Overreporting Oil Reserves*

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This Draft: September 2009

Abstract

An increasing number of oil market experts argues that OPEC members substantially overstate their oil reserves. While the implied outlooks for the world economy are disastrous, the incentives for overreporting remain unclear. This paper shows that oil exporting countries may rationally overreport to raise expected future supply, discourage oil-substituting R&D, and hence improve their future market conditions. Yet, credible overreporting must be backed by observable actions and therefore induces costly distortions of supply. Surprisingly, these distortions can cancel with other distortions, arising as reaction to technological change regardless of information asymmetries. In this case, overreporting is rational, credible, and cheap.

Keywords: Exhaustible Resource, Substitution Technology, Signaling

JEL Classifications: F10, F16, D82.

*I would like to thank Raphael Auer, Martin Brown, Eddie Dekel, Andreas Fischer, Elhanan Helpman, Werner Herrmann, Eugene Kandel, Marcel Peter, Andrea Pescatori, Rick van der Ploeg, Hosny Zoabi, and the participants at Tel Aviv University and Hebrew University for helpful comments. All remaining errors are mine.

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1 Introduction

The recent hike of crude oil prices has revived old fears about energy security. In addition to political and geological imponderability, a small but growing number of market pundits points out that overreporting of crude oil reserves is one, possibly dramatic, source of uncertainty.[1] Thus, Bentley (2002) observes that "Saudi Arabia and Iran may well have significantly smaller reserves than listed" publicly. The Economist (2006) reports warnings that suppliers as "Kuwait might have only half of the [...] oil reserves" officially reported. The Energy Watch Group, a Germany-based think tank claims that, when applying "the same criteria which are common practice with western companies, ...[Saudi Arabia's] statement of proven reserves should be devalued by 50%" (Energy Watch Group (2007)). As a consequence, the privately funded UK Industry Taskforce on Peak Oil & Energy Security reckons that "the world's supposedly proved reserves of 1,200 billion barrels are probably overstated by at least 300 billion barrels" (ITPOES (2008)). The International Energy Agency (IEA) expresses doubts "about the reliability of official MENA [Middle Eastern and North African] reserves estimates, which have not been audited by independent auditors" for decades (IEA (2005)) and The Wall Street Journal (2008) reports that "[f]uture crude oil supplies could be far tighter than previously thought."

In combination these sources suggest the following picture: opaque national oil companies hold private information on major parts of world crude oil reserves, the amount of which they grossly overreport to the rest of the world. The economic consequences of an abrupt correction of expected future supply could indeed be dramatic. The actual motives of overreporting, however, remain unclear.[2] To the economist, unfamiliar with the technical details of the oil market but trained to handle rational expectations, the following type of questions occurs: Why would oil suppliers overreport their reserves? When would this be credible and do the necessary conditions hold? After all, shouldn't oil suppliers underreport reserves, since anticipated shortages raise current prices?

The present paper addresses these questions. It shows that in a standard setting of exhaustable resources, incentives to overreport emerge naturally through the following mechanism. Market participants rationally decide to engage in resource-saving R&D whenever expected future supply of conventional oil is sufficiently low. Oil-exporters, in turn, overreport their reserves to raise the expected future oil supply, discourage resource-saving R&D and thereby improve their future market conditions.

[1] Many definitions of *crude oil reserves* exist. Quotations refer to the standard definitions *proven* and *proven and probable* reserves (conventional crude oil in place with 90% and 50% probability, respectively).

[2] It is sometimes argued that OPEC members overreport reserves to increase their allotted production quotas. In fact, OPEC's quota system was formally established in March 1983, around the time when many OPEC members substantially increased their reported oil reserves (see Campbell and Laherrèr (1998) and Bentley (2002)). If a strategic quota game was the obvious and sole reason, however, market participants would correct such spurious reporting. In absence of this natural correction the central question recurs whether oil exporters have motives to strategically deceive the markets.

