, as in (2.11).

2.2 Production Technologies and Optimal Pricing Rules

There are two sectors producing intermediate goods: a non-traded sector and a traded sector. In each sector, there is a continuum of firms producing differentiated products indexed in the interval $[0, 1]$. To produce intermediate goods in each sector requires labor input, with constant-returns-to-scale (CRS) technologies

$$Y_{Nt}(i) = A_{Nt} L_{Nt}(i), \quad i \in [0, 1],$$

and

$$Y_{Ht}(j) + Y^*_H(j) = A_{Tt} L_{Tt}(j), \quad j \in [0, 1],$$

where $Y_{Nt}(i)$ is the output of type-$i$ non-traded intermediate goods; $Y_{Ht}(j)$ is the output of type-$j$ traded intermediate goods sold in the domestic market, and $Y^*_H(j)$ that exported to the foreign market; $A_{Nt}$ and $A_{Tt}$ are productivity shocks in the two sectors; and $L_N$ and $L_T$ are labor inputs in the non-traded and in the traded sector respectively. The logarithms of the productivity shocks in each sector follows a random-walk process, that is,

$$\ln(A_{k,t+1}) = \ln(A_{k,t}) + \varepsilon_{k,t+1}, \quad k \in \{N, T\},$$

where $\varepsilon_{Nt}$ and $\varepsilon_{Tt}$ are mean-zero, iid normal processes with finite variances given by $\sigma^2_N$ and $\sigma^2_T$, respectively. We allow the shocks to be correlated across sectors, with a correlation coefficient given by $\rho_{TN}$ (they need not be perfectly correlated). The productivity shocks in the foreign country follow similar processes, and are potentially correlated with the shocks in the home country, with correlation coefficients denoted by $\rho_{TT}$ for traded sectors and $\rho_{NN}$ for non-traded sectors.

There is a CES aggregation technology that transforms intermediate goods produced in each sector into final consumption goods according to

$$C_{Nt} = \left[ \int_0^1 Y_{Nt}(i)^{\theta_N-1} \frac{d}{di} \right]^{\theta_N-1}_{\theta_N}, \quad C_{Ht} = \left[ \int_0^1 Y_{Ht}(j)^{\theta_T-1} \frac{d}{dj} \right]^{\theta_T-1}_{\theta_T},$$

where $\theta_N$ and $\theta_T$ denote elasticities of substitution between differentiated products in the two sectors. To ensure equilibrium existence, we assume that the $\theta$’s both exceed unity (e.g., Blanchard and Kiyotaki, 1987).

By solving the cost-minimizing problem of the aggregation sector, we obtain the demand functions for each type of intermediate goods:

$$Y^d_{Nt}(i) = \left[ \frac{P_{Nt}(i)}{P_{Nt}} \right]^{-\theta_N} C_{Nt}, \quad Y^d_{Ht}(j) = \left[ \frac{P_{Ht}(j)}{P_{Ht}} \right]^{-\theta_T} C_{Ht},$$

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where $P_{Nt}(i)$ is the price of type-$i$ non-traded intermediate goods, $P_{Ht}(j)$ is the price of type-$j$ traded intermediate goods, and $\bar{P}_{Nt} = \left[ \int_0^1 P_{Nt}(i)^{1-\theta_N} dj \right]^{1/1-\theta_N}$ and $\bar{P}_{Ht} = \left[ \int_0^1 P_{Ht}(j)^{1-\theta_T} dj \right]^{1/1-\theta_T}$ are the corresponding price indices.

Firms are price takers in the input market and monopolistic competitors in the product markets. In each sector, firms stagger their pricing decisions in the spirit of Calvo (1983). Specifically, in each period of time, each firm receives an i.i.d. random signal that determines whether or not it can set a new price. The probability that a firm can adjust its price is $1 - \gamma_k$ in sector $k \in \{N, T\}$. By the law of large numbers, a fraction $1 - \gamma_k$ of all firms in sector $k$ can adjust prices, while the rest of the firms cannot.

If a firm who produces type-$i$ non-traded goods can set a new price, it chooses $P_{Nt}(i)$ to maximize its expected present value of profits

$$E_t \sum_{\tau = t}^{\infty} \gamma_N^{\tau - t} D_{t,\tau} [P_{Nt}(i)(1 + \tau_N) - V_{Nt}(i)] Y_{N_t}^d(i),$$

(2.17)

where $\gamma_N$ is a production subsidy, $V_{Nt}$ is the unit cost, which is identical across firms since all firms face the same input market, and $Y_{N_t}^d(i)$ is the demand schedule for type $i$ non-traded good described in (2.16). Regardless of whether a firm can adjust its price, it has to solve a cost-minimizing problem, the solution of which yields the unit cost function

$$V_{Nt} = W_t / A_{Nt},$$

(2.18)

and a conditional factor demand function

$$L_{Nt} = \frac{1}{A_{Nt}} \int_0^1 Y_{N_t}^d(i) di.$$  

(2.19)

The solution to the profit-maximizing problem gives the optimal pricing rule

$$P_{Nt}(i) = \frac{\mu_N}{(1 + \tau_N)} \frac{E_t \sum_{\tau = t}^{\infty} \gamma_N^{\tau - t} D_{t,\tau} V_{Nt} Y_{N_t}^d(i)}{E_t \sum_{\tau = t}^{\infty} \gamma_N^{\tau - t} D_{t,\tau} Y_{N_t}^d(i)},$$

(2.20)

where $\mu_N = \theta_N / (\theta_N - 1)$ measures the steady-state markup in sector $N$. Similarly, the optimal pricing rule for a firm that produces type-$j$ traded good is given by

$$P_{Ht}(j) = \frac{\mu_T}{(1 + \tau_T)} \frac{E_t \sum_{\tau = t}^{\infty} \gamma_T^{\tau - t} D_{t,\tau} V_{Ht} [Y_{H_t}^d(j) + Y_{H_t}^d(j)]}{E_t \sum_{\tau = t}^{\infty} \gamma_T^{\tau - t} D_{t,\tau} [Y_{H_t}^d(j) + Y_{H_t}^d(j)]},$$

(2.21)

where $\mu_T = \theta_T / (\theta_T - 1)$ measures the steady-state markup in sector $T$. From solving the firm’s cost-minimizing problem, we obtain the unit cost function

$$V_{Tt} = W_t / A_{Tt},$$

(2.22)

and a conditional factor demand function

$$L_{Tt} = \frac{1}{A_{Tt}} \int_0^1 [Y_{H_t}^d(j) + Y_{H_t}^d(j)] dj.$$  

(2.23)
An important point of departure of our analysis from the NOEM literature is that we introduce asymmetric trading structures across countries. Specifically, we assume that the share of traded good in the foreign consumption basket may differ from that in the home country. In particular, the foreign consumption basket is given by

\[ C_t^* = \bar{\alpha}^* C_{\tilde{T}_t}^* C_{N_t}^{\alpha^* - 1}, \quad \bar{\alpha}^* = \alpha^* - \alpha^* (1 - \alpha^*)^{\alpha^* - 1}, \]  

(2.24)

where \( \alpha^* \) may not equal \( \alpha \). As we have alluded to in the introduction, such asymmetry in trading structures captures some important cross-country differences and it turns out to be crucial for generating gains from international policy coordination.

The foreign country’s structure is otherwise similar to that of the home country’s. In what follows, we denote all foreign variables with an asterisk and assume that all other parameters are identical to their counterparts in the home country.

2.3 Risk Sharing, Market Clearing, and Equilibrium

Since the state-contingent nominal bond is traded in the international financial market, the foreign household’s optimal consumption-saving decision leads to

\[ D_{t,t+1} = \beta C_t^* P_t^* \frac{P_{t+1}^*}{\bar{\alpha}^* C_{\tilde{T}_{t+1}}^*}. \]  

(2.25)

By combining this equation with its home counterpart (2.4) and iterating with respect to \( t \), we obtain a risk-sharing condition

\[ Q_t = \phi_0 C_t^*, \]  

(2.26)

where \( Q_t = E_t P_t^* / P_t \) is the real exchange rate, and \( \phi_0 = Q_0 C_0^* / C_0 \). The risk-sharing condition links the real exchange rate to the marginal rate of substitution between consumption in the two countries, so that all households face identical relative price of consumption goods in the world market.

Since labor is mobile within each country (but not across countries), labor market clearing implies that

\[ L_{N_t} + L_{\tilde{T}_t} = L_t, \quad L_{N_t}^* + L_{\tilde{T}_t}^* = L_t^*. \]  

(2.27)

The nominal bonds are traded in the international asset market, and bond market clearing requires that \( B_t + B_t^* = 0 \).

Our goal is to analyze optimal monetary policy under two alternative policy regimes. One in which each national authority tries to maximize its own households’ welfare, taking the other country’s policy actions as given; and the other in which a world planner tries to coordinate the two countries’ policies so as to maximize their collective welfare. For this purpose, we do not specify a particular monetary policy rule. Instead, we solve for the optimal policy that maximizes the
welfare objective under each regime, subject to the private sector’s optimizing conditions. For any
given monetary policy, we can define an equilibrium for this world economy.

