Abstract

When all cross-border inefficiencies travel through international prices, trade negotiators need to negotiate terms of trade only. Starting from this plain principle, Bagwell and Staiger (2006) argue that the ban of production subsidies by the WTO legislation is inefficient. The present paper shows that their logic fails when trade agreements need to be self-enforceable. Two observation are key. First, self-enforcement constraints do not restrict the actions of individual countries but abridge the set of sustainable policies - they are thus boundary conditions for trade negotiators. Second, via their effect on national output, subsidies impact the self-enforcement constraints. Consequently, subsidies become targets in the negotiators’ optimization problem. Finally, I show that under reasonable conditions the WTO’s ban of subsidies is efficient.

Keywords: Trade Agreement, Subsidy, Self-enforceability.

JEL Classifications: F10, F13.

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1 Introduction

Production subsidies traditionally plague trade negotiations. Baldwin (2006) reports that "negotiations at Cancun collapsed [...] in the absence of greater commitments by the developed countries to reduce agricultural subsidies and lower import barriers on agricultural products." In fact, by blocking international cooperation, subsidies might induce economic losses that exceed even their huge direct costs (see Anderson (2004)). Despite the obvious importance of subsidies for trade talks, trade theory has, until very recently, neglected their role in trade agreements. In a laudable attempt to fill this gap, Bagwell and Staiger (2006) provide the first formal analysis of the issue and show that efficient trade agreements target market access only and that, consequently, the de facto prohibition of production subsidies put in place by the current WTO legislation is inefficient. Starting point is the assumption that all cross-border inefficiencies travel through world prices. Therefore, as world prices are uniquely determined by market access, the authors can conclude that negotiating mutual market access is sufficient to resolve all cross-border inefficiencies and that any additional restriction — e.g., a prohibition of subsidies — can only generate inefficiencies.

The present paper shows that this line of argument fails when trade agreements are required to be self-enforceable. By imposing self-enforceability one acknowledges the fact that sovereign countries cannot be forced into trade agreements but instead join and respect them only if that seems beneficial from each individual country’s perspective. This assumption is truly restrictive, since, by the optimal tariff argument, large countries have incentives to defect from trade agreements by raising tariffs to distort world prices in their favour. Fortunately, however, countries refrain from doing so when the benefits of today’s defection come at the cost of tomorrow’s breakdown of international trade cooperation. Accordingly, the central precondition for trade agreements – formalized by the self-enforcement constraint – requires that the dynamic benefits of honoring an agreement outweigh defection temptation.

Under this self-enforcement requirement, optimal trade agreements generally target their members’ production subsidies for two fundamental reasons. First, the self-enforcement requirement does not constrain the individual country’s actions but, instead, the set of policies a trade negotiator can hope to implement successfully. Hence, the self-enforcement constraint must be addressed by the designers of trade agreements – e.g., by the WTO – and not by the individual countries. Formally, the self-enforcement constraints become part of the trade negotiators optimization problem. Second, the values of cooperation and defection – and hence the self-enforcement constraint – are affected by the output structures of countries and thus by their subsidies. The optimal trade agreement then fixes the mix of policies, which induces the least efficiency losses while still satisfying the self-enforcement constraint. This, in turn, implies that the optimal trade agreement requires and specifies a particular amount of production subsidies.

Generally speaking, the self enforcement requirement makes a case for trade agreements to address subsidies in their legal code. To exemplify this general principle and to evaluate the WTO subsidy rules in the light of self-enforceability,
the present paper develops a two-country two-good competitive general equilibrium model, where benevolent governments employ import tariffs and production subsidies in order to maximize their citizens’ welfare. Under the assumption that the self-enforcement constraints bind marginally, the model offers two new insights. First, optimal trade agreements ban production subsidies for import competing sectors. Second, and as a corollary, trade agreements that target market access only are necessarily inefficient.

The intuition for the optimal ban of production subsidies is the following. As production capacities require time-to-build, output reacts to prices changes only with a delay. If a country uses subsidies to create a continuous flow of import-competing output, it is less vulnerable to sudden import disruption than in absence of such intervention. Thereby, the subsidizing country mitigates the hardship of a potential breakdown of trade cooperation (which constitute the cost of defection) and thereby increases it’s own defection temptation. To compensate this increase in defection temptation, tariffs must be raised to inefficient levels.

By focussing on the role of production subsidies in trade agreements, the present paper closely connects to Bagwell and Staiger (2006). Very much in contrast to this earlier work, however, the present study provides a rationale for directly addressing subsidies in trade agreements and, in particular, for prohibiting them altogether. Thus, it may serve as a useful complement to Bagwell and Staiger (2006) in a balanced evaluation of the prevailing WTO legislation.

Since the early work of Yarbrough and Yarbrough (1986) and Dixit (1987), trade theory has generally understood trade agreements as a set of rules that encourage trade integration but which, in the absence of a supra-national executive authority, must be self-enforcing. A list of prominent contributions includes Devereux (1997), Maggi (1999), Bagwell and Staiger (2000), Park (2000), Ederington (2001), and Bond and Park (2002). Some of this work highlights the effects of adjustment costs of output for optimal trade agreements. In presence of such rigidities, a change of regime from cooperative to non-cooperative policies is generally prolonged and more costly. In this case, the consequences of a defection on trade agreements are typically harsher, which, in turn, makes a defection less attractive. Via this channel adjustment costs can generate endogenous gradualism in trade liberalization when output changes sluggishly (Staiger (1994) and Furusawa and Lai (1999)). At the same time, adjustment costs induce endogenous shifts in outside options and bargaining positions, which, in turn, can generate an aggravated version of a hold-up problem. McLaren (1997) presents such a scenario and concludes that trade can make a "small country worse off than it would have been if its trade partner did not exist". Crucial to these results is the assumption that the supply side of the world economy is entirely decentralized. The present paper departs from this assumption to analyze how and when governments should intervene in decentralized production in order to alleviate efficiency losses. Moreover, it argues that in some cases the relevant subsidy rules must be fixed in the legal code of trade agreements.

Finally, Ederington (2001) argues that trade agreements need not target domestic policies (such as subsidies), showing that "trade policy is the most efficient means of countering the temptation to defect" on trade agreement and efficient trade
agreements must use only tariffs to keep domestic policies at their individually optimal level. This statement, however only applies under a set of specific assumptions, which includes, in particular, the absence time-to-build requirements when production reacts to price changes instantaneously. The present paper, in contrast, builds precisely on the feature of sluggish production changes, which is both, arguably realistic and supported by empirical work (see Montgomery (1995) and Koeva (2000)).

The remainder of the paper is organized as follows. Section 2 introduces the general setup and formalizes the incentives to defect from trade agreements. Section 3 characterizes the optimal self-enforcing trade agreements and presents the main results of the paper. Section 4 concludes.

2 The Model

The model uses a well-established framework of trade agreements to formalize two arguments. First, it shows that production subsidies, via their impact on self-enforcement constraints, generally become a choice variable in the optimization problem of trade negotiators. Second, it is to lay out conditions, under which the WTO’s effective ban of production subsidies is, indeed, efficient.

Throughout the paper political motives for subsidies are abstracted from. This shall by no means imply that motives for the imposition of production subsidies, as incorporated in Bagwell and Staiger (2006), are generally irrelevant. Instead, it the assumption reflects the present paper's focus on an additional mechanism, which the previous authors neglect.

2.1 The Basic Setup

There are two countries, Home (no *) and Foreign (*). Individuals in both countries consume two final goods $X_1$ and $X_2$.

Preferences. Consumers are infinitely-lived and derive lifetime utility

$$U^{(*)} = \sum_{t \geq 0} \beta^t \ u\left(c_{1,t}^{(*)}, c_{2,t}^{(*)}\right)$$

(1)

where $c_{i,t}^{(*)}$ is consumption of good $X_i$ at time $t$. The momentary utility $u$ (referred to as "utility" in the following) is continuously differentiable and gives rise to constant and equal expenditure shares on the goods $X_1$ and $X_2$. Aggregate consumption in either country is thus

$$c_{1}^{(*)} = I^{(*)}/2p_{1}^{(*)}, \ c_{2}^{(*)} = I^{(*)}/2p_{2}^{(*)}$$

(2)

where $I^{(*)}$ are national incomes and $p_{i}^{(*)}$ are local prices. Here and whenever there is no risk of confusion, time indices are omitted.