The two assumptions underlying this mechanism are common in the literature of exhaustible resource: first, technological change is the outcome of directed and costly R&D. This assumption implies that high expected future oil supply depresses resource-saving R&D. Second, oil supply is not competitive. Consequently, oil suppliers internalize the impact of their supply on the market conditions and can actively manipulate them.

The paper's argument is framed with a signalling game – a standard and suitable tool to analyze information rents. An oil-supplier holds private information about the realization of its total stock of oil and decides how to allocate it between two periods. An oil-consumer, on the other side, decides whether or not to invest in time- and resource-intensive oil-substituting R&D. Now, the oil-supplier is said to successfully misreport (*i.e.*, over- or underreport) if (i) the observable signal – current supply – is uninformative about remaining reserves and if (ii) the resulting pooling equilibrium generates strictly higher benefits than the respective full information equilibrium. By this definition, misreporting and beneficial pooling are one and the same thing.

The requirement that overreporting be credible means that overreporting must be backed by observable actions and is, in generally, costly. More precisely, observable contemporaneous supply must correspond to officially reported reserves and hence does not coincide with supply under full information. The thus induced costs of overreporting will generally limit the supplier's willingness to overreport.

The present paper shows, however, that the cost of overreporting can be negligible and indeed even negative for a wide parameter range. The intuition for this surprising result is the following. Regardless of information asymmetries, the key problem of the oil-supplier is how to allocate the resource over time. In the absence of technological change, it is dynamically optimal to smooth supply across periods. Under the threat of resource-saving R&D, however, the smooth supply rule is no longer optimal. Instead, the supplier decreases current and increases future supply to discourage oil-substituting R&D. This effect is present under full information. Now, overreporting under private information generally requires an increase of current supply and hence brings the overreporting country closer to its unconstrained optimal – the smooth supply rule. Consequently, the distortions from overreporting eliminate the earlier distortions due to technological change, in which case the costs of overreporting can be said to be negative.

The paper thus predicts that oil exporters tend to overreport if, first, substitution R&D reacts significantly to expected future oil supply, second, if oil supply is not competitive and finally, if the costs of overreporting are limited. The last requirement, however, is no meaningful qualitative criterion since, as argued above, theory is consistent with zero or even negative costs of signaling. Hence, the attention rests on preconditions one and two. Concerning the first, empirical work shows that substitution R&D has indeed been responsive to oil prices (see Newell et al (1999) and Popp (2002)). Figure 1 illustrates