An equilibrium consists of allocations \( C_t, C_{Nt}, C_{Tt}, L_t, B_{t+1} \) for the home household and \( C^*_t, C^*_{Nt}, L^*_t, B^*_{t+1} \) for the foreign household; allocations \( Y_{Nt}(i) \), and \( L_{Nt}(i) \), and price \( P_{Nt}(i) \) for non-traded intermediate good producer \( i \in [0, 1] \) in the home country and \( Y^*_{Nt}(i) \), and \( L^*_{Nt}(i) \), and price \( P^*_{Nt}(i) \) for non-traded intermediate good producer \( i \in [0, 1] \) in the foreign country; allocations \( Y_{Ht}(j), Y^*_{Ht}(j) \), and \( L_{Tt}(j) \), and price \( P_{Ht}(j) \) for traded intermediate good producer \( j \in [0, 1] \) in the home country and \( Y^*_{Ft}(j), Y_{Ft}(j) \), and \( L^*_{Tt}(j) \), and price \( P^*_{Ft}(j) \) for traded intermediate good producer \( j \in [0, 1] \) in the foreign country; together with prices \( D_{t,t+1}, E_t, Q_t, P_t, P^*_{Ht}, P^*_{Ht}, P^*_{Ft}, P^*_{Ft}, \bar{P}^*_{Nt}, \bar{P}^*_{Tt}, \bar{P}^*_{Ft}, \) and wages \( W_t \) and \( W^*_t \), that satisfy the following conditions: (i) taking the prices
and the wage as given, the household’s allocations in each country solve its utility maximizing
problem; (ii) taking the wage and all prices but its own as given, the allocations and the price of
each non-traded intermediate good producer in each country solve its profit maximizing problem;
(iii) taking the wage and all prices but its own as given, the allocations and the price of each traded
intermediate good producer in each country solve its profit maximizing problem; and (iv) the world
market for bonds and the domestic markets for labor clear.

3 Equilibrium Dynamics

To facilitate analysis of optimal monetary policy, we first examine a useful benchmark in which
price adjustments are flexible, and then describe the equilibrium dynamics under sticky prices. We
call the allocations in the flexible-price equilibrium the “natural rate” allocations, and deviations
of the allocations in the sticky-price equilibrium from their natural rate levels the “gaps.” In
analyzing the equilibrium dynamics, we focus on log-deviations of equilibrium variables from their
steady-state values (denoted by hatted variables).

3.1 The Balanced-Trade Steady State and the Current Account

We begin by describing a balanced-trade steady state, in which all shocks are shut off (i.e., \( A_k = A^*_k = 1 \) for \( k \in \{N, T\} \)) and the net export is zero. The net export in the home country is given
by \( NX_t = \bar{P}_{Ht}C^*_{Ht} - \mathcal{E}_tP^*_{Ft}C_{Ft} \). Using the demand functions for non-traded goods (2.8) and for
traded goods (2.10), and the international risk-sharing condition (2.26), we obtain

\[
NX_t = (1 - \omega)\alpha^*\mathcal{E}_tP^*_{Ft}C^*_t \left[ 1 - \frac{\alpha}{\alpha^*}\phi_0^{-1} \right].
\]

In the balanced-trade steady state, \( NX = 0 \). This occurs only if \( \phi_0 = \alpha/\alpha^* \), and the risk-sharing
condition (2.26) becomes

\[
Q_t = \frac{\alpha}{\alpha^*} \frac{C_t}{C^*_t}.
\]
Clearly, if the countries have symmetric structures, that is, if $\alpha = \alpha^*$, then we have $\phi_0 = 1$ and the risk-sharing condition implies that $C_t = Q_tC_t^*$. Since the real exchange rate $Q_t$ represents the relative price of foreign consumption basket in terms of home consumption, it follows that, under symmetric structures, international risk-sharing leads to equalized aggregation consumption (measured in identical units) across countries for each period $t$.

Yet, with asymmetric structures where $\alpha \neq \alpha^*$, we have $\phi_0 \neq 1$, so that the risk-sharing condition contains a “wedge” that reflects the cross-country differences in initial wealth levels. To see this connection, recall that $\phi_0 = Q_0C^*_0/C_0$. If $\alpha > \alpha^*$, or $\phi_0 > 1$, then consumption and wealth in the foreign country (which has a smaller traded sector) exceed those in the home country when the relative price (i.e., the real exchange rate) is taken into consideration. It turns out, as we show below, the conditions under which the risk-sharing wedge arises also lead to sizable welfare gains from international monetary policy coordination.\footnote{Pesenti and Tille (2004) emphasize the importance of the risk-sharing wedge in analyzing gains from international monetary policy coordination in a one-sector open economy model with preset prices. The risk-sharing wedge in our model is somewhat different from theirs in that it is determined here by the balanced-trade steady-state conditions, so that it is independent of monetary policy; whereas in the Pesenti-Tille world, the wedge is given by the ratio of the expected marginal utility of consumption in the two countries, and is endogenous to policy. Such difference stems mainly from the different assumptions about the timing of portfolio choice decisions.}

Given that the steady state requires that $\phi_0 = \alpha/\alpha^*$, equation (3.1) implies that the net export is zero not only in the steady state, but for all $t \geq 0$. With zero net export, along with the assumption that neither country has an initial outstanding debt, the equilibrium current account would be zero for all $t$. This result greatly simplifies our analytical derivations of the welfare criteria.

### 3.2 The Flexible-Price Equilibrium and the Natural Rate

When price adjustments are flexible, firms’ pricing decisions are synchronized, so that the optimal price set by a firm is a constant markup over its contemporaneous marginal cost and that the price index in each sector coincides with the pricing decision of a typical firm in that sector. We now describe the flexible-price equilibrium allocations, which we call the natural rate allocations.

Let $Y_{Tt} = C_{Ht} + C^*_{Ht}$ denote aggregate demand for home-produced traded intermediate goods. We call $Y_{Tt}$ the aggregate traded output. Similarly, let $Y_{Nt} = C_{Nt}$ denote the aggregate non-traded output. As we show in Appendix A.2, the natural-rate levels of sectoral outputs, in log-deviation forms, are given by

$$\hat{y}^T_{Tt} = \hat{a}_{Tt}, \quad \hat{y}^N_{Nt} = \hat{a}_{Nt}. \quad (3.3)$$

Thus, under flexible prices, each sector’s output responds one-for-one with the sector-specific shocks, and there is no inter-sectoral or international spillover effects of shocks on production. Equation (3.3) also implies that the natural rates of sectoral employment are constant, that is, $\hat{l}^T_{Tt} = \hat{l}^N_{Nt} = 0$. 
Next, let $S_t = E_t P_{t}^s / P_{t} H_t$ denote the home country’s terms of trade. In Appendix A.2, we show that the natural rate of the terms of trade is given by

$$\hat{s}_t^n = \hat{a} T_t - \hat{a}^* T_t.$$  
(3.4)

Thus, an increase in the relative productivity in home’s traded sector (relative to the foreign traded sector) tends to lower the relative price of traded goods produced in the home country, and thus leads to worsened terms of trade for that country.

Third, the natural-rate level of aggregate consumption is given by

$$\hat{c}_t^n = \alpha \hat{a} T_t + (1 - \alpha) \hat{a} N_t - \alpha (1 - \omega) \hat{s}_t^n.$$  
(3.5)

Thus, aggregate consumption responds not only to domestic sectoral shocks, but also to movements in the terms of trade since part of the consumption basket consists of imported goods. An improved domestic productivity or terms of trade would raise the natural rate level of consumption. It follows from the intertemporal Euler equation (2.5) that the real interest rate in the flexible-price equilibrium is given by

$$\hat{r}_t^n = E_t \Delta \hat{c}_t^n + 1 = 0,$$  
(3.6)

where we have used the solution for $\hat{c}_t^n$ in (3.5) and the random-walk properties of the shock processes. The solutions for foreign consumption and real interest rate are similar.

Finally, the relative price of non-traded goods in terms of traded goods in the flexible-price equilibrium can be obtained by using the pricing decision equations and the solution for the terms of trade:

$$\hat{q}_{N_t} = \hat{P}_{N_t} - \hat{P}_T = \hat{a} T_t - \hat{a} N_t - (1 - \omega) \hat{s}_t^n.$$  
(3.7)

Hence, in the flexible-price equilibrium, the relative price of non-traded goods decreases with the relative productivity of the non-traded sector. Further, an improvement in the terms of trade (i.e., a fall in $\hat{s}_t^n$) would make imported goods relatively cheaper, so that the price of the traded basket would fall and the relative price of non-traded goods would rise.