Production. Firms in both countries produce the goods $X_i$ out of a set of factors that does not need to be further specified. Instead, aggregate output is
only assumed to react to domestic prices. Formally, national output of either
good is described by the functions

\[ x_i = H_i(p_1, p_2) \quad \text{and} \quad x_i^* = H_i^*(p_1^*, p_2^*) \]  

(3)

where \( p_i^* \) are local price. The functions \( H_i \) and \( H_i^* \) are assumed to be semi-
differentiable and to satisfy

\[
\frac{d}{dp_i} H_i \geq 0 \quad \frac{d}{dp_i} H_i^* \geq 0 \quad i = 1, 2. \\
\frac{d}{dp_k} H_i \leq 0 \quad \frac{d}{dp_k} H_i^* \leq 0 \quad i \neq k.
\]  

(4)

for the left and the right derivative. As the nominal price level is irrelevant, the
functions \( H_i \) and \( H_i^* \) are homogeneous of degree zero (HD0) so that

\[
\frac{d}{dp_i} H_i = \frac{d}{dp_i} H_i^* = 0 \\
\frac{d}{dp_k} H_i = \frac{d}{dp_k} H_i^* = 0
\]

(5)

There are differences in \( H_i \) and \( H_i^* \), which constitute a motive for international
specialization and trade. At any given relative price \( p = p_2/p_1 \), Home is assumed
to have a comparative advantage in \( X_1 \)-production and

\[
\frac{H_1(1, p)}{H_2(1, p)} > \frac{H_1^*(1, p)}{H_2^*(1, p)}
\]  

(6)

holds whenever the denominators are non-zero. Production of \( X_2 \) (\( X_1 \)) in Home
(Foreign) may drop to zero if the respective good prices are too low. In particular,
assume that there are \( \bar{\pi}, \underline{\pi} \in \mathbb{R} \) with

\[
\frac{p_2^*}{p_1^*} \geq \bar{\pi} \Leftrightarrow H_1^*(p_1^*, p_2^*) = 0 \quad \text{and} \quad \frac{p_2}{p_1} \leq \underline{\pi} \Leftrightarrow H_2(p_1, p_2) = 0
\]

Finally, assume that there is a positive range of prices that induce full interna-
tional specialization, i.e.

\[
\bar{\pi} > \underline{\pi}
\]  

(7)

**Integrated Economy.** In the integrated – or closed – economy, equilibrium
prices are determined by demand (2) and the technologies (3). Denoting the
relative price with \( p = p_2/p_1 = p_2 \), one has \( p = c_1/c_2 \) or

\[
p = \frac{H_1(1, p)}{H_2(1, p)}
\]  

(8)

This condition pins down the equilibrium price, which is, by (4), unique.

**Free Trade.** Whatever the underlying source of comparative advantage, it is
assumed to be strong enough to generate full specialization, given that trade and
production are undistorted. With (3) and (6) this amounts to assuming
\[ \exists \, p \in (\pi^*_2, \pi_1) \quad \text{s.t.} \quad \frac{H_1(1, p)}{H_2(1, p)} = p \]  
(9)
This restriction shall reflect that in an undistorted world, each country covers its consumption of at least one good entirely through imports. This feature emerges in a great variety of standard trade models (e.g. Dornbusch et al (1977), Dornbusch et al (1980), Acemoglu and Ventura (2002), and Romalis (2004)) and is, in this sense, a natural one.

2.2 Government Policies

Governments are assumed to charge import tariffs and to hand out production subsidies so as to maximize their citizens’ utility.

**Import Tariffs.** Gross ad valorem import tariffs are denoted by \( T \) and \( T^* \). Throughout the paper Home’s domestic price of good \( X_1 \) is taken as the numéraire. Thus, when denoting relative world prices with \( \pi \) the local prices in Home and Foreign are, respectively,

\[ p_1 = 1 \quad p_2 = Tp \quad \text{and} \quad p_1^* = T^* \quad p_2^* = p. \]  
(10)

Home’s national income includes tariff revenues and equals, in terms of domestic prices, \( I = x_1 + Tx_2 + (T - 1)p(c_2 - x_2) \) or with \( c_2 = I/(2Tp) \) from (2)

\[ I = \frac{2T(x_1 + px_2)}{T + 1}. \]  
(11)

Similarly, Foreign’s income is \( I^* = T^*x_1^* + px_2^* + (T^* - 1)(c_1^* - x_1^*) \) or

\[ I^* = \frac{2T^*(x_1^* + px_2^*)}{T^* + 1}. \]  
(12)

The trade balance \( p(c_2 - x_2) = c_1^* - x_1^* \) together with (2), (11), and (12) determines relative prices

\[ p = \frac{x_1(T^* + 1) + x_1^*T^*(T + 1)}{x_2T(T^* + 1) + x_2^*(T + 1)}. \]  
(13)

**Production Subsidies.** Governments subsidize local firms, handing out rewards per unit of output. Competitive firms perceive these subsidies as a corresponding increase in prices of final good. When per-unit subsidies to sector \( i \) are denoted by \( s_i - 1 \) and \( s^* - 1 \) (\( s_i^{(*)} = 1 \) indicating zero subsidies), aggregate output becomes with (3)

\[ x_1 = H_1(s_1, s_2Tp) \quad x_1^* = H_1^*(s_1^*T^*, s_2^*p) \]
\[ x_2 = H_2(s_1, s_2Tp) \quad x_2^* = H_2^*(s_1^*T^*, s_2^*p) \]

Notice that HD0 of \( H_i^{(*)} \) implies that the effect of subsidies on output – and hence on the world economy – depends only on the ratios \( s = s_2/s_1 \) and \( s^* = s_1^*/s_2^* \).
and one can write
\[
x_1 = H_1 (1, sTp) \quad x_1^* = H_1^* (s^*, p/T^*)
\]
\[
x_2 = H_2 (1, sTp) \quad x_2^* = H_2^* (s^*, p/T^*)
\]
(14)

The one important difference between subsidies and tariffs is that a subsidy alters the domestic production structure without affecting consumer prices (other than though world supply). Given that full specialization prevails under no subsidies, one can, by (4) and (14), identify a subsidy to the import-competing sector with the import-competing output it generates.

Observe finally that a negative subsidy (tax) on \( \mathcal{P}_1 \) is equivalent to a positive subsidy on \( \mathcal{P}_2 \) and vice versa. Therefore, the effective subsidies \( s \) and \( s^* \) can be chosen as a mix of a positive and a negative subsidies so that the total bill on national subsidies is zero.\(^1\) By focussing on these specific pairs of production subsidies and production taxes, the following analysis avoids considering transfers between government and consumers.

Market clearing under (2) and (3) requires that the world price \( p \) satisfies
\[
p = \frac{H_1 (1, sTp) + H_1^* (s^*, p/T^*)}{H_2 (1, p) + H_2^* (s^*, p/T^*)}.
\]
By (4) this equation uniquely determines the world price.

3 Trade Agreements

It is well known that trade agreements are efficient as long as relative (consumer and producer) prices equalize across countries (see Mayer (1981) or Dixit (1987)). With (10) and (14) these conditions become \( TT^* = 1 \) and \( s^{(*)} = \max \{1, 1/T^{(*)} \} \).

In this case, the tariffs, which may be non-zero, merely serve to transfer income from one country to another, without distorting neither supply nor demand.\(^2\)\(^3\)

The amount and the direction of these transfers generally depend on the countries’ gains from trade, outside options and bargaining power.

Now, the present paper starts from the premises that the first best trade agreement is not self-enforceable but countries would, instead, defect on this unconstrained optimum. Fundamental to the concept of self-enforceable trade agreements is the optimal tariff argument, according to which large countries benefit from unilaterally charging tariffs, thus distorting the terms of trade to their favor. In the context of trade agreements, this means that each (large) country has the incentive to defect, or cheat, on a trade agreement by charging tariffs unilaterally. Such a defection, however, typically comes at the cost of a breakdown of future cooperation. The self-enforcement constraint, now, requires that the

\(^1\)In the case of full international specialization, any production tax on the imported good is permissible.

\(^2\)Denote the world price of a zero-tariff, zero-subsidy world \( p \). Then the world price \( p \) sustains complete specialization as in the undistorted economy. Together with, undistorted demand this implies (8). Hence, \( p \) is the (unique) equilibrium world price.

\(^3\)Transfers can be realized through side-payments without involving negative net tariffs.
total value of respecting a trade agreement be higher than the value of defecting on it. This constraint will be discussed next.

3.1 Self Enforcement Constraint

In a self enforceable trade agreement each country needs to receive weakly more utility from respecting the agreement than from defecting on it. To formalize this concept, let $\bar{u}^{(s)}$ represent the utility under cooperation, $u^D(u^{D,(s)})$ under one-sided defection of Home (Foreign), and $u^N(u^{N,(s)})$ under uncooperative (or Nash) policies on both sides. With this notation and with total utility (1) the self-enforcement requirement can be written as:

$$
\sum_{s \geq t} \beta^{s-t} \bar{u}^{(s)} \geq u^D_{t,(s)} + \sum_{s \geq t+1} \beta^{s-t} u^N_{s,t}, \quad \forall \ t \geq 0
$$

This sequence of constraints collapses to just two constraints – one for Home and one for Foreign – when all parameters and equilibrium policies are time-invariant. The following analysis will be restricted to this case of time-invariant trade agreements. Hence, the self-enforcement constraint in period $t = 0$ stands for the self-enforcement constraint of all periods.

The impact of tariffs and subsidies on the different components of (15) is central to the following analysis. In particular, the net gains from defection crucially depend on the timing of actions, which will be specified in turn.

**The Timing.** Defection is the unilateral deviation to the individually optimal strategy in a one-shot game by one player at a given period, the defection period. This (off-equilibrium) action is unanticipated by other players, none of whom can react within the defection period. Consequently, the tariffs of a country which is defected on remain at levels set by the trade agreement. Similarly, private firms cannot react within the period of defection but need time to adapt their output. Since this principle applies to firms in both countries, the world output structure, summarized by the vector

$$X = (x_1, x_2, x^*_1, x^*_2) \in \mathbb{R}^4_+$$

is fixed and taken as given within the defection period. Production subsidies, as specified above, do not resolve this rigidity, since they, too, operate via price incentives for private firms.