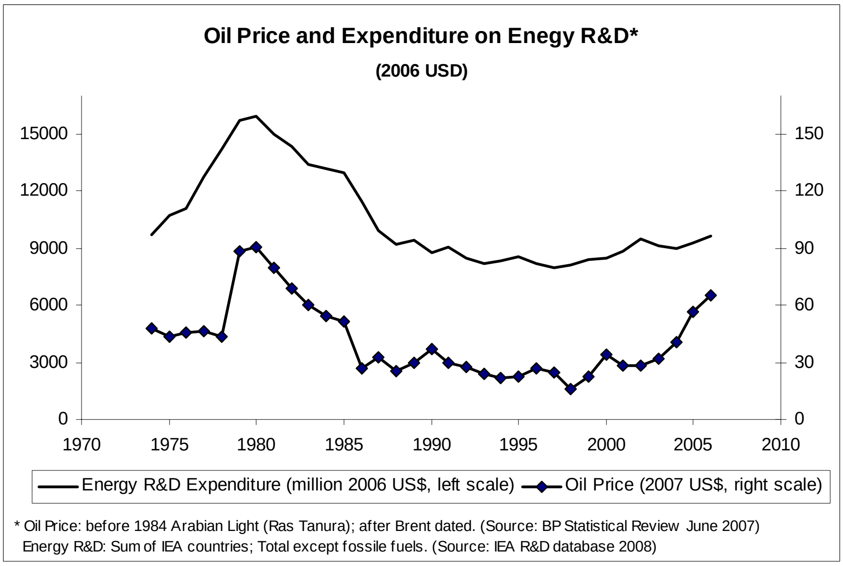

Figure 1: Oil Price and Expenditure on Energy R&D*

this relation in a suggestive way, plotting of crude oil prices and total R&D expenditure on non-oil energy sources in IEA member countries between 1973 and 2006.[3] The second requirement of non-competitive oil markets seems intriguing. Contrary to conventional wisdom, however, empirical evidence on OPEC's market power is mixed. Nevertheless, some recent studies make the case that the OPEC successfully sacrificed supply following the counter oil shock of 1986 (*e.g.* Smith (2003)). In this case, a rough and tentative application of the model cannot refute that OPEC member states overreport their crude oil reserves. This general finding calls for further quantitative research of the matter.

The paper contributes to the rich literature on the economics of exhaustible resources. Since the seminal article by Hotelling (1931), this literature has highlighted the suppliers' market power. Not surprisingly, the oil shocks of the 1970s intensified the focus on the effects of monopoly power and cartel formation on aggregate supply (see Stiglitz (1976), Salant (1976), Pindyck (1978), Ulph and Folie (1980), and Gaudet and Moreaux (1990)), paralleled by empirical work on the collusive behavior of world oil suppliers (e.g. Griffin (1985), for recent contributions see Smith (2003), Almoguera and Herrera (2007), Lin (2007), and the references therein). About the time of the first oil shock Dasgupta and Heal (1974) sparked a line of research on substitution of exhaustible resources analyzing either the exploration (see Burt and Cummings (1970), Arrow and Chang (1982) and Quyen (1988)) or the closely related directed technical change for substitution (e.g., Davidson (1978) and Deshmukh and Pliska (1983)). The present paper rests on both prominent streams of the literature – the one concerned with monopolistic power, the other with

[3] Throughout the period, contemporaneous prices comove with expected future prices (see Lynch (2002)). This latter is the key statistic for R&D expenditure according to this paper's argument.

substitution efforts – as basic elements to analyze the motives of the misreporting of natural resource reserves. It also connects to earlier work on private information in natural resource markets from Gaudet et al (1995) and Osmundsen (1998), who analyze information asymmetries about the reserves of natural resources and show that firms have incentives to underreport reserves: given that extraction costs are higher for lower reserves, underreporting of reserves means overreporting of costs, which, finally, saves taxes on profits. In contrast to these earlier studies, the present paper addresses private information of sellers vis-à-vis the buyers and draws entirely different conclusions. Recently, Gerlagh and Liski (2007) analyze a setup where asymmetric information about natural resource reserves impacts the consumers' decision to invest in substitution technologies. In a rich dynamic setting, their study focuses on the buyer's exposure to uncertainty of supply. The present paper, instead, concentrates in particular on the seller's potential and limitations to conceal or to signal its type.

The remainder of the paper is organized as follows. Section 2 outlines the model economy, describes the action of economic agents and sets up the strategic game involving the governments' decisions. Sections 3 and 4 solve the strategic game under full information and under asymmetric information, respectively, and discuss the main results. Finally, Section 5 concludes.

2 The Model

To analyze the incentives of an oil-exporter to misreport reserves, this section develops a two-country model with international trade in two goods, one of which represents oil or a natural resource in general. The setup reflects the dichotomy between oil exporters and importers and captures consumption smoothing motives in oil exporting countries that affect the exporter's optimal intertemporal supply rule.

2.1 General Setup

The world economy consists of two countries O (*) and W (no *), each of which are populated by individuals of mass one. These countries engage in cross-border trade in two consumption goods within each of two periods, $t = 1, 2$. After the second period, the world ends. The two periods are meant to represent long time intervals, defined by the time it takes to develop a technology with which to substitute the natural resource.[4]

Production. In the periods $t = 1, 2$, country W produces y_t units of a perishable consumption good Y. Country O is endowed with N^* units of a second consumption

[4] In a recent study Lovins et al (2005) reckon that US oil demand projected for 2025 can be cut to half by the use of substitutes and energy-saving technologies. In this sense, periods are "long".