### 3.3 The Sticky-Price Equilibrium

The sticky price equilibrium is characterized by the optimizing conditions derived in Section 2. Denote $\hat{x}_t = \hat{x}_t - \hat{x}_t^n$ the deviation of equilibrium variable $\hat{x}_t$ under sticky prices from its own natural rate $\hat{x}_t^n$, that is, the gap. After log-linearizing, the private sector’s optimizing conditions in
the home country can be summarized below:

\[ \pi_{Nt} = \beta E_t \pi_{N,t+1} + \kappa_N \tilde{y}_{Nt}, \quad (3.8) \]
\[ \pi_{Ht} = \beta E_t \pi_{H,t+1} + \kappa_T \tilde{y}_{Tt}, \quad (3.9) \]
\[ \Delta \tilde{y}_{Nt} = \Delta \tilde{y}_{Tt} - \pi_{Nt} + \pi_{Ht} - \Delta \hat{a}_{Nt} + \Delta \hat{a}_{Tt}, \quad (3.10) \]
\[ \alpha \tilde{y}_{Tt} + (1 - \alpha) \tilde{y}_{Nt} = E_t [\alpha \tilde{y}_{T,t+1} + (1 - \alpha) \tilde{y}_{N,t+1}] - \{ \hat{r}_t - E_t [\alpha \pi_{H,t+1} + (1 - \alpha) \pi_{N,t+1}] \}, \quad (3.11) \]

where the \( \pi \)'s denote the sectoral inflation rates, the \( \tilde{y} \)'s denote the sectoral output gaps, and \( \kappa_i = (1 - \beta \gamma_i)(1 - \gamma_i) \) is a constant that measures the responsiveness of the pricing decisions to variations in the real marginal cost gap in sector \( i \in \{ N, T \} \). The foreign optimizing conditions are analogous.

Equations (3.8) and (3.9) describe the Phillips-curve relations in the two sectors. These relations are forward-looking in that a sector’s period-\( t \) inflation rate depends solely upon current and expected future marginal cost gaps, which coincide here with the sectoral output gaps. Equation (3.10) describes the relation between changes in the expenditures on the two sectors’ outputs. Given the Cobb-Douglas aggregation technologies, these expenditures are proportional to each other, as reflected in (3.10). Equation (3.11) is derived from log-linearizing the intertemporal Euler equation (2.5) for the home household, with the consumption gap replaced by the output gaps using the constant-expenditure-share relations. Note that the terms-of-trade gap cancels out as we replace the consumption gap by the output gaps and the consumer price inflation rate by the sectoral inflation rates.

It is instructive to examine the relation between the marginal cost gaps (which equal the output gaps here) and the consumption gap, the terms-of-trade gap, and the relative-price gap. In log-deviation forms, these relations are given by

\[ \tilde{y}_{Tt} = \tilde{c}_t + (1 - \alpha) \tilde{q}_{Nt} + (1 - \omega) \tilde{s}_t, \quad (3.12) \]
\[ \tilde{y}_{Nt} = \tilde{c}_t - \alpha \tilde{q}_{Nt}. \quad (3.13) \]

The marginal cost gap in each sector depends positively on the consumption gap but negatively on the sector’s relative price gap. Additionally, the marginal cost in the home country’s traded sector depends positively on its terms-of-trade gap, so that a terms-of-trade improvement (i.e., a fall in \( \tilde{s}_t \)) leads to a fall in the real marginal cost in the home traded sector, but has no direct effect on the marginal cost in the non-traded sector.

Before we proceed to characterize optimal monetary policy, we examine whether or not, in a two-sector model like this, the national monetary authority faces a policy trade-off in stabilizing the output gaps and inflation rates. In the absence of such trade-offs, optimal independent monetary policy would be able to replicate the efficient flexible-price allocations and there would be no need
for cooperation. Woodford (2003, Chapter 6) shows that, in a closed economy with two sectors, if the degree of price stickiness is identical across sectors, then the sectoral Phillips curve relations can be reduced to an aggregate Phillips-curve that is identical to that in a one-sector model, so that the trade-off between price stability and stabilizing output gap fluctuations disappears, regardless of whether or not the sectoral shocks are correlated.

Is this still the case in our two-sector open economy environment? To answer this question, consider the special case with $\kappa_N = \kappa_T = \kappa$ so that the two sectors have identical durations of price contracts. Define a domestic inflation index as $\hat{\pi}_{Dt} = \alpha \hat{\pi}_{Ht} + (1 - \alpha) \hat{\pi}_{Nt}$. Then, by taking a weighted average of the sectoral Phillips curves in (3.8) and (3.9), and use (3.12) and (3.13) to replace the output gaps, we obtain

$$\pi_{Dt} = \beta E_t \pi_{D,t+1} + \kappa \tilde{c}_t + \kappa \alpha (1 - \omega) \tilde{s}_t.$$  \hspace{1cm} (3.14)

In the special case of a closed-economy (with $\omega = 1$), this relation reduces to an aggregate Phillips curve that implies no trade-off between output stability and price stability: the national policymaker is able to close the output gap by simply setting the domestic inflation index $\pi_{Dt} = 0$. In an open economy as the one presented here, however, fluctuations in the terms-of-trade gap act as an endogenous “cost-push shock” that introduces a trade-off between stabilizing the output gap and the domestic inflation index, unless there is no trade (i.e., with either $\omega = 1$ or $\alpha = 0$). It turns out that it is in general not possible to implement the flexible price allocations in this open economy.

**Proposition 1.** In the presence of nominal rigidities in both sectors and sector-specific shocks, it is not possible to implement the flexible-price allocations unless the domestic sectoral shocks are perfectly correlated.

**Proof:** By contradiction. Suppose that the flexible-price allocations could be replicated. Then the output gaps would both be closed, that is, $\bar{y}_{Tt} = \bar{y}_{Nt} = 0$ for all $t$. It follows from (3.8) and (3.9) that $\pi_{Nt} = \pi_{Ht} = 0$ for all $t$. But then, given that the output gaps are all closed, (3.10) implies that $\Delta \hat{a}_{Tt} - \Delta \hat{a}_{Nt} = \pi_{Nt} - \pi_{Ht}$, contradicting $\pi_{Nt} = \pi_{Ht} = 0$ unless $\Delta \hat{a}_{Tt} = \Delta \hat{a}_{Nt}$ for all $t$. \hspace{1cm} Q.E.D.

Even if the flexible-price equilibrium allocations are made Pareto optimal, the existence of the trade-off between stabilizing the gaps and inflation rates stated in Proposition 1 renders optimal monetary policy second best. In the next section, we define the optimal monetary policy problems and characterize allocations under cooperative and non-cooperative policies.

## 4 Optimal Monetary Policy

Optimal monetary policy entails maximizing a social objective function subject to the private sector’s optimizing conditions. A natural welfare criterion in our model is the representative households’ expected life-time utility. Following the approach described in Benigno and Woodford (2005),
we derive an analytical, quadratic expression for the welfare criterion based on second-order approximations to the representative households’ utility functions and to the private sectors’ optimizing conditions (except for those exact log-linear relations). We substitute all relevant second-order relations into the objective function to obtain a quadratic expression for the welfare objective. Finally, upon obtaining this objective, we solve for the allocations under optimal monetary policy by maximizing the quadratic objective subject to the set of log-linearized equilibrium conditions (3.8)-(3.11) and their foreign counterparts. In this final step, we are essentially solving a linear-quadratic (LQ) problem with rational expectations. The LQ approach has become a popular tool in studying optimal monetary policy in closed economy models with a single sector (e.g., Rotemberg and Woodford, 1997) or multiple sectors (e.g., Erceg, et al., 2000; Huang and Liu, 2005), and in open economy models with a single traded sector (e.g., Clarida, et al., 2002; Benigno and Benigno, 2003; Gali and Monacelli, 2005; Pappa, 2004). We are the first to derive an analytical expression for the welfare objective in an open economy model with multiple sectors and multiple sources of nominal rigidity, for both a regime with independent national policymakers (i.e., the Nash regime) and one with policy cooperation (i.e., the cooperating regime).5

4.1 The Nash Regime

A Nash regime is one in which each national policymaker seeks to maximize the welfare of its own representative household, subject to the private sector’s equilibrium conditions, taking the other country’s policy as given.

There are two layers of policy-making issues. The first involves each national policymaker’s optimal choice of fiscal instruments (i.e., the subsidy rates) in the (long-run) steady state equilibrium, taking as given the other country’s policy choice. The second involves each national policymaker’s optimal choice of monetary policy instruments (i.e., the nominal interest rate) in the (short-run) dynamic equilibrium, taking as given the other country’s policy choice. A Nash equilibrium in such policy games is then the joint best responses.

The steady-state problem involves each national policymaker choosing its domestic subsidy rates to maximize domestic household’s welfare, taking as given the domestic resource constraints and the other country’s subsidy rates. As we show in the Appendix, the Home’s optimal subsidy

5Our approach differs slightly from that adopted in the open-economy papers mentioned here (e.g., Clarida, et al., 2002; Benigno and Benigno, 2003; Gali and Monacelli, 2005; Pappa, 2004) in that we do not limit ourselves from the outset to taking first-order approximations to the private sectors’ optimizing conditions. An alternative solution method is to take second-order approximations throughout the model and then to compute approximate optimal policy rules through non-linear simulations of the second-order system (e.g., Pesenti and Tille, 2004; Sutherland, 2002). A main advantage of our approach, and the standard LQ approach described by Woodford (2003) and Benigno and Woodford (2005) as well, is that it allows us to obtain an analytical and explicit description of the objective function for optimal policy.
rates are given by

\[ 1 + \tau_T = \omega \mu_T, \quad 1 + \tau_N = \mu_N. \]  

(4.1)

Evidently, the optimal subsidy rate for the non-traded sector exactly offsets the steady-state markup distortion in that sector, but that for the traded sector does not. The subsidy rate for the traded sector is chosen to lower the effective markup distortion, but not all the way to neutralize it. As discussed in Benigno and Benigno (2003), a national policymaker under a non-cooperative regime may be tempted to create surprise deflation (or surprise currency appreciation). Allowing for some steady-state markup in the traded sector would create an incentive for surprise inflation that, by an appropriate choice of the subsidy scheme, exactly offsets the incentive for surprise deflation.