A strictly positive reaction time of the economic agents who face unanticipated defection is an essential feature of the present paper’s setup. Indeed, this reaction time defines the length of the game’s periods. In standard trade models,
where the import tariffs are the only policies that governments set, this definition is unambiguous and clear. In the present model, however, where both import tariffs and output adapt to price changes, the question arises whether tariffs and outputs are equally fast to change. Notably, the time a country needs to adapt its import tariffs generally differs from the time required to bring into being a new industry in the country. On the micro level, firms tend to have extended adjustment periods to realize substantial increases in output plans. Capacity-building or start-up periods substantially limit private firms' short-term expansion of production when prices increase. Not surprisingly, empirical literature finds that adjustment of output is sluggish. Thus, Montgomery (1995) estimates firms' "construction periods average five to six quarters" and Koeva (2000) estimates "that the average construction lead time for new plants is around two years in most industries." Compared to time spans of these dimensions tariffs are set and adapted rather quickly.7

These facts are incorporated in the model through the assumption that governments can change the tariffs on a period by period basis while firms need more periods to increase output capacity.8 This assumption has important implications for the cause of events in case of a defection on trade agreements: after an unanticipated defection firms in both countries need more periods to expand production. Consequently, following a defection in period $t_0$, national output cannot expand for the periods $t \in \{t_0, t_0 + 1, \ldots, t_0 + M\}$ in both countries. From period $t_0 + M + 1$ onward, the output structures in both economies adapt according to production incentive (i.e., prices and subsidies). In sum, a defection is followed by two qualitatively different punishment phases: the first punishment phase (the medium term) during which uncooperative behavior is limited to tariffs while output is still at its cooperation levels and the second phase (the long term) characterized by uncooperative tariffs and subsidies with private firms producing accordingly.

With these assumptions about the timing, the self-enforcement constraints (15) take a particular structure, which is formalized with the following notation. First, let $\bar{\pi}^\ast$ denote the tariffs set by the trade agreement and $\bar{X}$ the equilibrium world output structure under tariffs and subsidies of the trade agreement. Further, write $T^{BR}(T^\ast, X)$ and $T^{BR, \ast}(T, X)$ for Home's and Foreign's unilaterally optimal tariff given the respective other country's tariffs and given the world output structure. Moreover, let $T^{N,\ast}(X)$ be the tariffs of the Nash Equilibrium of the tariff game at given output $X$ (i.e. $T^{N}(X) = T^{BR}(T^{BR, \ast}(T, X), X)$ and $T^{N,\ast}(X) = T^{BR, \ast}(T^{BR}(T^\ast, X), X)$).9 Finally, define Home's and Foreign's utilities as functions of tariffs $T$ and $T^\ast$ and output $X$ by

$$w^\ast(T, T^\ast, X) \equiv u(c^1_1(p(T, T^\ast, X), T, X), c^2_2(p(T, T^\ast, X), T, X))$$

Notice that, since consumption (2), incomes (11), (12), and prices (13), are

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7Even under a lengthy WTO dispute settlement process the standard procedure takes about a year (see WTO (2007)). A clear-cut defection in the game-theoretic sense is likely to generate much quicker reactions.

8The assumption that the reaction time for output is a multiple of the reaction time of tariffs is mild and irrelevant for the findings below.

9It is shown that these Nash strategies are unique.
smooth functions of $T^{(s)}$ and $X$ and since $u$ is continuously differentiable the function $w$ is continuously differentiable as well.

With a convenient notation thus established, the utilities under a trade agreement are $w^{(s)}(T, T^*, X)$ and Home’s and Foreign’s defection utilities are, respectively, $w(T^{BR}(T^*, X), T^*, X)$ and $w^{(s)}(T, T^{BR,*}(T, X), X)$. Utilities in the first punishment phases are $w^{(e)}(T^N(X), T^{N,*}(X), X)$, while those in the second punishment phases are denoted by $w^{N,(s)}$. Thus, Home’s and Foreign’s self-enforcement constraints (15) in a time-invariant trade agreement can be written as

$$w(\bar{T}, \bar{T}^*, \bar{X}) \geq (1 - \beta) w(T^{BR}(\bar{T}^*, \bar{X}), \bar{T}^*, \bar{X})...$$

$$+ (\beta - \beta^M) w(T^N(\bar{X}), T^{N,*}(\bar{X}), \bar{X}) + \beta^M w^N$$

$$w^*(\bar{T}, \bar{T}^*, \bar{X}) \geq (1 - \beta) w^*(\bar{T}, T^{BR,*}(\bar{T}, \bar{X}), \bar{X})...$$

$$+ (\beta - \beta^M) w^*(T^N(\bar{X}), T^{N,*}(\bar{X}), \bar{X}) + \beta^M w^{N,*}$$

For given output the best-response tariffs $T^{BR}(T^*, X)$ and $T^{BR,*}(T, X)$ generally involve lengthy expressions. It thus seems clear that incorporating subsidies and the reaction of decentralized firms in a game between governments can amount to a demanding task. Fortunately, however, Dixit (1987) establishes the existence of at least one Nash equilibrium in pure strategies, characterized by prohibitive tariffs on all sides. Applying this idea to the framework of the present model, the corresponding equilibrium consists of infinite tariffs $T = T^* = \infty$ and zero subsidies. The corresponding equilibrium is merely a replication of the respective autarkic economies. For the following analysis it is of little importance which of the (potentially many) equilibria prevails in the second – the long run – punishment phase. The long-run punishment utilities will be simply referred to as $w^N$ and $w^{N,*}$ without further specification. It is, however, important to notice that the potential long-run equilibria are neither affected by tariffs nor by the subsidies of the trade agreement and are, in particular, independent of output $\bar{X}$.

### 3.2 Best-Response Tariffs

As output changes require lengthy adjustment periods, the short- and medium-term policies can be calculated taken output structure as given. Thus, deflection tariffs and medium-term punishment tariffs are computed under constant output $X = (x_1, x_2, x_1^*, x_2^*)$. It is shown in the appendix that in this case the tariffs that maximize the citizens’ welfare are

$$T^{BR}(T^*, X) = \sqrt{x_1(T^* + 1) / (T^* x_1^* + 1) / [x_2(T^* + 1) / x_2^* + 1]}$$

### Footnotes

10 See Kennan and Riezman (1988) and Devereux (1997).

11 Subsidies are zero by the first fundamental welfare theorem.
These expressions show, first, that at constant $\pi$ the interior best-response tariffs are unique and, second, that these tariffs $T^{BR}$ and $T^{BR,*}$ have singularities at $x_1^* = 0$ and $x_2 = 0$, respectively. More precisely, in the limit of diminishing import competing production, the exporter’s best-response tariff grows unbounded and
\[
\lim_{x_1^* \to 0} \sqrt{x_1^* T^{BR}} \in (0, \infty) \quad \text{and} \quad \lim_{x_2 \to 0} \sqrt{x_2 T^{BR,*}} \in (0, \infty)
\]
holds. The expressions (18) and (19) imply further that the interior Nash Equilibrium of the tariff game at constant output is described by
\[
T^N(X) = \sqrt{\frac{x_1 + x_1^*}{x_1^*} \frac{x_2}{x_2 + x_2^*}} \quad \text{and} \quad T^{N,*}(X) = \sqrt{\frac{x_1^* + x_2}{x_2} \frac{x_1}{x_1^* + x_1}}
\]
which finally implies
\[
\lim_{x_1^* \to 0} \sqrt{x_1^* T^N} \in (0, \infty) \quad \text{and} \quad \lim_{x_2 \to 0} \sqrt{x_2 T^{N,*}} \in (0, \infty)
\]
and
\[
\lim_{x_1^* \to 0} \sqrt{x_1^*} \frac{dT^N}{dx_1} \in (-\infty, 0) \quad \text{and} \quad \lim_{x_2 \to 0} \sqrt{x_2} \frac{dT^{N,*}}{dx_2} \in (-\infty, 0)
\]
With the best response functions (18) and (19) the Nash tariffs (21) and the properties (20), (22), and (23) one can turn to the different components of the self-enforcement constraint. The first of those are the defection utilities.

### 3.3 Defection Utilities

In the limit $x_1^* \to 0$ ($x_2 \to 0$) Home’s (Foreign’s) defection utility exhibits a singularity, the degree of which is classified by the following

**Claim 1** If $T^*$ is constant, then Home’s defection utility satisfies
\[
\lim_{x_1^* \to 0} \sqrt{x_1^*} \frac{d}{dx_1^*} w(T^{BR}(T^*, X), T^*, X) \in (-\infty, 0)
\]
\[
\lim_{x_2 \to 0} \frac{d}{dx_2} w(T^{BR}(T^*, X), T^*, X) \in \mathbb{R}
\]
Similarly, if $T$ is constant Foreign’s defection utility satisfies
\[
\lim_{x_2 \to 0} \sqrt{x_2} \frac{d}{dx_2} w^*(T, T^{BR,*}(T, X), X) \in (-\infty, 0)
\]
\[
\lim_{x_1^* \to 0} \frac{d}{dx_1^*} w^*(T, T^{BR,*}(T, X), X) \in \mathbb{R}
\]

**Proof.** See Appendix. 

Property (24) and its counterpart (26) show that the first unit of a country’s import-competing output induces an unbounded loss of its trade partner’s defection utility. Thus, small amounts of $X_1$-output in Foreign are very effective
in depressing Home’s defection incentives. Intuitively, the first unit of Foreign’s $X_1$-production heavily reduces Home’s market power in its export market and thus strongly curbs Home’s one-shot gains from defection.

Compared to these effects, the impact of either country’s first unit of import-competing output on its own defection utility are shown to be small ((25) and (27)).

### 3.4 Punishment Utilities

As discussed above, punishment decomposes in two phases, reflecting the short and the long run after a defection. It has been pointed out already that the strategies in the second phase (the long run) are independent of $\bar{X}$ (the prevailing output structure under cooperation).