good N. Good N represents a natural resource and N^* is country O's total reserve of it. Hence, when supplying $n_1^* \geq 0$ units of it in period one, country O's maximal supply in period two is $N^* - n_1^*$. The mining costs of N are negligible, yet once N is mined, it becomes perishable. This assumption reflects prohibitive storage costs.[5] Before period one, country O's total reserves of N are uncertain and distributed according to

$$N^* = \begin{cases} \bar{N} & \text{with probability } \pi \\ \xi\bar{N} & \text{with probability } 1-\pi \end{cases} \tag{1}$$

with $\xi \in (0, 1)$. Notice that total reserves do not depend on prices.

All of the model's parameters are common knowledge except for the realization of N^*, the information of which is private to country O. More specifically, uncertainty about total reserves is costlessly resolved to the government of country O, which controls world supply of N in both periods, supplying n_t^* units in $t = 1, 2$ under the constraints $n_t^* \geq 0$ and $n_1^* + n_2^* \leq N^*$.

Preferences. The individuals' preferences are reflected by total utility

$$U^{(*)} = \sum_{t=1,2} u\left(c_{n,t}^{(*)}, c_{y,t}^{(*)}\right) \tag{2}$$

where $c_{x,t} \geq 0$ are consumed quantities of good $x = n, y$ at time $t = 1, 2$. The sub-utility takes the form

$$u(c_n, c_y) = \ln(c_n + 1) + c_y \tag{3}$$

This sub-utility has a number of properties, which substantially simplify the analysis of the model. First, the quasi-linear form implies that income is transferable across periods[6]. This feature generates particularly clean solutions for both countries' trade-off between foregone consumption in period one and their revenues in the period two. In particular, the logarithmic specification including the additive term in the argument ensures that export taxes are bounded and have explicit analytic solutions[7].

Government policies. Since consumers and firms are atomistic, the countries' governments are the only strategic players. For simplicity "the strategy of country X's government" will simply be referred to as "country X's strategy."

With this terminology, country O is said to supply n_t^* in periods $t = 1, 2$. In addition, country O sets gross export taxes T_t^*.

[5] In the case of oil production, the storage cost are substantial, impeding storage of quantities needed to cover supply for the "long" periods.

[6] The condition for this statement to hold is $c_{y,t}^{(*)} > 0$, which will be satisfied throughout the paper.

[7] The additive term in the argument can be read as a flow of a perishable substitute to the natural resource in each country.

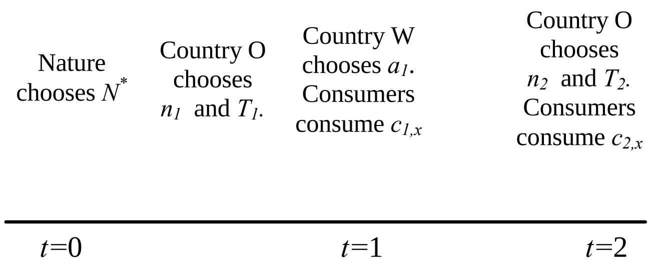

Figure 2: Timing of actions.

Country W's strategies are limited to the development[8] of a *substitution technology*, which enables it to produce a substitute of the natural resource N, using good Y as input. More precisely, country W may incur $A > 0$ units of good Y in period $t = 1$ to develop a constant returns to scale technology that becomes available in period $t = 2$ and enables country W to convert good Y into a perfect substitute of good N with the marginal rate of transformation $B > 1$. Country W's substitution technology can be summarized by

$$b_1 = 0 \qquad \text{and} \qquad b_2 = \begin{cases} 0 & \text{if } \ a_1 = 0 \\ B & \text{if } \ a_1 = A \end{cases} \tag{4}$$

For a convenient notation let country W's first period output and second periods investment be denoted by $n_1 = 0$ and $a_2 = 0$, respectively.