As the Nash subsidy policies do not eliminate steady-state markup distortions in the traded sectors, equilibrium allocations are efficient from each individual country’s view, but are socially suboptimal. This possibility of suboptimal allocations under the Nash regime stems from a national policymaker’s incentive to manipulate the terms of trade in its own favor, and at the cost of its trading partner. It represents a form of market failure, and is known as the “terms-of-trade externality.”

To characterize the optimal monetary policy problem in the short-run dynamic equilibrium with uncertainties, we first derive a welfare objective for a Nash policymaker by taking second-order approximations to the domestic household’s utility function. In the Appendix, we show that the objective function for the Home policymaker is given by

\[
W^{Nash} = E_0 \sum_{t=0}^{\infty} \beta^t U_t = -\frac{1}{2} E_0 \sum_{t=0}^{\infty} \beta^t L_t^{Nash} + t.i.p. + O \left( \| \xi \|^3 \right),
\]

(4.2)

where the term \( t.i.p. \) denotes terms independent of policy, and the period loss function is given by

\[
L_t^{Nash} = (1 - \alpha)(\tilde{y}_{Nt}^2 + \theta_N \kappa_{Nt}^{-1} \pi_{Nt}^2) + \alpha \omega(\tilde{y}_{Tt}^2 + \theta_T \kappa_T^{-1} \pi_{Tt}^2).
\]

(4.3)

The objective function here reveals that optimal monetary policy is inward-looking. Specifically, the Nash policymaker seeks to minimize variations in domestic marginal cost gaps (or output gaps) and inflation rates, so as to bring equilibrium allocations close to the natural rate. Yet, as we learn from Proposition 1, the policymaker faces a trade-off between stabilizing domestic marginal cost gaps and inflation rates, so that it cannot implement the flexible-price allocations, unless the size of one sector approaches zero (i.e., \( \alpha = 1 \) or \( \alpha = 0 \)), or there is only one source of nominal rigidity (i.e., \( \gamma_T = 0 \) or \( \gamma_N = 0 \)), or the shocks are perfectly correlated (i.e., \( \Delta \hat{a}_{Tt} = \Delta \hat{a}_{Nt} \)).

In general, allocations under Nash optimal monetary policy are second best, and the welfare depends on the relative weights in front of each of the four variables that the policymaker cares about. The weights assigned to non-traded inflation and output gap are proportional to the sector’s size \( (1 - \alpha) \), while the weights assigned to traded sector inflation and output gap depend both on the sector’s size \( (\alpha) \) and on the degree of steady-state bias towards domestic traded goods \( (\omega) \). Since
\[ \omega < 1, \] the weights assigned to the traded-sector variables are smaller than the sector’s size. This reflects a tradeoff between stabilizing output gap in the traded sector and using the terms-of-trade adjustment to insulate the economy from fluctuations originated from domestic and foreign shocks: by not stabilizing output gap in the traded sector as much as it would in a closed economy, the policymaker is able to alter the prices in that sector—hence the terms of trade. This is possible exactly because the policymaker has some monopoly power on its own terms of trade, as is evident in the optimal subsidy scheme (4.1).

The loss function (4.3) also reveals that, holding the size of each sector fixed, the weight assigned to a sector’s inflation rate increases with the elasticity of substitution between differentiated goods produced in that sector (i.e., increases with \( \theta_j \)) and with the sector’s price-rigidity (i.e., decreases with \( \kappa_j \)). Yet, a sector with more rigid prices needs not receive a larger weight for its inflation in the loss function, since the weight here is scaled by the relative size of the sector.

Under the Nash regime, a country’s optimal monetary policy problem is to maximize the quadratic welfare objective function (4.2) subject to the domestic private sector’s optimizing conditions (3.8)-(3.11) and the foreign counterparts, taking as given the other country’s policy instrument (i.e., the nominal interest rate). A Nash equilibrium is the joint “best responses” in the space of policy instruments.

For the purpose of solving the optimal monetary policy problem under the Nash regime, we first note that the foreign policy-relevant variables (i.e., those variables that enter the policymaker’s objective) are functions of the foreign nominal interest rate only. More formally, we have

**Proposition 2.** In the optimal monetary policy problem facing the home policymaker under the Nash regime, taking the foreign policy instrument (i.e., the foreign nominal interest rate) as given is equivalent to taking the foreign policy-relevant variables as given. These variables include foreign sectoral output gaps and inflation rates, which are the only variables that enter the foreign policymaker’s objective function.

**Proof:** The loss function (4.3) reveals that the only policy-relevant variables under the Nash regime are domestic sectoral inflation rates and output gaps. From the private sector’s optimizing conditions (3.8)-(3.11), these policy-relevant variables would be determined —independent of foreign variables, as long as a domestic monetary policy instrument (i.e., the nominal interest rate) is set. The same logics apply to the foreign country. \( Q.E.D. \)

Thus, the home optimal monetary policy problem under the Nash regime is indeed self-oriented: it involves maximizing the welfare objective (4.2) subject to the four domestic optimizing conditions (3.8)-(3.11). Similar in the foreign country.

An important issue of concern, in the spirit of Obstfeld and Rogoff (2002), is then: From a global perspective, would the lack of international monetary policy coordination incur substantive welfare losses? Obstfeld and Rogoff find that the answer is perhaps “no” in their model with a single
source of nominal rigidity. We revisit this issue below in a context with multiple sources of nominal rigidities and with potential structural differences across countries in the form of asymmetric sizes of traded sectors. For this purpose, we first derive a welfare objective function for the policymakers under the cooperating regime in the next subsection, and then examine the quantitative welfare gains from coordination in Section 5.

4.2 The Cooperating Regime

A cooperating regime is one in which monetary policy decisions are delegated to a supranational monetary institution (i.e., a social planner), who seeks to maximize a weighted average of national welfare in the two countries, subject to the private sectors’ optimizing conditions in both countries. Unlike a Nash policymaker, the planner does not take any country’s variables as given. Since the population size is equal across countries, we assume that the planner assigns equal weights (half) to each member country’s national welfare.⁶

Under the cooperating regime, the social planner chooses the steady-state subsidy rates in the two countries to maximize the countries’ collective welfare

$$\frac{1}{2} [U(C) - V(L) + U(C^*) - V(L^*)],$$

subject to the national resource constraints. As we show in the Appendix, the optimal subsidy rates that support the Pareto optimal steady-state allocations are given by

$$1 + \tau_T = \frac{\mu_T}{\alpha} [\alpha \omega + \alpha^*(1 - \omega)], \quad 1 + \tau_N = \mu_N,$$

(4.4)

$$1 + \tau_T^* = \frac{\mu_T^*}{\alpha^*} [\alpha^* \omega + \alpha(1 - \omega)], \quad 1 + \tau_N^* = \mu_N^*.$$

(4.5)

Evidently, in the case with symmetric structures, that is, with $\alpha = \alpha^*$, the optimal subsidy rates in the traded sectors exactly offset the steady-state markup distortions, as do the subsidies in the non-traded sectors. With asymmetric structures, the optimal subsidy rates in the traded sector do not offset the monopolistic markups, although the resulting steady-state allocations are Pareto optimal since they solve the social planner’s steady-state problem. The optimality of the steady-state allocations greatly simplifies our derivation of the welfare objective facing the social planner when we study the optimal monetary policy problem in the dynamic equilibrium with sticky prices.

To gain some intuition about the planner’s optimal subsidy schedule in the presence of structural asymmetry, suppose, without loss of generality, that $\alpha > \alpha^*$. Then, the optimal subsidy rates are such that $1 + \tau_T < \mu_T$ and $1 + \tau_T^* > \mu_T^*$. The planner’s optimal subsidy schedule thus entails redistributing some monopoly markup power from foreign firms in the traded-sector to their counterparts in the home country, so as to maintain higher prices of home traded goods relative to foreign traded goods than under symmetric structures. In other words, with structural asymmetry,

⁶We have also examined the gains from coordination with more general welfare weighting schemes (unreported). Although the numbers that we obtain are different from the case with equal weights, it remains true that the welfare gains become larger when the two countries become more dissimilar.
the planner’s optimal subsidy rates involve an efficient terms-of-trade bias in favor of the country that has a larger traded sector.