In the first punishment phase, however, the output structure of both economies is inherited form the trade agreement and hence the utilities shortly after a defection period do, indeed, depend on $\bar{X}$. The properties of these punishment utilities will be analyzed next. Similarly to the defection utilities, these punishment utilities exhibit singularities at $\bar{x}_1^* = 0$ and $\bar{x}_2 = 0$. The degree of these singularities is classified by the following

**Claim 2** Home’s punishment utility in the first phase satisfies

\[
\lim_{\bar{x}_2 \to 0} \sqrt[2]{\bar{x}_2} \frac{d}{d\bar{x}_2} w(T^N(X), T^{N,*}(X), X) = \infty
\]  

\[
\lim_{\bar{x}_1^* \to 0} \sqrt[\bar{x}_1^*]{} \frac{d}{d\bar{x}_1^*} w(T^N(X), T^{N,*}(X), X) \in \mathbb{R}
\]  

*Foreign’s punishment utility in the first phase satisfies*

\[
\lim_{\bar{x}_1^* \to 0} \sqrt[\bar{x}_1^*]{} \frac{d}{d\bar{x}_1^*} w^*(T^N(X), T^{N,*}(X), X) = \infty
\]  

\[
\lim_{\bar{x}_2 \to 0} \sqrt[2]{\bar{x}_2} \frac{d}{d\bar{x}_2} w^*(T^N(X), T^{N,*}(X), X) \in \mathbb{R}
\]

**Proof.** See Appendix. ■

Property (28) and its counterpart (30) show that the adverse impact of the uncooperative tariffs in the first punishment phase can be substantially reduced by small amounts of import competing production. Indeed, comparing these equations with (24) and (26) show that the positive effect of the first unit of import competing production on a country’s punishment utility is of higher degree than its adverse effect on the trade partner’s defection utility. This qualitative difference is quite intuitive. Thus, at $\bar{x}_1^* = 0$ Home can extract the maximum share of Foreign’s income by driving its export price to infinity and its export quantity to zero. According to (24) small positive amounts of $\bar{x}_1^*$ curb this ability, which, in terms of income change, hurts Home as much as it helps Foreign. In addition to this income effect, however, small amounts of the import competing
production raise Foreign’s consumption of $X_1$ from zero to strictly positive levels, which, by the standard properties of utility functions, adds infinite utility gains. This qualitative difference will be important for the design of the efficient trade agreement presented below. Together, (28) - (31) show that the first unit of import-competing production has a stronger impact on the country’s medium-term punishment utility than on that of the trade partner.

In sum, Claims 2 and 3 imply that small amounts of import-competing production have a huge effect on the defection utilities and the utilities of the first punishment phase and, therefore, on the respective defection temptation. These results suggest already that, via their potential to generate import competing output, subsidies may become important policies for trade agreements. Notice, however, that not all policies that affect incentives to defect automatically need to be addressed in trade agreements. Indeed, countries may find it individually optimal to set them at their globally optimal level, so that explicit legislation on these policies are obsolete.

With these preparatory observations, one can turn to the main objective of the paper, the characterization of optimal trade agreements.

3.5 Constrained Optimal Trade Agreements

This subsection characterizes the marginal impact of subsidies on the self-enforcement constraints and, thus, on trade agreements. In order to avoid complications, the focus will be on time-invariant trade agreements – and the term optimal trade agreement will be used for the Pareto optimal time-invariant trade self-enforceable trade agreement.

Exploiting the fact that, by (4) and (14), a subsidy can be identified by the import-competing output it generates, a trade agreement will formally be defined by the vector $(\bar{T}, \bar{T}^*, \bar{\xi}_2, \bar{\xi}_1^*)$. Now, an optimal trade agreement maximizes the weighted sum the countries’ welfare subject to both self-enforcement constraints, i.e., $(\bar{T}, \bar{T}^*, \bar{\xi}_2, \bar{\xi}_1^*)$ solves

$$\max_{\bar{T}^{(*)}, x_2, x_1^*} \sigma w(\bar{T}, \bar{T}^*, \bar{X}) + (1 - \sigma)w^*(\bar{T}, \bar{T}^*, \bar{X}) \quad s.t. \quad (16) \text{ and } (17)$$

where $\sigma \in (0, 1)$. The self-enforcement constraints of Home and Foreign will be rewritten as non-negativity restrictions on the functions $\Gamma^{(s)}$, which are defined, respectively, by

$$\Gamma = w(\bar{T}, \bar{T}^*, \bar{X}) - (1 - \beta)w(T^{BR}(\bar{T}^*, \bar{X}), \bar{T}^*, \bar{X})...$$
$$\quad - (\beta - \beta^M) w(T^N(\bar{X}), T^N^*(\bar{X}), \bar{X}) - \beta^M w^N$$

and

$$\Gamma^* = w^*(\bar{T}, \bar{T}^*, \bar{X}) - (1 - \beta)w^*(\bar{T}, T^{BR^*}(\bar{T}, \bar{X}), \bar{X})...$$
$$\quad - (\beta - \beta^M) w^*(T^N(\bar{X}), T^{N^*}(\bar{X}), \bar{X}) - \beta^M w^{N^*}$$

Combining these expression renders the Lagrangian of the optimization problem

$$L = \sigma w(\bar{T}, \bar{T}^*, \bar{X}) + (1 - \sigma)w^*(\bar{T}, \bar{T}^*, \bar{X}) + \lambda \Gamma + \lambda^* \Gamma^* + \nu \bar{\xi}_2 + \nu^* \bar{\xi}_1^*$$
where $\lambda \geq 0$ and $\lambda^* \geq 0$ stand for the Lagrange multipliers on Home’s and Foreign’s self-enforcement constraint and $\nu, \nu^* \geq 0$ are the Lagrange multipliers for the non-negativity constraints on output $x_2$ and $x_1^*$. It has been discussed at the start of this section that unconstrained efficient trade agreements implement a pair of tariffs satisfying $TT^* = 1$ and enforces zero subsidies ($x_1^* = \bar{x}_2 = 0$), so that full specialization prevails by assumption (7). As a point of reference, define now $T_\sigma$ as Home’s tariff of the unconstrained optimal trade agreement $((T_\sigma, 1/T_\sigma, 0, 0)$ solve the problem (32) in absence of (16) and (17)). Define further $\bar{\beta}_\sigma$ as the minimum discount factor for which none of the self-enforcement constraints binds under $\bar{x}_2 = \bar{x}_1^* = 0$, i.e.

$$\bar{\beta} \equiv \min \{ \beta \in [0,1] \mid (16), (17) \text{ holds under } (\bar{T}, \bar{T}^*, \bar{x}_2, \bar{x}_1^*) = (T_\sigma, 1/T_\sigma, 0, 0) \}$$

As the self-enforcement constraints are trivially satisfied in the limit $\beta \to 1$, $\bar{\beta} \in (0,1)$ holds. By construction of $\bar{\beta}$, the agreement $(\bar{T}, \bar{T}^*, \bar{x}_2, \bar{x}_1^*) = (T_\sigma, 1/T_\sigma, 0, 0)$ solves the constrained problem (32) for all $\beta \in [\bar{\beta}, 1]$. With these definitions one can formulate a central result in the following proposition.

**Proposition 1** There is an $\varepsilon > 0$ so that for all $\beta \in (\bar{\beta} - \varepsilon, \bar{\beta})$ the optimal trade agreement prohibits subsidies to import-competing sectors of the countries whose self-enforcement constraints bind.

**Proof.** Part I shows: a positive subsidy induces a discrete welfare loss. Recall first with (14) that $TT^* = 1$ and $s^{(*)} = \max\{1, 1/T^{(*)}\}$ imply full specialization. Now take $\delta_0 > 0$ and define the compact set

$$T = \left\{ (T, T^*) \mid T_\sigma + \delta_0 \geq T \geq T_\sigma, T_\sigma > T^* \geq T_\sigma, \quad s^{(*)} = \max\{1, 1/T^{(*)}\} \right\}.$$

Let $p$ be the equilibrium price under $T = T_\sigma$, $T^* = 1/T_\sigma$ and $s^{(*)} = \max\{1, 1/T^{(*)}\}$. By (9) marginal increases in tariffs preserve full specialization so that $T \neq \{(T_\sigma, 1/T_\sigma)\}$. Next, since $\beta < \bar{\beta}$, one can assume wlog $\lambda > 0$. For $(T, T^*) \in T$, the derivative of the Lagrangian (35) wrt. $x_2$ is

$$\frac{d\mathcal{L}}{dx_2} = \left[ (\sigma + \lambda) \frac{dw(T, T^*, \bar{X})}{dx_2} + (1 - \sigma + \lambda^*) \frac{dw^*(T, T^*, \bar{X})}{dx_2} \right] \ldots$$

$$- (1 - \beta) \left[ \lambda \frac{dw(T^{BR}(\bar{T}^*, \bar{X}), \bar{T}^*, \bar{X})}{dx_2} + \lambda^* \frac{dw^*(T, T^{BR}, \bar{T}^*, \bar{X})}{dx_2} \right] \ldots$$

$$- (\beta - \beta^M) \left[ \lambda \frac{dw^*(T^N(\bar{X}), T^{N,*}(\bar{X}), \bar{X})}{dx_1} + \lambda^* \frac{dw^*(T^N(\bar{X}), T^{N,*}(\bar{X}), \bar{X})}{dx_1} \right] + \nu_2$$

where tariffs $\bar{T}^{(*)}$ and $\bar{x}_1^*$ are constant, while $T^{BR,(*)}$ and $T^{BR,(*)}$ are from (18) - (19) and (21), and $d\bar{x}_1/d\bar{x}_2 = dH_1/dH_2 \in (-\infty, 0)$ by (5). Since $w$ is differentiable

$$\left| \frac{dw^{(*)}(T, T^*, \bar{X})}{dx_2} \right| < \infty$$
holds. Thus, by (5), (25), (26), (28), (31) and \( \lambda > 0 \)

\[
\lim_{x_2 \to \infty} \frac{dL}{dx_2} - \nu_2 = -\infty
\]  

(36)

holds. Consequently, there is a \( \hat{x}_2 > 0 \) so that \( dL/d\hat{x}_2 = 0 \) implies \( \hat{x}_2 \geq \bar{x}_2 \) for all \((T, T^*) \in T \) (\( T \) being compact). As unconstrained optimality requires \( \bar{x}_2 = 0 \), any trade agreement with \( \bar{x}_2 \geq 0 \) (and thus \( \hat{x}_2 \geq \bar{x}_2 \)) generates a welfare loss with a positive lower bound \( \Delta W' > 0 \).