Substitution R&D has two key characteristics. First, it is defined as a binary process (4), so that engagement in R&D is either zero or big time. This assumption is common in the literature (see, e.g., Dasgupta and Heal (1974), Deshmukh and Pliska (1983), Quyen (1988), and Barrett (2006)) and shall reflect a major technological breakthrough in substitution technologies. Further below I argue that this first feature is no crucial. Second, there is the time-lag between R&D expenditure and availability of the substitution technology. This assumption reflects that major technological innovations need substantial time to be developed and become applicable[9]. Now, this second assumption is crucial to the paper's mechanism, as the argument relies on the fact that technological progress cannot be generated instantaneously, *i.e.* the day a supply shortage materializes but instead needs some time to evolve. Clearly, under instantaneous technological progress, R&D decisions would be delayed until uncertainty is resolved; in such a scenario, information rents would vanish.

[8] It will become clear that the profits of the substitution R&D are non-appropriable so that private firms do not engage in substitution R&D without according subsidies. Hence, subsidizing substitution R&D is country W's strategic decision. R&D subsidies are financed by lump-sum taxes.

[9] Lovins et al (2005) reckon that US oil demand projected for 2025 can be cut to half by when beginning to adopt substitutes and energy-saving technologies today.

Timing. The timing of actions in the game between governments is the following. First, nature chooses the realization of N^*, which is revealed to country O only. Next, country O supplies the quantity n_1^* and sets the export tax T_1^*. Then, country W chooses the investment a_1 and first period consumption is realized. Finally, country O sets the second period's export tax T_2^*, supplies $n_2^* \leq N^* - n_1^*$ and the second period's equilibrium quantities are consumed. Figure 2 summarizes this time-line.

It is noteworthy that country W chooses its strategy a_1 after n_1 and T_1 are set. This assumption implies that country W can condition its strategy a_1 on T_1 and on n_1. If, instead, a_1, n_1, and T_1 were taken simultaneously, country O's overreporting via observable actions could not influence country W's strategies and would thus be irrelevant for the equilibrium. A signaling game has, after all, no strategic content if the receiver of the signal cannot condition her actions on the observed signal.

It is intuitive that country W's substitution R&D, harms country O by generating competition of N-supply in period $t = 2$. To avoid such losses, country O may use its supply and private information, to discourage R&D activity. The costs and benefits from doing so are central to the analysis below. Before turning to the strategic aspects of the game, however, demand will be derived from (3).

2.2 Consumers' Optimization

At each point in time, consumers maximize sub-utilities (3) subject to the respective budgets constraints

$$T_t^* p_t c_{n,t} + c_{y,t} \leq E_t \qquad \text{and} \qquad p_t c_{n,t}^* + c_{y,t}^* \leq E_t^*$$

taking as given domestic prices and the quantities n_t^* and n_t. $E_t^{(*)}$ is respective net per capita income. Provided that interior solutions prevail ($c_{x,t}^{(*)} > 0$ for $x = n, y$), optimal quantities are

$$\begin{aligned} c_{n,t} &= 1/(T_t^* p_t) - 1 \qquad & c_{y,t} &= E_t - 1 + T_t^* p_t \\ c_{n,t}^* &= 1/p_t - 1 \qquad & c_{y,t}^* &= E_t^* - 1 + p_t \end{aligned}$$

When the natural resource market clears, i.e., under $c_{n,t} + c_{n,t}^* = n_t^* + n_t \equiv \bar{n}_t$, the relative price p_t is (remember $n_1 = 0$ so that $\bar{n}_1 = n_1^*$)

$$p_t = \frac{1 + 1/T_t^*}{\bar{n}_t + 2} \tag{5}$$

With this expression of the price, country W's expenditure on consumption goods equals the value of its production in domestic prices minus the value of inputs for R&D plus N-production, *i.e.*

$$E_t = y_t + n_t \frac{T_t^* + 1}{\bar{n}_t + 2} - \frac{n_t}{B} - a_t$$

Country O's revenue from the export tax $(T_t^* - 1)p_t(n_t^* - c_{n,t}^*)$ is distributed lump-sum so that its citizens income is

$$E_t^* = (T_t^* + 1)\frac{n_t^* + 1}{\bar{n}_t + 2} - \left(\frac{T_t^* + 1}{T_t^*}\right)\frac{1}{\bar{n}_t + 2} - T_t^* + 1$$