Why? When $\alpha > \alpha^*$, a larger share of consumption expenditure goes to traded consumption in the home country than in the foreign country. Since the expenditure share of imported goods in the traded consumption basket is constant and equal across countries, the home household needs to import more traded goods than does the foreign household. Yet, since the trade balance is zero in the steady-state equilibrium, the only way to enable the home household to afford more imported goods is to lower the relative price of foreign traded goods, that is, to adjust the terms-of-trade in favor of the home country. This terms-of-trade bias also mirrors the differences in the countries’ initial wealth levels. With $\alpha > \alpha^*$, the risk-sharing wedge is given by $\phi_0 = \alpha/\alpha^* > 1$, so that the initial wealth level (and consumption) in the foreign country (who has a smaller traded sector) is higher than that in the home country. Under the planner’s optimal subsidy scheme, the terms of trade is tilted in favor of the home country, which has a larger traded sector and a smaller initial wealth level.

It is important to emphasize that, although the terms-of-trade bias arises for a similar reason as the terms-of-trade externality—that is, the willingness of the policymakers to maintain some monopoly power on the terms of trade, their implications for policy making are different: the planner would always like to correct for the market failure associated with independent policy making regardless of whether or not the two countries have symmetric structures; but the planner would tilt the terms of trade in favor of the country that has a larger traded sector only in the presence of structural asymmetry.

In the Appendix, we show that the welfare objective for the optimal monetary policy problem facing the social planner is given by

$$W_{\text{Planner}} = \frac{1}{2} E_0 \sum_{t=0}^{\infty} \beta^t [U_t + U^*_t] = -\frac{1}{4} E_0 \sum_{t=0}^{\infty} \beta^t L_{t}^{\text{Planner}} + \text{t.i.p.} + O \left( \| \xi \|^3 \right),$$

(4.6)

where

$$L_{t}^{\text{Planner}} = (1 - \alpha)(\tilde{y}_{Nt}^2 + \theta_N \kappa_N^{-1} \pi_{Nt}^2) + \tilde{\alpha}(\tilde{y}_{Tt}^2 + \theta_T \kappa_T^{-1} \pi_{Ht}^2)$$

$$+ (1 - \alpha^*)(\tilde{y}_{Nt}^*2 + \theta_N^* \kappa_N^*^{-1} \pi_{Nt}^2) + \tilde{\alpha}^*(\tilde{y}_{Tt}^* + \theta_T^* \kappa_T^*^{-1} \pi_{Ht}^2),$$

(4.7)

with $\tilde{\alpha} = \alpha \omega + \alpha^* (1 - \omega)$ and $\tilde{\alpha}^* = \alpha^* \omega + \alpha (1 - \omega)$. Note that, although the optimal steady-state allocations under the cooperating regime differ from those under the Nash regime, the natural rate allocations are, to a first-order approximation, independent of policy regimes, as we have described in Section 3.3. The gaps in the welfare objective functions are thus deviations of the allocations under each policy regime from the same natural rate allocations.
Optimal monetary policy under the cooperating regime is obtained by maximizing the welfare
objective (4.6) subject to the private sector’s optimizing conditions summarized in (3.8)-(3.11) and
their foreign counterparts.

5 Gains from Coordination

We have so far established that, in our model with two sources of nominal rigidities and imperfectly
correlated domestic shocks within each country, neither independent policymakers under the Nash
regime nor the social planner under the cooperating regime can replicate the Pareto optimal natural-
rate allocations. There are potential gains from coordination as the social planner internalizes the
terms-of-trade externality. Further, if the countries have asymmetric structures, the planner also
engineers a terms-of-trade bias in favor of the country with a larger traded sector, giving rise to
further welfare gains relative to the Nash regime. A natural question is then: How large are the
welfare gains from policy coordination? In this section, we provide an answer to this question under
calibrated parameters. We also study the sensitivity of the welfare gains to changes in the values
of some key parameters.

5.1 Parameter Calibration

The parameters to be calibrated include $\beta$, the subjective discount factor; $\alpha$ and $\alpha^*$, the shares
of traded goods in the two countries’ consumption baskets; $\gamma_T$ and $\gamma_N$, the Calvo probabilities of
price non-adjustment in the two sectors; $\omega$, the share of domestic goods in the traded basket; $\theta_T$
and $\theta_N$, the elasticities of substitution between differentiated products in the two sectors; $\rho_{ij}$, the
cross-correlations of productivity shocks between sector $i \in \{T, N\}$ and sector $j \in \{T, N\}$; and
finally, $\sigma_T$ and $\sigma_N$, the standard deviations of the shocks. The calibrated parameter values are
summarized in Table 1.

Since we have a quarterly model, we set $\beta = 0.99$, so that the steady-state annualized real inter-
est rate is 4 percent. We set $\omega = 0.7$ to capture steady-state home bias in the traded consumption
baskets. Since the steady-state share of imports in the home country’s GDP is given by $\alpha(1 - \omega)$, if
we consider a traded-sector share of $\alpha = \alpha^* = 0.3$ as a benchmark value, then $\omega = 0.7$ implies that
the steady-state share of imports in GDP is 0.09, roughly corresponding to the sample average of the
import share in the U.S. in its trade with the European Union. We set $\theta_T = \theta_N = 10$, so that
the steady state markup is 11 percent; and $\gamma_T = \gamma_N = 0.75$, so that the Calvo pricing contracts
in each sector last for four quarters on average. We set the standard deviation of the innovations
to sectoral productivity shocks to 0.01. In our baseline experiment, we assume that the shocks are
uncorrelated across sectors and across countries, so that $\rho_{ij} = 0$ for $i, j \in \{T, N\}$. 

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5.2 Gains from Coordination under Symmetric Structures

We first consider the symmetry case with \( \alpha = \alpha^* \). In this case, the countries have identical initial wealth and consumption levels (in comparable units), and the risk-sharing wedge in (2.26) disappears. Thus, the planner has no incentive to tilt the terms of trade in favor of either country, and the optimal subsidy rates under the cooperative regime exactly offset the steady-state markup distortions in all sectors. For this reason, the gains from coordination arise solely from internalizing the terms-of-trade externality.

Figure 1 plots the social welfare losses associated with optimal monetary policy under the Nash regime (the solid line) and those under the cooperating regime (the dashed line) for a range of values of \( \alpha \), where we assume the two countries have symmetric structures. The welfare loss here is measured by the percentage of steady-state consumption equivalence, that is, the percentage increase in steady-state consumption required to keep the households indifferent between living in a world with flexible prices and in one with sticky prices and optimal monetary policy. The gains from coordination are measured by the difference between the welfare losses under the Nash regime and those under the cooperating regime.

The figure reveals that the welfare gain from coordination (i.e., the difference between the two curves) is small relative to the welfare losses that arise from inefficient fluctuations in domestic relative prices under optimal policy (i.e., the scale of each curve). The size of the traded sector matters. When \( \alpha = 1 \), the model reduces to a one-sector open-economy model similar to a version of the model studied by Corsetti and Pesenti (2005).\(^7\) In this case, as these authors point out, optimal independent monetary policy can replicate the natural-rate allocations, leaving no scope for welfare gains from coordination. When \( \alpha = 0 \), the model corresponds to a one-sector closed economy model, where the natural-rate allocations can be replicated by following a policy that achieves price stability.

In the more general case where \( \alpha \) lies between 0 and 1, nominal rigidities in both the traded and the non-traded sectors become relevant for optimal policy. Proposition 1 suggests that inward-looking policy cannot replicate the efficient natural rate allocations, and thus there are potential gains from coordination. As shown in the figure, the gains from coordination are larger when \( \alpha \) takes less extreme values. The welfare gain (measured by the difference between the two lines in the figure) reaches its peak at \( \alpha = 0.5 \), with a maximum gain of about 0.14% of consumption equivalence. When \( \alpha \) moves away from 0.5, the gain diminishes. In Liu and Pappa (2005) we show that the hump-shaped relation between the welfare losses and \( \alpha \) primarily reflects the effectiveness of domestic relative-price adjustments in face of imperfectly correlated domestic sector-specific shocks.

\(^7\)In particular, a version of their model with producer currency pricing.
5.3 Gains from Coordination under Asymmetric Structures

When the countries have asymmetric structures (i.e., $\alpha \neq \alpha^*$), the international risk-sharing equation (2.26) appropriately contains a wedge that reflects differences in the initial wealth and consumption levels across countries. Facing this wedge, the planner has incentive to tilt the terms of trade in favor of the country that has a larger traded sector (and smaller initial wealth and consumption), giving rise to the possibility of further welfare gains beyond those attained through internalizing the terms-of-trade externality. A natural question is then: How large are such gains?

Figure 2 provides an answer. There, we plot the relative welfare losses under the Nash regime relative to those under cooperation in the $(\alpha, \alpha^*)$ space. The relative losses here measure the gains from coordination. The gains are small along the diagonal of the space, where $\alpha = \alpha^*$. When we move away from the diagonal so that the difference between $\alpha$ and $\alpha^*$ enlarges, the gains also increase, until reaching a maximum of 0.62 percent of steady-state consumption at the edges of the grid.