**Part II shows: increases in tariffs relax self-enforcement constraints.**

Note that under full specialization \( c_1 = I/2 = x_1T/(T + 1) \) is increasing in \( T \) (and constant in \( T^* \)) and (with (13)) \( c_2 = I/(2pT) = x^*_2/(T + 1) \) is constant in \( T \) (and decreasing in \( T^* \)). Hence, with \( X_0 = (x_1, 0, x^*_2) \)

\[
\frac{dw(T, T^*, X_0)}{dT} \bigg|_{(T, T^*)=(T_\sigma, 1/T_\sigma)} > 0 \quad \text{and} \quad \frac{dw(T, T^*, X_0)}{dT^*} \bigg|_{(T, T^*)=(T_\sigma, 1/T_\sigma)} < 0
\]

Therefore, there are \( \Delta, \Delta^* > 0 \) so that

\[
\frac{d}{d\delta} w(T_\sigma + \delta \Delta, 1/T_\sigma + \delta \Delta^*, X_0) \bigg|_{\delta=0} = 0.
\]

This implies, together with the FOCs of the unconstrained optimal agreement,

\[
\frac{d}{dT} \left[ \sigma w(T, T^*, \bar{X}) + (1 - \sigma)w^*(\bar{T}, \bar{T}^*, \bar{X}) \right] \bigg|_{(T, T^*)=(T_\sigma, 1/T_\sigma)} = 0
\]

\[
\frac{d}{dT} \left[ \sigma w(T, T^*, \bar{X}) + (1 - \sigma)w^*(\bar{T}, \bar{T}^*, \bar{X}) \right] \bigg|_{(T, T^*)=(T_\sigma, 1/T_\sigma)} = 0
\]

that

\[
\frac{d}{d\delta} w^*(T_\sigma + \delta \Delta, 1/T_\sigma + \delta \Delta^*, X_0) \bigg|_{\delta=0} = 0
\]

Notice now that, when Home defects \((T \to \infty)\) its consumption is, according to (2) and (13) \( c_1 = I/2 = x_1 \) and \( c_2 = 2x^*_2/(T^* + 1) \). Hence, its defection utility \( w(T^{BR}(T^*, X_0), T^*, X_0) \) is strictly decreasing in \( T^* \). Together, this shows that \( \Gamma^{(\sigma)} \) from (33) and (33) satisfy

\[
\frac{d}{d\delta} \Gamma(T_\sigma + \delta \Delta, 1/T_\sigma + \delta \Delta^*, X_0) \bigg|_{\delta=0} = -\left(1 - \beta \right) \frac{dw(T^{BR}(T_\sigma, X_0), T^*, X_0)}{dT^*} \Delta > 0
\]

\[
\frac{d}{d\delta} \Gamma^*(T_\sigma + \delta \Delta, 1/T_\sigma + \delta \Delta^*, X_0) \bigg|_{\delta=0} = -\left(1 - \beta \right) \frac{dw^*(T, T^{BR}(T, X_0), X_0)}{dT} \Delta^* > 0
\]

This implies that there is a \( \bar{\delta} > 0 \) so that for \( \delta \in (0, \bar{\delta}) \) (i) \( \Gamma \) and \( \Gamma^* \) are strictly increasing in \( \delta \) (ii) functions of \( \delta \), \( \Gamma^{(\sigma)} \) are of order 1 at \( \delta = 0 \) and (iii) \((T_\sigma + \delta \Delta, 1/T_\sigma + \delta \Delta^*) \in T \) holds. Moreover, \( \Gamma^{(\sigma)} \) are decreasing in \( \beta \) and of the order 1 in \( \beta \) at \( \beta = \bar{\beta} \). Therefore,

\[
\forall \delta \in (0, \bar{\delta}) \exists \varepsilon > 0 \text{ so that } \left\{ T_\sigma + \delta \Delta, 1/T_\sigma + \delta \Delta^*, 0, 0 \right\} \Rightarrow \Gamma^{(\sigma)} \geq 0. \quad (37)
\]
Part III shows: tariffs are more efficient than subsidies in relaxing self-enforcement constraints. Finally, let $\Delta W(\delta)$ stand for the welfare loss of the agreement $(T_\sigma, 1/T_\sigma + \delta, 0, 0)$ relative to unconstrained optimal agreement $(T_\sigma, 1/T_\sigma, 0, 0)$. By the Envelope Theorem, $\Delta W(\delta)$ is increasing in $\delta$ of order 2 ($\Delta W(\delta) = O(\delta^2)$). Hence, there is a $\delta \in (0, \bar{\delta})$ so that $\Delta W' > \Delta W'(\bar{\delta})$. With the corresponding $\varepsilon > 0$ from (37) this proves that $(T_\sigma + \delta \Delta, 1/T_\sigma + \delta \Delta^*, 0, 0)$ $\bar{x}_2 = 0$ induces less losses than any $(T, T^*, \bar{x}_2, \bar{x}_2^*)$ with $\bar{x}_2 > 0$. ■

The proposition shows that output of import-competing sectors are optimally zero in all countries whose self-enforcement constraints bind marginally. The intuition of this result is the following. Subsidies to import-competing sectors affect output of some periods ahead, thereby generating domestic supply of the import good in the periods that follow potential defection. Doing so, they provide some degree of self-sufficiency, which mitigates the consequences of defection for the subsidizing country. By reducing its dynamic costs of defection, subsidies therefore increase this country’s temptation to defect and, hence introduce inefficiencies.\textsuperscript{12} This is the dominant effect of production subsidies and the reason why they undermine the trade agreement.

Proposition 1 proves that the above-mentioned effect dominates other effects of subsidies that could, potentially, stabilize trade agreements. Such effects actually exist. Thus, consider a situation where one country’s – say Home’s – self-enforcement constraint binds while Foreign’s does not. In this case, subsidizing import-competing production in Foreign reduces Home’s incentive to defect, as the effect in (24) dominated.\textsuperscript{13} At the same time, Foreign’s subsidies do not necessarily bring its own self-enforcement to bind. In such a situation, subsidies in Home harm the trade agreement (Proposition 1 applies) while Foreign’s subsidies relax Home’s self-enforcement constraint and enhance the efficiency of the agreement. Consequently, the optimal trade agreement would not implement general rules on subsidies but would, instead, tailor individual subsidy schemes according to individual defection incentives. (The practicability of such tailored agreements may of course be questioned.)

In sum, the proposition has shown that production subsidies play an important role for optimal trade agreements whereas Bagwell and Staiger (2006) argue that efficient trade agreements shall not address them. Both results point in opposite directions but, nevertheless, do not contradict each other. In particular, Proposition 1 does not show that optimal trade agreements require subsidy rules included in their legal text. More precisely, the proposition is consistent with a situation, where it is in each country’s individual interest to set subsidies to the (optimal) level that generates international specialization. In that case, subsidy rules would merely be superfluous in the legal code of trade agreements – Proposition 1 would be interesting but without practical value. Put differently, trade agreements that target tariffs (or, as in Bagwell and Staiger (2006), market access) would grant efficiency, while impeding subsidies did neither good nor

\textsuperscript{12}Sustainable cooperation under positive subsidies would necessarily entail higher import tariffs to reduce the defection incentives. These tariff increase would induce the efficiency loss relative to the optimal agreement.

\textsuperscript{13}Foreign is compensated for subsidizing by adjustments in tariff.
harm.\textsuperscript{14} It will be shown next that this is not the case.

3.6 GATT versus WTO Legislation

Bagwell and Staiger (2006) analyze the differences between the GATT and the WTO subsidy rules. They find that the current WTO legislation, which essentially impedes production subsidies, generates inefficiencies, whereas the former GATT rules, which regulated tariffs and market access only, were efficient. Proposition 1 conveys a different message without explicitly contradicting this finding. To directly compare the GATT and the WTO legislation within the current framework, the respective subsidy rules will be mapped in a reduced form to the current framework by the following definition.

Definition

\textit{(i)} WTO - type trade agreements fix \((\bar{T}, \bar{T}^*, \bar{x}_2, \bar{x}_1^*)\) with \(\bar{x}_2 = \bar{x}_1^* = 0\).

\textit{(ii)} GATT - type trade agreements fix a world price \(p\) and require

\textit{Homets (Foreigns) policies not to decrease net import prices \(p(1/p)\).}

These definitions heavily simplify the respective rules but capture their essence.