These expressions lead to equilibrium consumption

$$\begin{aligned} c_{n,t} &= \frac{\bar{n}_t + 2}{T_t^* + 1} - 1 & \quad c_{y,t} &= y_t + (T_t^* + 1)\frac{n_t + 1}{\bar{n}_t + 2} - 1 - \tfrac{n_t}{B} - a_t \\ c_{n,t}^* &= T_t^* \frac{\bar{n}_t + 2}{T_t^* + 1} - 1 & \quad c_{y,t}^* &= 1 - (T_t^* + 1)\frac{n_t + 1}{\bar{n}_t + 2} \end{aligned} \tag{6}$$

and the sub-utilities

$$\begin{aligned} u_t &= \ln\left(\frac{\bar{n}_t + 2}{T_t^* + 1}\right) + y_t + (T_t^* + 1)\frac{n_t + 1}{\bar{n}_t + 2} - 1 - \tfrac{n_t}{B} - a_t \\ u_t^* &= \ln\left(T_t^* \frac{\bar{n}_t + 2}{T_t^* + 1}\right) + 1 - (T_t^* + 1)\frac{n_t + 1}{\bar{n}_t + 2} \end{aligned} \tag{7}$$

Countries employ their respective policies (n_t^*, T^*, and a_1) to maximize the sum of their citizen's sub-utilities (7).

2.3 Optimal Export Tax

The governments of both countries use their policies to maximize the respective total utilities (2) so that, in principle, the policies has an intertemporal component and involve a trade-off between periods. By expression (7), however, country O's optimal export taxes T_t^* lack this intertemporal trade-off and maximize sub-utilities u_t^* from (7) for the following reason. Equation (7) implies that the first period's export tax T_1^* does not affect the cost of R&D investment A. The export tax T_1^* neither impacts country W's returns to R&D, which accrue in the second period only. Hence, a_1 is independent of T_1^* and so is u_2^*. Further, when T_2^* is set in $t = 2$, all first-period variables are taken as given, so that u_2^* is independent of T_2^*. Consequently, the next paragraphs compute optimal export taxes T_t^* as the maximizer of u_t^*.

No substitution technology. In case $b_t = 0$, country W is not capable of N-production, $n_t = 0$ holds trivially and maximization of u_t^* from (7) leads to[10]

$$T_t^* = \sqrt{9/4 + n_t^*} - 1/2 \tag{8}$$

Substitution technology. If, in contrast, $b_2 = B$ holds, the price ratio $T_2^* p_2$ cannot exceed the marginal rate of transformation $1/B$ and country O's equilibrium export tax

[10] Note that (5) and (8) imply $T_t^* p_t < 1$ so that consumed quantities $c_{n,t}^{(*)}$ from (6) are positive.

must satisfy the constraint $T_2^* p_2 \leq 1/B$. When this constraint binds, price (5) yields

$$T_2^* = \frac{\bar{n}_2 + 2}{B} - 1 \tag{9}$$

Equation (9) implies that any increase of the export tax T_2^* above the level $\frac{n_t^*+2}{b} - 1$ induces an increase in n_2 that keeps the relative price $T_2^* p_2$ constant. With (7) it is straight forward to check that, under (9) and at constant n_2^*, sub-utility u_2^* is decreasing in n_2 if and only if $T_2^* \geq 1$ or, equivalently, $\bar{n}_2 \geq 2(B-1)$. Consequently, under condition $b_2 = B$, country O's optimal export tax is $T_2^* = \max\left\{\left(n_2^* + 2\right)/B - 1, 1\right\}$ and the optimal export taxes are summarized by

$$T_t^*(n_t^*) = \begin{cases} \sqrt{\frac{9}{4} + n_t^*} - \frac{1}{2} & \text{if } b_t = 0 \\ \max\left\{\frac{n_t^*+2}{B} - 1, 1\right\} & \text{if } b_t = B \end{cases} \tag{10}$$