The welfare gains that arise from the terms-of-trade bias as we document here are sizable in light of the standard welfare calculations in the literature. For instance, Obstfeld and Rogoff (2002) find that, in a model with both traded and non-traded goods but with one source of nominal rigidities (sticky wages), the upper bound of the gain from coordination is smaller than 0.18% of output (for a relative risk aversion of 8) and, in the case with log-utility and Cobb-Douglas aggregation as we assume here, there is no gain from coordination (see their Table 1, p.524). In comparison, the welfare gain obtained in our model with structural asymmetry can be as high as 0.6% of consumption (or roughly 0.4% of output) even with log-utility and Cobb-Douglas aggregation. The size of the welfare gain that we document here is comparable to that obtained by Pappa (2004), who focus on the role of non-unitary intertemporal and intratemporal elasticities of substitution. Our result suggests that welfare gains from coordination can be sizable if the countries involved are dissimilar.

5.4 Sensitivity

We have so far considered the welfare gains from coordination with calibrated values of the parameters. We now examine the sensitivity of the welfare gains to a range of values for the key parameters.

5.4.1 Home bias

The parameter $\omega$ is an important determinant of the terms-of-trade externality and, hence, has implications on the gains from coordination. To understand how home bias in traded consumption
affects welfare under optimal policies, we revisit the optimal subsidy rates to traded-goods production for both the Nash policymakers, as in equation (4.1), and the social planner, as in equations (4.4)-(4.5). As \( \omega \) increases, the optimal subsidy rates to the traded sector become closer to the sector’s steady-state markup distortion; in the limit with \( \omega = 1 \), there is no trade and the optimal subsidy rates exactly offset the markup distortions regardless of the structural differences between the countries. Thus, with a larger value of \( \omega \), the welfare losses should be smaller under both the Nash regime and the cooperating regime; when \( \omega \) approaches 1, the loss under the Nash regime would approach zero, so would the gains from coordination.

Figure 3 confirms this intuition. The figure plots the welfare losses under the two alternative regimes (the upper panel) and the gains from coordination (the lower panel) for \( \omega \in [0.1, 0.9] \), where we have fixed \( \alpha = 0.3 \) and \( \alpha^* = 0.6 \) to capture the structural asymmetry between the two countries\(^8\). The figure shows that, as \( \omega \) increases (i.e., as the countries rely less on imported goods and are thus less exposed to international trade), not only do the welfare losses under both regimes become smaller, but the coordination gains also decline.

### 5.4.2 Correlation of Shocks

In our baseline experiments, we have assumed that shocks are uncorrelated both across sectors within a country and across countries. In contrast, the NOEM literature frequently assumes that shocks are perfectly correlated within each country but uncorrelated across countries. We now examine the sensitivity of our results to correlations between the shocks.

Figure 4 displays the welfare gains from coordination as the correlations between sectoral shocks vary in the interval \([-1, 1]\). Apparently, variations in cross-country correlations (i.e., \( \rho_T T \) and \( \rho_N N \)) do not generate visible effects on the welfare gains. The gains are much more sensitive to the correlations between domestic shocks (i.e., \( \rho_T N \)). As domestic shocks become more correlated, the welfare gains become unambiguously smaller. In the extreme case with perfectly correlated domestic shocks, optimal monetary policy under the Nash regime and the cooperating regime can both replicate the flexible-price allocations, and thus there are no gains from coordination, despite of the structural asymmetry across countries. In this sense, our results here extend the findings by Obstfeld and Rogoff (2002) (for the case with perfectly correlated domestic shocks) and by Canzoneri, et al. (2005) (for the case with imperfectly correlated domestic shocks) to an environment that allows for structural asymmetry across countries.

### 5.4.3 Price stickiness

In our baseline analysis, we have assumed that firms in different sectors face an identical duration of pricing contracts: they both last on average for 4 quarters. We now relax this assumption by

\(^8\)These values of the \( \alpha \)'s are also used in the rest of the sensitivity analysis.
allowing the relative length of the pricing contracts to vary. We examine the implications of varying the exogenous price-stickiness on the gains from coordination.

Figure 5 plots the welfare gains as the price-stickiness in one sector varies, holding the stickiness in the other sector fixed at its calibrated value. In particular, the solid line denotes the gains from coordination when $\gamma_T$ varies in the interval $[0.1, 0.8]$, while fixing $\gamma_N = 0.75$; and the dashed line represents the other case when $\gamma_N$ varies in the same interval while $\gamma_T$ is fixed at 0.75. Evidently, holding one sector’s price rigidity fixed, the welfare gains increase with the rigidity in the other sector. An exception seems to be that, when $\gamma_T$ is fixed at 0.75, the gains initially increase with $\gamma_N$, and then decline when $\gamma_N$ exceeds $\gamma_T = 0.75$. In general, the gains are more sensitive to variations in traded price rigidity than to non-traded price rigidity. This is so because the welfare gains stem mainly from movements in the terms-of-trade, so that larger nominal rigidities in the traded sector and the resulting greater price-dispersions among firms in that sector would lead to disproportionately larger distortions in the terms of trade, leaving more room for welfare gains from international monetary policy coordination.

6 Conclusions

We have revisited the issue of gains from international monetary policy coordination in a framework that generalizes the standard model in the NOEM literature by introducing both traded and non-traded goods, and more importantly, by allowing for a structural asymmetry across countries in the size of the traded sector. For this purpose, we obtain welfare measures through second-order approximations to the households’ utility functions and to the private sector’s optimizing conditions. The gains arise from two channels. The first channel is rather standard in the NOEM literature and is independent of the structural asymmetry in the model: if acting independently, a national policymaker does not care about the effects of terms-of-trade movements on the other country’s well being; whereas when the countries cooperate, this terms-of-trade externality would be properly recognized and efforts would be made to internalize it. The second channel is unique to our model and works only through structural asymmetries across countries: the planner’s optimal policy under the cooperating regime involves in general a terms-of-trade bias that favors the country with a larger traded sector; and this bias has to be balanced against the need to stabilize fluctuations in the terms-of-trade gap, among other variables in the policy objective. Absent structural asymmetry, the welfare gains from coordination are quantitatively small; as the degree of asymmetry enlarges, so do the welfare gains in general. With plausible structural asymmetries, there are sizable gains from policy coordination. Further, holding other things constant, the gains are larger if the countries have a greater share of imported goods in their traded basket, if the domestic shocks are less correlated, or if the duration of pricing contracts is longer.
Although, the terms-of-trade bias identified in this paper arises from the same principles as does the terms-of-trade externality described in the NOEM literature, their implications for the gains from coordination should not be confused. In the case with symmetric structures across countries, there is no terms-of-trade bias under cooperation and the welfare gains arise solely from internalizing the terms-of-trade externality. Under plausible parameters, such gains are quantitatively small. A stronger case for policy coordination can be made when the countries involved have asymmetric trading structures.

In our analysis, we have focused on a particular form of cross-country asymmetries. Of course, there are other forms of structural asymmetries that might be relevant for international monetary policy analysis. For instance, countries may have different trend components of traded-sector productivity, they may have different abilities to access international financial markets, or they may have different labor market institutions. Modeling structural differences along these dimensions may have important implications for understanding the consequences of international policy coordination.

For the purpose of studying optimal monetary policy, we assume that fiscal policies are “passive” in the sense that they are needed only to the extent that production subsidy rates are chosen to achieve optimal steady-state allocations. Further, the planner’s weighting scheme can be interpreted as a “bargaining” outcome between fiscal authorities in countries with asymmetric structures. A natural extension would be to study the implications of international coordinations in both fiscal policy and monetary policy in a model like ours. Future research along these lines should be both fruitful and promising. The current paper represents a first step toward this direction.

A Appendix

This Appendix (not to be published) contains derivations of some key results in the text. Most of the derivations here are also available in Liu and Pappa (2005).

A.1 Some Preliminary Aggregation Results

We first note that aggregate nominal demand for Home traded output is given by

$$\bar{P}_H Y_{tH} = P_tC_t\left[\omega \alpha + (1 - \omega)\alpha^* Q_t \frac{C_t^*}{C_t} \right] = \alpha P_t C_t,$$  \hspace{1cm} (A.1.1)

where the first equality follows from the demand functions for traded consumption as described in (2.8) and (2.10) and their foreign counterparts, and the second equality follows from the risk-sharing condition (3.2). Similarly, demand for foreign traded output is given by

$$\bar{P}_{Ft} Y_{tF} = \alpha^* P_t^* C_t^*.$$  \hspace{1cm} (A.1.2)
Let $S_t = \frac{\varepsilon_t P_{Ft}^*}{\bar{P}_{Ht}}$ denote the home country’s terms of trade. It follows from (A.1.1) and (A.1.2), along with (3.2), that the terms of trade is given by the relative traded outputs. That is,

$$S_t = \frac{Y_{Tt}}{Y_{Tt}^*}. \quad (A.1.3)$$

Under Cobb-Douglas aggregation technologies, expenditure on non-traded goods is a constant fraction of total consumption expenditures in each country. In particular, we have

$$\bar{P}_{Nt} Y_{Nt} = (1 - \alpha) P_t C_t, \quad \bar{P}_{Nt}^* Y_{Nt}^* = (1 - \alpha^*) P_t^* C_t^*. \quad (A.1.4)$$

Equation (A.1.1) and the price index relations $P_t = \bar{P}_{Nt}^{1-\alpha}$ and $\bar{P}_{Tt} = \bar{P}_{Ht} [\varepsilon_t \bar{P}_{Ft}]^{1-\omega}$ imply that the real demand for home-traded goods is given by

$$Y_{Tt} = \alpha C_t Q_t^{1-\alpha} S_t^{1-\omega}, \quad (A.1.5)$$

where $Q_{Nt} = \bar{P}_{Nt}/\bar{P}_{Tt}$ denotes the relative price of non-trade goods. Similarly, (A.1.4) implies that the real demand for home non-traded goods is given by

$$Y_{Nt} = (1 - \alpha) C_t Q_{Nt}^{-\alpha}. \quad (A.1.6)$$

Use (A.1.6) to eliminate $Q_{Nt}$ and (A.1.3) to eliminate $S_t$ from (A.1.5), and go through the same procedure for the foreign country, we obtain

$$C_t = \bar{\alpha} Y_{Nt}^{1-\alpha} [Y_{Tt}^* Y_{Tt}^{1-\omega}]^\alpha, \quad C_t^* = \bar{\alpha}^* Y_{Nt}^{1-\alpha^*} [Y_{Tt}^* Y_{Tt}^{1-\omega}]^\alpha. \quad (A.1.7)$$

Thus, aggregate consumption in each country is a weighted average of the non-traded output and a composite of traded outputs produced in the two countries.