On the one hand, the WTO Agreement on Subsidies and Countervailing Measures allows member countries to challenge and enforce the removal of other members’ production subsidies. Since any increase in import competing output due to subsidies adversely affects the trade partner’s relative export prices (compare (13)) and as, moreover, subsidies to import-competing sectors increase the subsidizing country’s defection temptation (compare Claim 1 and 2) all subsidies will be challenged within the present framework. Consequently, WTO subsidy rules \textit{de facto} implement \(\bar{x}_2 = \bar{x}_1^* = 0\).\textsuperscript{15}

On the other hand, the GATT subsidy rules rely on the concept of market access. According to Bagwell and Staiger (2006) GATT rules are violated whenever a "government has bound a tariff in a GATT negotiation [...] and then subsequently alters its domestic policies in a way that diminishes the market access implied by that original tariff negotiation." At given domestic policies, Home’s (Foreign’s) access to Foreign’s (Home’s) market is \(M = x_1 - c_1 (M^* = x_2^* - c_2^*)\). Export market access is thus determined through (2), (3) and (11) for Home (2), (3) and (12) for Foreign) and therefore by its net-of tariff import price \(p(1/p)\). Therefore, an upper bound on import prices is a sufficient target of GATT-type trade agreements.\textsuperscript{16}

With this specification of the WTO and the GATT rules, a clear separation is possible between the finding of Proposition 1 and those of Bagwell and Staiger (2006).

**Proposition 2** Assume \(\beta \in (\beta - \varepsilon, \beta)\) with \(\varepsilon > 0\) from Proposition 1. Then, within the class of time-invariant, self-enforcing trade agreements, GATT-type agreements are inefficient.

\textsuperscript{14}This is the main message of Ederington (2001).

\textsuperscript{15}This argument holds for negligible costs of non-violation claims only.

\textsuperscript{16}In particular, a reduction of tariffs that does not decrease the trade partner’s market access is not considered a defection.
Proof. Assume the trade agreement of the GATT-type is efficient. Let its world price be \( \bar{\rho} \) and its underlying policies be denoted by \((\bar{T}, \bar{T}^*, \bar{x}_2, \bar{x}_1^*)\). Since \( \beta < \beta \) assume wlog that \( \lambda > 0 \) (\( \lambda \) from (35)), i.e., Home’s constraint binds. Then, Proposition 1 implies \( \bar{x}_2 = 0 \). Further, \( \lambda > 0 \) implies \( \Gamma = 0 \) (\( \Gamma \) from (33)). Observe that Home’s net import price (13) is continuously decreasing in \( \bar{T} \) and \( \bar{x}_2 \). Hence, for each small \( \delta > 0 \) there are \( \Delta_T \) and \( \Delta_x \) with \( 0 \leq \Delta_T, \Delta_x < \infty \) so that the policies \((\bar{T} - \delta \Delta_T, \bar{T}^*, \delta \Delta_x, \bar{x}_1^*)\) imply the equilibrium price \( \rho = \bar{\rho} \).

By construction, Home’s strategies \((\bar{T} - \delta \Delta_T, \delta \Delta_x)\) do not violate the trade agreement. At the same time, properties (24) and (28) imply that the value of defection is strictly increasing in \( \delta \). In fact,

\[
\frac{d}{d\delta} \left[w(T^{BR}(\bar{T}^*, \bar{X}), \bar{T}^*, \bar{X}) + \frac{\beta - \beta M}{1 - \beta} w(T^{N}(\bar{X}), T^{N*}(\bar{X}), \bar{X}) + \frac{\beta M}{1 - \beta} w^N\right]_{\delta=0} = +\infty
\]

holds. Thus, at \( \delta > 0 \) small enough, defecting on \((\bar{T} - \delta \Delta_T, \delta \Delta_x)\) renders Home strictly higher utility than defecting on \((\bar{T}, 0)\). Finally, since Home’s self-enforcement constraint binds \( \Gamma = 0 \) this implies that defection on \((\bar{T} - \delta \Delta_T, \delta \Delta_x)\) renders higher utility than respecting the agreement. This contradicts self-enforceability of the agreement and proves the statement. ■

Loosely speaking, under the GATT rule with presumed full specialization, a country constrained by self-enforcement may slip along the line defined by the self-enforcement constraint on the \((\bar{x}_2, \bar{T})\)-plane. Starting from \( \bar{x}_2 = 0 \), this move comes at negligible efficiency costs but allows the country, at \( \bar{x}_2 > 0 \), to reap large utility gains by defection. The proposition thus shows that it is important through which policy mix a country grants market access to its trade partner.

Proposition 2 marks a clear line between Proposition 1 and the main message of Bagwell and Staiger (2006). While the latter study argues that efficient trade agreements essentially target market access only, Proposition 2 shows that this limitation necessarily creates inefficiencies.\(^{17}\)

Obviously, a fixed level of market access is compatible with wide a range policy combinations – as long as there is more than one policy instrument. Naturally, countries pick their favorite mix among these policy combinations, for which the corresponding defection temptation must be checked by the rules of the trade agreement. But this requirement constitutes a tougher restriction than checking the defection incentives of the one optimal combination of policies only. Therefore, in the presence of a binding self-enforcement constraint, GATT-type trade agreements can enforce a more limited degree of cooperation than WTO-type trade agreements and are thus less efficient.

Interestingly, both findings, those of Bagwell and Staiger (2006) as well as Proposition 2, build on the fact that those agreements, which target market access leave countries a degree of freedom to choose the optimal policy mix that is compatible with the specified market access. However, while Bagwell and Staiger (2006) stress that leaving combination of policies to individual countries enhances global

\(^{17}\)The present paper’s mechanism can, of course, be applied to a richer setting, where subsidies positively affect domestic welfare when holding trade cooperation constant. In such a case, it is a quantitative question which of the antagonistic effect dominates.
welfare, the present paper points out that this freedom introduces additional defection incentives, which limit the degree of international cooperation.

The key element that generates these diametrically opposed results is the self-enforcement requirement. Absent in Bagwell and Staiger (2006), it drives the results of the present study. In the current setup, subsidies impact the countries’ defection incentives and hence the self-enforcement constraints via their effect on international production patterns. Since, moreover, the self-enforcement constraints need to be accounted for by the designers of the trade agreement (and not by individual countries) the subsidies have to be hard-wired into the legal code of trade agreements. These findings thus contradict those in Bagwell and Staiger (2006) and, at the same time, make a case for the current WTO subsidy rules.

3.7 Discussion of the Results

The main results of the paper, formulated in Proposition 1 and 2 describe subsidy rules of Pareto optimal, time-invariant and self-enforcing trade agreements. They have been derived under a number of assumptions that may appear restrictive and which deserve some words of justification. At the same time, the relevance of the findings and their link to the literature is briefly discussed. This discussion proceeds along the list of the model’s assumptions.

**Full Specialization.** Proposition 1 and 2 rely on full international specialization (assumption (9)), a potentially severe restriction. As discussed in connection with Proposition 1, key to the mechanism is that, under free trade, each country covers its consumption of at least one good entirely by imports, and attains a certain degree of self-sufficiency in that sector by subsidizing. This type of international specialization arises in most trade models under a variety of setups (see, e.g., Dornbusch et al (1977), Dornbusch et al (1980), Acemoglu and Ventura (2002), and Romalis (2004)). In this sense, the assumption of full specialization is a natural one.

**Marginally Binding Constraints.** The propositions require that the discount factor $\beta$ fall marginally short of the level that grants an undistorted world economy (i.e., $\beta \in (\bar{\beta} - \varepsilon, \bar{\beta})$). Thanks to successive rounds of trade negotiations over the past decades, average import tariffs have substantially dropped and were about 5% around the turn of the century (see e.g., Subramanian and Weil (2007)). While some countries set tariffs to zero, most charge positive but moderate import tariffs. From the point of view of self-enforcing trade agreements, these observations suggest that trade agreements are almost, but not quite, unconstrained. To put it differently, most self-enforcement bind but only marginally, meaning that Propositions 1 and 2 apply. – On a more general note, the beneficial effects of deliberate mutual economic dependence between countries seem to operate in other scenarios, as. economic history illustrates: according to standard interpretation of the European Coal and Steel Community, its foundation relied on a mutual dependence on trade that raised the dynamic costs of defective actions and aimed to foster the adherence to international cooperation. In this historic case, the pooling of steel, coal, and, to some extent, wheat
was meant to create a mutual dependence between the six Western European member nations and aimed to make cooperation indispensable (see Gillingham (1991)).

Preferences. A third important assumption concerns the preferences, which are assumed to give rise to constant expenditure shares. This assumption implies that best-response tariffs, and hence on utilities, react strongly on import-competing output. The set of goods with the key property – positive expenditure shares at unbounded prices – seems quite exclusive. Yet again, the findings should not be prematurely discounted. Thus, a pronounced dependence on imports and hence strong vulnerabilities to import shortages can arise from vertical international specialization and is likely to become more important under an increasingly unbundled international production chain. In particular, national dependence and vulnerability comes along with imports and exports of highly specialized and relation-specific intermediate goods, giving support to this third assumption.\(^{18}\) – Finally, and more generally, one has to keep in mind that the model’s strong assumptions are meant to exemplify the general principle that subsidies may play an important role in trade agreements. While changes to this specific setup can change the way that optimal subsidy-rules should be designed, but they will not affect the basic principle this paper aims to highlight.

Time to Build. The paper’s findings crucially depend on the substantial time-lag with which output reacts to price jumps. More specifically, industry-wide output cannot increase from zero to positive numbers at short notice. The empirical studies by Montgomery (1995) and Koeva (2000) have already been quoted in support of this assumption. It should moreover be stressed that this assumption does not concern small adjustment in firm output but reflects time-to-build of plants and the set-up of entire industries. The according time-lags are likely to be far larger than capacity-building on an intra-firm project basis and are thus assumed to exceed the time required for tariff-changes.