Equation (10) has been derived assuming that the constraint $T_2^* p_2 \leq 1/B$ binds. With (5) and (8), this is the case if $T_2^*|_{b=0} < B$, or if B is large compared to country O's second period's supply, i.e.

$$B^2 + B - 2 > n_2^* \tag{11}$$

holds. In that case, equilibrium consumption (6) of both countries depends on country W's substitution technology. In particular, if $b_2 = 0$ quantities are

$$\begin{array}{ll} c_{n,t} = \frac{n_t^*+2}{T_t^*+1} - 1 & c_{y,t} = y_t + \frac{T_t^*+1}{n_t^*+2} - 1 - a_t \\ c_{n,t}^* = T_t^* \frac{n_t^*+2}{T_t^*+1} - 1 & c_{y,t}^* = 1 - \frac{T_t^*+1}{n_t^*+2} \end{array} \tag{12}$$

while under $b_2 = B$ equations (6) and (10) lead to

$$\begin{array}{ll} c_{n,2} = B - 1 & c_{y,2} = y_2 - \frac{B-1}{B} \\ c_{n,2}^* = \min\{n_2^*, B-1\} & c_{y,2}^* = \left(B - 1 - n_2\right)/B \end{array}$$

Combining both cases, sub-utilities (7) are

$$\begin{aligned} u_t &= \begin{cases} \ln\left(\frac{n_t^*+2}{T_t^*+1}\right) + y_t + \frac{T_t^*+1}{n_t^*+2} - 1 - a_t & \text{if } n_t = 0 \\ \ln(B) + y_2 - \frac{B-1}{B} & \text{if } t = 2 \text{ and } a_1 = A \end{cases} \\ u_t^* &= \begin{cases} \ln\left(T_t^* \frac{n_t^*+2}{T_t^*+1}\right) + 1 - \frac{T_t^*+1}{n_t^*+2} & \text{if } n_t = 0 \\ \ln\left(\min\{n_2^* + 1, B\}\right) + \frac{B-1-n_2}{B} & \text{if } t = 2 \text{ and } a_1 = A \end{cases} \end{aligned} \tag{13}$$

Together, equations (10) and (13) determine the equilibrium utility for a given supply n_t^* and investment a_1 and imply the following observations.

First, the sub-utility u_t^* is increasing in n_t^*, hence the resource constraint $n_1^* + n_2^* \leq N^*$

binds and the second period's supply equals the residual $n_2^* = N^* - n_1^*$. This is a simple verification of Walras' Law.

Second, conditional on $b_t = 0$ (i.e. $a_1 = 0$), u_t^* is concave in n_t^* because

$$\left.\frac{d^2 u_t^*}{(dn_t^*)^2}\right|_{b_t=0} = \frac{-2}{(T_t^*)^3(2T_t^*+1)} < 0 \tag{14}$$

holds. Thus, conditional on country W not investing in R&D, country O's total utility (2) is maximized when the natural resource is supplied evenly across the periods

$$n_1^* = n_2^* = N^*/2 \tag{15}$$

This supply rule is a degenerate version of Hotelling's optimal rule, which states that – correcting for time preference rates – optimal supply is smooth. Any deviation from optimal supply rules can be attributed to substitution R&D activity through direct or indirect effects.

Third, combining equations (11) and (15) shows that county W's R&D constraints world prices of good N if and only if

$$B^2 + B - 2 > N^*/2 \tag{16}$$

holds. Intuitively, technology B affects the price of N if either the substitution technology is very efficient (B is large) or country O's endowment N^* is small. For the rest of the paper condition (2) will be assumed to hold. In this case, county O's utility falls short of its utility in absence of oil-substituting R&D.