From (2.16) and (2.19), aggregate demand for labor in the non-traded sector is given by

$$L_{Nt} = \frac{1}{A_{Nt}} \int_0^1 Y_{Nt}^d(j) dj = \frac{G_{Nt}}{A_{Nt}} Y_{Nt}, \quad (A.1.8)$$

where $G_N = \int_0^1 (P_N(j)/\bar{P}_N)^{-\theta_N} dj$ measures the price-dispersion within the sector. Similarly, aggregate demand for labor in the traded-sector is given by

$$L_{Tt} = \frac{G_{Ht}}{A_{Tt}} Y_{Tt}, \quad (A.1.9)$$

where $G_H = \int_0^1 (P_H(j)/\bar{P}_H)^{-\theta_H} dj$. Expressions for $L_N^*$ and $L_T^*$ can be obtained in a similar way for the foreign country.

### A.2 The Flexible-Price Equilibrium Allocations

Using the aggregate expenditure relation for traded goods (A.1.1) together with the labor supply equation (2.3) and the steady-state version of the optimal pricing decision (2.21) for firms in the
traded sector, we obtain the natural-rate level of traded output given by \( Y_{Tt} = A_{Tt} \). Similarly, using the aggregate expenditure relation for non-traded goods (A.1.4) together with the labor supply equation (2.3) and the steady-state version of the optimal pricing decision (2.20) for firms in the non-traded sector, we obtain the natural-rate level of the non-traded output given by \( Y_{Nt} = A_{Nt} \). The foreign counterparts of the natural-rate output levels are similarly obtained. The log-linearized outputs in the flexible-price equilibrium are thus given by (3.3) in the text.

The solutions for the natural rate levels of outputs, together with the labor-demand relations (2.19) and (2.23) and the fact that the price dispersion terms \( G_{jH} = 1 \) in the flexible-price equilibrium for \( j \in \{N, H\} \), imply that the natural-rate levels of employment are constant.

The natural-rate of the terms of trade in (3.4) in the text follows from (A.1.3) and (3.3); the natural-rate level of consumption in (3.5) in the text follows from the solutions for the sectoral outputs and the terms of trade.

### A.3 Deriving the Welfare Objective under the Nash Regime

We now derive the welfare objective function facing a national policymaker under the Nash regime. To do this, we first describe the steady-state subsidy policy, and then derive the welfare objective for the optimal monetary policy problem through second-order approximations to the representative household’s utility function and to the private sector’s optimizing conditions.

#### A.3.1 Steady State Subsidies under the Nash Regime

In a steady state, the shocks are shut off and pricing decisions are synchronized, so that \( A_N = A_T = 1 \), \( G_H = G_N = 1 \), and \( L_j = Y_j \) for \( j \in \{N, T\} \). The home policymaker chooses \( (\tau_T, \tau_N) \) to maximize \( U(C) - V(L) \), subject to the resource constraint (A.1.7) and the labor market clearing condition \( L = L_N + L_T \), taking foreign policy instruments \( (\tau_T^*, \tau_N^*) \) as given. In the steady-state equilibrium, \( L_j = Y_j \) so that the resource constraint can be rewritten as \( C = \bar{\alpha}L_N^{1-\alpha}[L_T^{n-1-\omega}]^{\alpha} \). Further, it is easy to show that \( Y_T^* = \alpha^*(1 + \tau_T^*)/(\mu_T^*\Psi) \), that is, the foreign traded output in the steady-state equilibrium is solely determined by foreign policy instruments. Thus, when choosing its steady-state subsidies, the home policymaker takes \( Y_T^* \) as given.

Solving the home steady-state problem yields optimal steady-state labor allocations:

\[
\Psi L_N = 1 - \alpha, \quad \Psi L_T = \alpha \omega, \quad \Psi L = 1 - \alpha + \alpha \omega. \tag{A.3.1}
\]

It is then straightforward to show that the subsidy rates consistent with the optimal steady-state allocations are given by (4.1) in the text.
A.3.2 Welfare Objectives under the Nash Regime

We characterize the welfare objective for a Nash policymaker by taking second-order approximations to the representative household’s period utility function. A second order approximation to the home household’s period utility function is given by

\[ U_t - U_{ss} = \hat{c}_t - \Psi L \left( \hat{l}_t + \frac{1}{2} \hat{l}_t^2 \right) + O \left( \|\xi\|^3 \right), \]  

(A.3.2)

where \( U_{ss} \) denotes the steady-state period utility, and \( O \left( \|\xi\|^3 \right) \) represents terms that are of third or higher order in an appropriate bound on the amplitude of the shocks.

The first component of the approximated utility function is deviations of consumption from steady state, which are related to deviations of outputs through the aggregate resource constraint (A.1.7). The relation is given by

\[ \hat{c}_t = (1 - \alpha)\hat{y}_{Nt} + \alpha \omega \hat{y}_{Tt} + \alpha (1 - \omega)\hat{y}_{\ast Tt}. \]  

(A.3.3)

The second part of the approximated period utility involves second-order approximations to the labor market clearing condition \( L_t = L_{Nt} + L_{Tt} \), which is given by

\[ \hat{L}_t = L_N \hat{I}_N + L_T \hat{I}_T + \frac{1}{2} L \left[ \frac{L_N}{L} \hat{l}_N^2 + \frac{L_T}{L} \hat{l}_T^2 - \hat{l}_t^2 \right] + O \left( \|\xi\|^3 \right). \]

(A.3.4)

Using this result, we obtain

\[ \Psi L \left( \hat{l}_t + \frac{1}{2} \hat{l}_t^2 \right) = (1 - \alpha)\hat{I}_N + \alpha \omega \hat{I}_T + \frac{1}{2} \left( (1 - \alpha)\hat{I}_N^2 + \alpha \omega \hat{I}_T^2 \right) + O \left( \|\xi\|^3 \right), \]

\[ = (1 - \alpha)(\hat{y}_{Nt} + \hat{G}_{Nt} - \hat{a}_{Nt}) + \alpha \omega(\hat{y}_{Tt} + \hat{G}_{Ht} - \hat{a}_{Tt}) + \frac{1}{2} \left( (1 - \alpha)(\hat{y}_{Nt} + \hat{G}_{Nt} - \hat{a}_{Nt})^2 + \alpha \omega(\hat{y}_{Tt} + \hat{G}_{Ht} - \hat{a}_{Tt})^2 \right) + O \left( \|\xi\|^3 \right), \]

(A.3.5)

where the first equality follows from the approximated labor market clearing condition (A.3.4), replacing the steady-state values \( \Psi L_N = 1 - \alpha \) and \( \Psi L_T = \alpha \omega \), and the second equality follows from the labor demand equations (A.1.8) and (A.1.9).

Subtract (A.3.5) from (A.3.3), and using Proposition 2 (i.e., foreign output \( \hat{y}_{\ast Tt} \) is solely determined by foreign nominal interest rate, which is taken as given by the home policymaker), we obtain

\[ U_t = -(1 - \alpha)\hat{G}_{Nt} - \alpha \omega \hat{G}_{Ht} - \frac{1}{2} \left[ (1 - \alpha)(\hat{y}_{Nt} + \hat{G}_{Nt} - \hat{a}_{Nt})^2 + \alpha \omega(\hat{y}_{Tt} + \hat{G}_{Ht} - \hat{a}_{Tt})^2 \right] + t.i.p + O \left( \|\xi\|^3 \right), \]

\[ = -(1 - \alpha)\hat{G}_{Nt} - \alpha \omega \hat{G}_{Ht} - \frac{1}{2} \left[ (1 - \alpha)\hat{y}_{Nt}^2 + \alpha \omega \hat{y}_{Tt}^2 \right] + t.i.p + O \left( \|\xi\|^3 \right), \]
where, in obtaining the second equality, we have used the definition of the output gaps \( \tilde{y}_{jt} - \tilde{y}_{jt}^n \) with \( \tilde{y}_{jt}^n = \hat{a}_{jt} \) being the natural rate output in sector \( j \in \{N, T\} \), and the fact that the price-dispersion terms \( \hat{G}_{jt} \) are of second order. The notation \( t.i.p \) represents terms independent of policy, including steady-state terms, shocks, and foreign outputs.