In sum, the key preconditions of this paper’s results, formulated in Propositions 1 and 2, give a rough but after all a not too exotic description of today’s world economy and its trading system. Consequently, the paper’s basic message is encouraging for the WTO subsidy legislation and its \textit{de facto} prohibition of production subsidies. It argues, in particular, that production subsidies for import-competing sectors can be an indispensable part of the legal code of optimal trade agreements.\(^{19}\)

Similar to Bagwell and Staiger (2006), the present paper addresses the role of subsidies in trade agreements. With its theoretical support for a prohibition of the WTO subsidy rules, however, the present paper strongly contradicts the results of Bagwell and Staiger (2006), who argue that impeding subsidies in trade agreements "may ultimately do more harm than good to the multilateral trading system." These diametrically oppose findings ultimately go back to the operating of the self-enforcement requirement. But there are more differences

\(^{18}\)Yi (2003) estimates high levels and growth rates of cross-border vertical integration.

\(^{19}\)This interpretation of the results obviously follows the standard assumption that trade agreements need to explicitly address the self-enforceable rules and policies and that cooperation is not automatically achieved as in the case of tacit collusion.
of importance. First, the key insights of Bagwell and Staiger (2006) rely on a degree of policy redundancy, which seems to be absent from the present paper’s the model. The present paper’s framework can, however, be directly translated to Bagwell and Staiger (2006): assuming that tariffs where tariffs have been fixed in a previous stage, the main questions question then revolves around the right mix of consumption taxes and subsidies. Second, the present paper does not model non-violation claims under the GATT or WTO rules in detail. This choice has been made due to the complexity of both models – the multi-stage game in the setup of Bagwell and Staiger (2006) and the repeated game in the present one. As argued at the beginning of subsection 3.6, however, the mapping of the respective subsidy rules to the present paper’s setting is reasonable. Third, and finally, the present paper abstracts entirely from all potential positive effects of production subsidies on national welfare. In particular, its model is stripped of the governments’ objectives for policy intervention (other than those arising under standard welfare functions) and the economic and political motives for non-trivial subsidies, which drive the results in the general setup of Bagwell and Staiger (2006), are assumed to be absent. I do not apologize for the bias, however. It should go without saying that the aim of the current study is not to disprove the validity of Bagwell and Staiger (2006) but, instead, to present a mechanism that may justify the inclusion of subsidies in trade agreements and, thereby, rationalizes the tough standing of the WTO subsidy rules. A balanced evaluation of the WTO rules will take both arguments into account and weigh their relative importance.

In a broader perspective, the paper stresses the role of mutual dependence in self-enforcing trade agreements. Whenever mutual dependence prevails in periods of potential punishment, it increases the threat of punishment for the deviant country and effectively increases room for cooperation. (See also Staiger (1994) and Devereux (1997) on this point.) On the base of this argument, it might generally be argued that policies such as export subsidies that have the potential to enhance mutual dependence should be embraced. In this sense, the production subsidy is merely a representative – albeit a natural one – for a broader range of policies that could be included in trade agreements.

4 Conclusion

This paper uses the framework of self-enforcing trade agreements to analyze the role of subsidies in trade agreements. A number of new results emerge. First, trade negotiators who aim to maximize national welfare must, in general, address subsidies in their agreements. Second, production subsidies are either large or zero. Consequently, when self-enforcement constraints bind only marginally, the optimal trade agreements ban production subsidies. This last scenario, which is argued to be a rough, but fair, description of today’s world economy, stands in stark contrast to the findings of Bagwell and Staiger (2006), who argue that the WTO’s ban on all subsidies is too much of a good thing and the "WTO subsidy
rules may ultimately do more harm than good to the multilateral trading system." In this sense the present paper’s findings contradict the results presented in Bagwell and Staiger (2006). At the same time, it makes a strong case for the WTO subsidy rules, providing a useful complement for a balanced evaluation of the prevailing WTO legislation.

A Appendix

Proof. Best Response Tariffs (18) and (19). Write the indirect utility of Home as \( v(\pi, I) \) with \( \pi = pT \) the price of Home’s import good. \( e(\pi, u) \) is the according expenditure function. Take derivatives wrt. \( \pi \) of the identity \( v(\pi, e(\pi, u)) = u \) to get \( v_\pi + v_\pi e_\pi = 0 \). (Subscripts stand for partial derivatives.) Combining this equation with the optimality condition for \( \pi \) leads to

\[
\frac{d}{dT} v(\pi, I) = v_I \frac{dI}{dT} + v_\pi \frac{d\pi}{dT} = v_I \left( \frac{dI}{dT} - e_\pi \frac{d\pi}{dT} \right) = 0
\]

Using Shephard’s Lemma \((e_\pi(\pi, u) = c_2)\), and \(c_2 = I/2\pi\) leads to

\[
\frac{d}{dT} \ln(I) = \frac{1}{2} \frac{d}{dT} \ln(\pi)
\]

(The optimality condition thus depends on income and prices only and the solution presented in Kennan and Riezman (1988) generalizes to the present scenario. The rest of the proof could thus be skipped but is provided here for completeness.) With \( p \) from (13), \( \pi = pT \), and \( I = (Tx_1 + \pi x_2)/\tau \) from (11) this gives

\[
\frac{x_1 - \pi x_2}{Tx_1 - \pi x_2} = \frac{T + 1}{2} \frac{d}{dT} \ln(\pi)
\]

With the shorthand \( \mu_1 = 1 + (x_1^*/x_1)T^*/\tau^* \) and \( \mu_2 = 1 + (x_2^*/x_2)(T+1)/(T(T^* + 1)) \) rewrite Home’s relative price \( \pi = pT \) as \( \pi = \mu_1 x_1/(\mu_2 x_2) \) and the optimality condition as

\[
\frac{\mu_2 - \mu_1}{T\mu_2 - \mu_1} = \frac{T + 1}{2} \frac{d}{dT} \ln(\pi)
\]

With the derivatives \( d\mu_1/dT = (\mu_1 - 1)/(T+1) \) and \( d\mu_2/dT = - (\mu_2 - 1)/(T(T + 1)) \) the optimality condition is 0 = 1 − 1/\( \mu_1 \) + 1/\( T \) (1 − 1/\( \mu_2 \)) − 2(\( \mu_2 - \mu_1 \))/(\( T \mu_2 - \mu_1 \)) or

\[
0 = \left[ \mu_2 (\mu_1 - 1) + \frac{1}{T} \mu_1 (\mu_2 - 1) \right] (T\mu_2 - \mu_1) - \mu_1 \mu_2 (\mu_2 - \mu_1)
\]

\[
= \left\{ \mu_2 + \frac{\mu_1}{T} \right\} [T(\mu_1 - 1)\mu_2 - (\mu_2 - 1)\mu_1]
\]
implying \( T(\mu_1 - 1)\mu_2 = (\mu_2 - 1)\mu_1 \). With the definitions of \( \mu_i \) this can be written as
\[
T^2 \left[ (T^* + 1) x_2 + x_2^* \right] - \frac{x_2^*}{x_1^*} \left( x_1 \frac{T^* + 1}{T^*} + x_1^* \right) = 0
\]
and proves the (18). Expression (19) follows by symmetry. ■

**Proof of Claim 1.** Define Home’s domestic relative prices as \( \pi(T^*, X) = p T^{BR}(T^*, X) \), where \( p \) is from (13) under \( T = T^{BR} \). Use (2) to compute the partial derivatives \( c_{1,1} = 1/2, c_{2,1} = 1/2\pi, c_{1,\pi} = 0, \) and \( c_{2,\pi} = -I/2\pi^2 \). At constant \( T^* \), this implies, using the optimality condition \( u_2 = \pi u_1 \),
\[
\frac{dw(T^{BR}, T^*, X)}{dx_1^*} = u_1(c_1, c_2) \left\{ \frac{dI}{dx_1^*} - \frac{I}{2} \frac{d\ln(\pi)}{dx_1^*} \right\} = u_2(c_1, c_2) \left\{ \frac{dI}{dx_1^*} - \frac{I}{2} \frac{d\ln(\pi)}{dx_1^*} \right\}
\]
(A1)

Note that in (A1) \( T^{BR} \) can be treated as a constant (Envelope Theorem). The superscripts \( T^{BR} \) will be omitted in the following.

(i) **Show (24).** Use (11) to derive \( dI/dx_1^* = 2x_2/(T+1) (d\pi/dx_1^*) \). Hence (A1) renders
\[
\frac{dw(T^{BR}, T^*, X)}{dx_1^*} = u_1(c_1, c_2) \left\{ \frac{2x_2}{T+1} \pi - \frac{I}{2} \right\} \left( \frac{d}{dx_1^*} \ln(\pi) \right)
\]

Now use (13) to get
\[
\frac{d\ln(\pi)}{dx_1^*} = \frac{T^*(T+1)}{x_1(T^* + 1) + x_1^* T^*(T+1)} + \frac{\bar{A}*(T+1)/T}{x_2(T^* + 1) + x_2^* (T+1)/T}
\]
where \( \bar{A}^* \) is Foreign’s marginal rate of transformation between \( X_1 \) and \( X_2 \). By (5) \( \bar{A}^* \) is finite. Combining these expressions implies with (11), (13), and (20)
\[
\lim_{x_1^* \to 0} \frac{1}{T^{BR}} \frac{dw(T^{BR}, T^*, X)}{dx_1^*} = \left( \lim_{x_1^* \to 0} u_1(c_1, c_2) \right) \left\{ -x_1 \right\} \left( \frac{T^*}{x_1(T^* + 1)} \right)
\]
The limits \( \lim_{x_1^* \to 0} c_1 > 0 \) imply \( \lim_{x_1^* \to 0} u_1(c_1, c_2) \in (0, \infty) \). This proves (24).