Finally, following Woodford (2003, p.400), we can show that the price dispersion terms \( \hat{G}_{Nt} \) and \( \hat{G}_{Ht} \) can be related to variabilities in the sectoral inflation rates. In particular, we have

\[
\sum_{t=0}^{\infty} \beta^t \hat{G}_{jt} = \frac{1}{2} \frac{\theta_j \gamma_j}{(1 - \beta \gamma_j)(1 - \gamma_j)} \sum_{t=0}^{\infty} \beta^t \pi_{jt}^2 + t.i.p + O \left( \|\xi\|_3^2 \right), \quad j = N, H.
\]

Thus, the home policymaker’s welfare objective under the Nash regime is given by

\[
W_{Nash} = E_0 \sum_{t=0}^{\infty} \beta^t U_t = -\frac{1}{2} E_0 \sum_{t=0}^{\infty} \beta^t L_{Nash}^t + t.i.p + O \left( \|\xi\|_3^2 \right),
\]

where the period loss function is given by

\[
L_{Nash}^t = (1 - \alpha)(\tilde{y}_{Nt}^2 + \theta_N \kappa_N^{-1} \pi_{Nt}^2) + \alpha \omega (\tilde{y}_{Tt}^2 + \theta_T \kappa_T^{-1} \pi_{Ht}^2),
\]

where \( \kappa_j = \gamma_j / ((1 - \beta \gamma_j)(1 - \gamma_j)) \) for \( j \in \{N, T\} \). These correspond to (4.2)-(4.3) in the text.

The foreign policymaker’s objective under the Nash regime can be similarly derived.

### A.4 Deriving the Welfare Objective Under the Cooperating Regime

We now describe the the derivations of the welfare objective function under the cooperative regime through second-order approximations. First, the steady state.

#### A.4.1 Steady State Subsidies under the Cooperating Regime

Under the cooperating regime, the social planner chooses the steady-state subsidy rates in the two countries to maximize the countries’ collective welfare \( \frac{1}{2} U(C) - V(L) + U(C^*) - V(L^*) \), subject to the national resource constraints (A.1.7) and the labor market clearing conditions \( L_T + L_N = L \) and \( L_T^* + L_N^* = L^* \), with \( Y_j = L_j \) and \( Y_j^* = L_j^* \) imposed. The optimal steady-state labor allocations are given by

\[
\Psi L_N = 1 - \alpha, \quad \Psi L_T = \alpha \omega + \alpha^*(1 - \omega), \quad (A.4.1)
\]

\[
\Psi L_N^* = 1 - \alpha^*, \quad \Psi L_T^* = \alpha^* \omega + \alpha(1 - \omega). \quad (A.4.2)
\]

It is then easy to show that the Pareto optimal steady-state allocations can be implemented by appropriate choice of the subsidy rates, which are given by (4.4)-(4.5) in the text.
A.4.2 Welfare Objective under the Cooperative Regime

The welfare objective for the social planner is given by

$$W_{\text{Planner}} = \frac{1}{2} E_0 \sum_{t=0}^{\infty} \beta^t [U_t + U_t^*], \quad (A.4.3)$$

where $U_t = \log C_t - \Psi L_t$ and $U_t^* = \log C_t^* - \Psi L_t^*$ are the representative households’ period-utility functions.

A second-order approximation to the home household’s period utility function is given by (A.3.2), the same as in the Nash case. The $\hat{c}_t$ term is also the same as in the Nash case, and is given by (A.3.3).

The terms involving employment, however, is different from the Nash case since the steady-state allocations are different. In particular, the approximated employment terms in the home household’s period utility function are given here by

$$\Psi L \left( \frac{i_t + \frac{1}{2} I_t^2}{2} \right) = (1 - \alpha) \hat{y}_{Nt} + \hat{\alpha} \hat{y}_{Tt} + \frac{1}{2} \left( (1 - \alpha) \hat{y}_{Nt}^2 + \hat{\alpha} \hat{y}_{Tt}^2 \right) + O \left( \|\xi\|^3 \right),$$

$$= (1 - \alpha) (\hat{y}_{Nt} + \hat{G}_{Nt} - \hat{\alpha} \hat{Nt}) + \hat{\alpha} (\hat{y}_{Tt} + \hat{G}_{Ht} - \hat{\alpha} \hat{Tt})$$

$$+ \frac{1}{2} \left( (1 - \alpha) (\hat{y}_{Nt} + \hat{G}_{Nt} - \hat{\alpha} \hat{Nt})^2 + \hat{\alpha} (\hat{y}_{Tt} + \hat{G}_{Ht} - \hat{\alpha} \hat{Tt})^2 \right) + O \left( \|\xi\|^3 \right), \quad (A.4.4)$$

where we have used the steady state conditions (A.4.1) and the labor demand functions (A.1.9), and we have defined a constant $\hat{\alpha} = \alpha \omega + \alpha^* (1 - \omega)$. Subtracting (A.4.4) from the expression for consumption in (A.3.3), we obtain the approximated period utility for the home country:

$$U_t = -(1 - \alpha) \hat{G}_{Nt} - \hat{\alpha} \hat{G}_{Ht}$$

$$- \frac{1}{2} \left( (1 - \alpha) (\hat{y}_{Nt} + \hat{G}_{Nt} - \hat{\alpha} \hat{Nt})^2 + \hat{\alpha} (\hat{y}_{Tt} + \hat{G}_{Ht} - \hat{\alpha} \hat{Tt})^2 \right) + t.i.p + O \left( \|\xi\|^3 \right),$$

$$= -(1 - \alpha) \hat{G}_{Nt} - \hat{\alpha} \hat{G}_{Ht} - \frac{1}{2} \left( (1 - \alpha) \hat{y}_{Nt}^2 + \hat{\alpha} \hat{y}_{Tt}^2 \right) + t.i.p + O \left( \|\xi\|^3 \right),$$

where $t.i.p.$ denotes terms independent of policy, including constant terms and shocks. Similarly, the approximated period utility function for the foreign country can be obtained as follows:

$$U_t^* = -(1 - \alpha^*) \hat{G}_{Nt}^* - \hat{\alpha}^* \hat{G}_{Ft}^*$$

$$- \frac{1}{2} \left( (1 - \alpha^*) (\hat{y}_{Nt}^* + \hat{G}_{Nt}^* - \hat{\alpha} \hat{Nt})^2 + \hat{\alpha}^* (\hat{y}_{Tt}^* + \hat{G}_{Ft}^* - \hat{\alpha} \hat{Tt})^2 \right) + t.i.p + O \left( \|\xi\|^3 \right),$$

$$= -(1 - \alpha^*) \hat{G}_{Nt}^* - \hat{\alpha}^* \hat{G}_{Ft}^* - \frac{1}{2} \left( (1 - \alpha^*) \hat{y}_{Nt}^2 + \hat{\alpha}^* \hat{y}_{Tt}^2 \right) + t.i.p + O \left( \|\xi\|^3 \right),$$

where $\hat{\alpha}^* = \alpha^* \omega + \alpha (1 - \omega)$.

Finally, replacing the $G$-terms using (A.3.6), we obtain the planner’s welfare objective:

$$W_{\text{Planner}} = \frac{1}{2} E_0 \sum_{t=0}^{\infty} \beta^t [U_t + U_t^*] = -\frac{1}{4} E_0 \sum_{t=0}^{\infty} \beta^t L_t^\text{Planner} + t.i.p. + O \left( \|\xi\|^3 \right), \quad (A.4.5)$$
where the period loss function is given by

\[
L^\text{Planner}_t = (1 - \alpha)(\tilde{y}^2_{N_t} + \theta^2_{N} N_t^2) + \tilde{\alpha}(\tilde{y}^2_{T_t} + \theta^2_{T} T_t^2) + (1 - \alpha^*)((\tilde{y}^2_{N_t} + \theta^2_{N} N_t^2) + \tilde{\alpha}^* (\tilde{y}^2_{T_t} + \theta^2_{T} T_t^2)),
\]

(A.4.6)

where \( \tilde{\alpha} = \alpha \omega + \alpha^*(1 - \omega) \) and \( \tilde{\alpha}^* = \alpha^* \omega + \alpha(1 - \omega) \). These expressions correspond to (4.6)-(4.7) in the text.

References


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<td>$\rho_{TN} = 0$, $\rho_{TT} = 0$, $\rho_{NN} = 0$</td>
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Figure 1: Welfare losses of alternative monetary policy regimes: symmetric structures.
Figure 2:—Welfare gains from coordination: sizes of the traded sectors
Figure 3:—Welfare losses under alternative regimes and gains from coordination: home bias.
Figure 4:—Welfare gains from coordination: correlations of sectoral shocks.
Figure 5:—Welfare gains from coordination: sectoral price stickiness.