(ii) **Show (25).** Check with (13) and \( T = \sqrt{x_1(T^* + 1)/(x_1^* T^*)} + 1 \) from (18)
\[
\lim_{x_2 \to 0} \pi = \frac{T x_1(T^* + 1) + x_1^* T^* (T+1)}{x_2^*(T+1)}
\]
\[
\lim_{x_2 \to 0} \frac{d\ln(\pi)}{dx_2} = \frac{-\bar{A}(T^* + 1)}{x_1(T^* + 1) + x_1^* T^*(T+1)} - \frac{T(T^* + 1)}{x_2^*(T+1)}
\]
where \( \bar{A} \) is Home’s equivalent to \( \bar{A}^* \). Both expressions are bounded for all \( x_1^* \geq 0 \).
Thus, (11) implies
\[
\lim_{x_2 \to 0} I = \frac{2T}{T+1} x_1 \quad \lim_{x_2 \to 0} \frac{dI}{dx_2} = \frac{2}{T+1} (-\bar{A}T + \pi).
\]
Again, both expressions are bounded for all \( x_1^* \geq 0 \). Combining these expressions
Further, (11) and (21) lead to
\[
\lim_{x_2 \to 0} \frac{d\mu(T^{BR}, T^*, X)}{dx_2} = \left( \lim_{x_2 \to 0} u_1(c_1, c_2) \right) \left\{ \frac{2 - \bar{A} + \pi}{T + 1} \right\} \left\{ \frac{T}{T + 1} x_1 \frac{d\ln(\pi)}{dx_2} \right\}
\]

By the reasoning above the expression in the slanted brackets is bounded for all \(x_1^* \geq 0\). Finally, since \(c_1 = I/2\) and \(c_1 = I/2\pi\) are both positive at \(x_2 \to 0\), \(\lim_{x_2 \to 0} u_1(c_1, c_2)\) is finite. This proves (25).

(iii) Show (26) and (27). They follow from (i) and (ii) by symmetry.

Proof of Claim 2. Define Home’s domestic relative price as \(\pi(T^N, T^{N,*}, X) = pT^N\) with \(p\) from (13) under \(T^{*} = T^{N,(*)}\). Equation (A1) still applies. By the Envelope Theorem \(T^N\) can be treated as a constant. The superscripts \(N\) will be omitted in the following.

(i) Show (28). If \(x_1^* > 0\) use (13) and (21) to compute
\[
\lim_{x_2 \to 0} \pi = \frac{x_1 + x_1^*(T + 1)}{x_2^*(T + 1)} \lim_{x_2 \to 0} T^*
\]
\[
\lim_{x_2 \to 0} \frac{d\ln(\pi)}{dx_2} = \frac{-\bar{A}(T^* + 1)}{x_1(T^* + 1) + x_1^*T^*(T + 1)} - \frac{T(T^* + 1)}{x_2^*(T + 1)} \ldots + \left( \frac{x_1 + x_1^*(T + 1)}{x_1(T^* + 1) + x_1^*T^*(T + 1)} - \frac{T x_2}{x_2^*(T + 1)} \right) \lim_{x_2 \to 0} \frac{dT^*}{dx_2}
\]

where \(\bar{A}^*\) is Foreign’s finite marginal rate of transformation between \(X_1\) and \(X_2\).

With (21) and (23) the second expression implies
\[
\lim_{x_2 \to 0} x_2 \frac{d\ln(\pi)}{dx_2} \in (-\infty, 0)
\]

(A2)

Further, (11) and (21) lead to
\[
\lim_{x_2 \to 0} I = \frac{2T x_1}{T + 1}
\]
\[
\lim_{x_2 \to 0} \frac{dI}{dx_2} = \frac{2(\pi - \bar{A}T)}{T + 1} + \frac{2}{T + 1} \lim_{x_2 \to 0} \left( \frac{x_2}{\pi} \frac{d\ln(\pi)}{dx_2} \right) = \frac{2(\pi - \bar{A}T)}{T + 1}
\]

Combining these expressions with (A1) leads to
\[
\lim_{x_2 \to 0} \sqrt{x_2} \frac{d\mu(T^N, T^{N,*}, X)}{dx_2} = \left( \lim_{x_2 \to 0} \frac{u_2(c_1, c_2)}{\sqrt{x_2} \pi} \right) \left\{ \frac{T x_1}{x_2^*(T + 1)} \lim_{x_2 \to 0} \frac{d\ln(\pi)}{dx_2} \right\}
\]

With (A2) the term in the slanted brackets is finite and positive; by (13) and (21) \(\lim_{x_2 \to 0} \sqrt{x_2} \pi = \frac{2}{\pi} \pi = \infty\) is finite. Since finally, \(\lim_{x_2 \to 0} c_1 = \lim_{x_2 \to 0} I/2 > 0\) and \(\lim_{x_2 \to 0} c_2 = \lim_{x_2 \to 0} I/2T = 0\) leads to \(\lim_{x_2 \to 0} \frac{u_2(c_1, c_2)}{\sqrt{x_2} \pi} = \infty\), which shows (28) for \(x_1^* > 0\).

For the case \(x_1^* = 0\) take (11), (13) and (21) to check
\[
\lim_{x_1^* \to 0} \pi = \frac{x_1(T^* + 1)}{x_2(T^* + 1) + x_2^*} \quad \text{and} \quad \lim_{x_1^* \to 0} I = 2x_1
\]

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Hence,
\[
\frac{dI}{dx_2} = -2\bar{A}
\]
\[
\frac{d\ln(\pi)}{dx_2} = -\frac{\bar{A}}{x_1} - \frac{T^* + 1}{x_2(T^* + 1) + x_2^*} + \left( \frac{1}{T^* + 1} - \frac{x_2}{x_2(T^* + 1) + x_2^*} \right) \frac{dT^{N,*}}{dx_2}
\]

With (21) this last expression shows that (A2) holds in the case \(x_1^* = 0\) as well. Thus
\[
\lim_{x_2 \to 0} \sqrt{x_2} \frac{dv(T^N, T^{N,*}, X)}{dx_2} = \left( \lim_{x_2 \to 0} \frac{u_2(c_1, c_2)}{\sqrt{x_2} \pi} \right) \left\{ -x_1 \lim_{x_2 \to 0} \frac{d\ln(\pi)}{dx_2} \right\}
\]
By (13) and (22) \(\lim_{x_2 \to 0} \sqrt{x_2} \pi\) is finite. Finally, observe that \(\lim_{x_2 \to 0} c_1 = x_1/2 > 0\) while \(\lim_{x_2 \to 0} c_2 = \lim_{x_2 \to 0} x_1/2\pi = 0\). Thus, \(\lim_{x_2 \to 0} u_2(c_1, c_2) = \infty\). This shows (28).

(ii) Show (29). If \(x_2 > 0\) equations (11) and (13) imply
\[
\lim_{x_1 \to 0} \frac{x_1(T^* + 1)}{x_2(T^* + 1) + x_2^*} = \frac{x_1}{2}
\quad \text{and} \quad \lim_{x_1 \to 0} I = 2x_1
\]
and
\[
\lim_{x_1 \to 0} \frac{d\ln(\pi)}{dx_1} = \left( \lim_{x_1 \to 0} \frac{T^*}{x_1(T^* + 1)} + \frac{A^*}{x_2(T^* + 1) + x_2^*} \right) \ldots + \left( \frac{1}{T^* + 1} - \frac{x_2}{x_2(T^* + 1) + x_2^*} \right) \frac{dT^*}{dx_1^*}
\]
Now, (21) implies \(dT^*/dx_1^* < \infty\) for \(x_2 > 0\) so that
\[
\lim_{x_1 \to 0} \sqrt{x_1} \frac{d\ln(\pi)}{dx_1} = \left( \lim_{x_1 \to 0} \sqrt{x_1} T^* \right) \frac{T^*}{x_1(T^* + 1)} \in (0, \infty)
\]
With
\[
\lim_{x_1 \to 0} \frac{dI}{dx_1} = 2 \lim_{x_1 \to 0} \frac{x_2}{x_1} \frac{d\pi}{dx_1} \in (0, \infty)
\]
and (A1) this leads to
\[
\lim_{x_1 \to 0} \sqrt{x_1} \frac{dv(T^N, T^{N,*}, X)}{dx_1} = \left( \lim_{x_1 \to 0} u_1(c_1, c_2) \right) \left\{ \left( 2 \lim_{x_1 \to 0} \frac{x_2 \pi}{T} - \frac{x_1}{2} \right) \frac{d\ln(\pi)}{dx_1^*} \right\}
\]
The expression in the slanted brackets is finite. Finally, \(\lim_{x_1 \to 0} c_1 = x_1 > 0\) shows \(\lim_{x_1 \to 0} u_1(c_1, c_2) \geq 0\), which proves (29) for \(x_2 > 0\).

For the case \(x_2 = 0\) take (11), (13) and (21) to check
\[
\lim_{x_2 \to 0} c_1 = \lim_{x_2 \to 0} I/2 = T/(T + 1) \quad \text{and} \quad \lim_{x_2 \to 0} c_2 = I/(2\pi) = 0
\]

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Hence,

\[
\lim_{x_1^* \to 0} \frac{dw(T^N, T^{N,*}, X)}{dx_1^*} = u_1 \left( \frac{T}{T+1}, 0 \right) \lim_{x_1^* \to 0} \left\{ \frac{\sqrt{x_1^*}}{(T+1)^2} \frac{dT}{dx_1^*} \right\}
\]

By (21) and (23) the term in slanted brackets is finite. As \( u_1 (c_1, c_2) \) is bounded for positive \( c_1 \) this proves (29).

(iii) **Show (30) and (31).** They follow from (i) and (ii) by symmetry.

**References**