Finally, if country O's supply in the second period is large enough ($n_2^* > 2(B-1)$), country W does not produce the substitute ($n_2 = 0$) even under $b_2 = B$. In this case the return to R&D activity consists of a reduction of prices $T_2^* p_2$ in the second period. The returns to costly substitution R&D are positive but non-appropriable and, hence, substitution R&D must be financed by country W's government.

The equilibrium of the supply and investment game is solved under full information next. This provides a benchmark case; the role of information asymmetries is addressed subsequently.

3 Full Information

This section analyzes the Nash equilibrium of the sequential game outlined in the previous section, assuming that the amount of total reserves N^* is common knowledge. It has been shown that all strategic interaction can be reduced to a two-stage game in which country

O first chooses n_1^* and then country W decides on a_1. Export tax and consumption choices follow from static optimization. The game is solved by backward induction.

2^{nd} stage: Optimal Investment a_1. Country W does not engage in substitution R&D if and only if the net gains fall short of the costs ($u_2|_{b_2=B} - u_2|_{b_2=0} \leq A$). With expressions (10) and (13) this condition is

$$\ln\left(B\frac{\sqrt{\frac{9}{4}+n_2^*}+\frac{1}{2}}{n_2^*+2}\right)+\frac{1}{B}-\frac{\sqrt{\frac{9}{4}+n_2^*}+\frac{1}{2}}{n_2^*+2}\leq A \tag{17}$$

and country W's optimal strategy is expressed by the rule

$$a_1 = \begin{cases} 0 & \text{if (17) holds} \\ A & \text{else} \end{cases}$$

This trade-off between costs and benefits of substitution R&D are trivial if the cost A is prohibitive and exceeds the gains even under zero supply of N in the second period. To rule out this case, assume that (17) is violated under $n_2^* = 0$, i.e.,[11]

$$\ln(B) + \frac{1}{B} - 1 > A \tag{18}$$

1^{st} stage: Optimal Supply n_1^*. In the first stage country O has two options: either to prevent country W's substitution R&D or to adjust to it. If it decides to prevent substitution R&D, it needs to satisfy condition (17). Since the term on the left of (17) is decreasing in n_2^*, condition (17) defines an upper bound on n_2^*, which satisfies (17) with equality, and which will be labeled n_P^*. Notice that the expression on the left of (17) is negative whenever $n_2^* > B^2 + B - 2$ so that, by assumption (18)

$$n_P^* \in (0, B^2 + B - 2) \tag{19}$$

By concavity of u^* (14), country O's optimal supply, conditional on $a_1 = 0$, is captured by

$$n_1^* = \min\{N^* - n_P^*, N^*/2\} \tag{20}$$

If country O aims to depress W's substitution R&D, it's supply of the natural resource N in the second period must be high enough to depress country W's gains from R&D below rentability. Whenever total resources N^* are large (i.e., $N^* > 2n_P^*$ holds), country O is not constrained by this requirement and plays its unconstrained optimal strategy $n_1^* = N^*/2$.

Country O may, alternatively, adjust to country W's R&D, in which case its optimal

[11] The expression on the left of (18) is positive by $B > 1$.

supply in the first period is (see Appendix)

$$n_C^* \equiv \begin{cases} \frac{1}{2}\left[N^* - B - \frac{1}{4} + \sqrt{\frac{N^*-B}{2} + \frac{33}{16}}\right] & \text{if } N^* \geq 3B + \sqrt{B} - 4 \\ B + \sqrt{B} - 2 & \text{if } N^* \in \left(2B - \frac{\sqrt{4B+5}+1}{2}, 3B + \sqrt{B} - 4\right) \\ \frac{1}{2}N^* + \frac{1}{8}\sqrt{8N^* + 25} - \frac{5}{8} & \text{if } N^* \leq 2B - \frac{\sqrt{4B+5}+1}{2} \end{cases} \tag{21}$$

Quantity n_C^* is the first period's optimal supply conditional on conceding to $a_1 = A$. One can check that $n_C^* \in (0, N^*)$ holds in all cases.[12]

In sum, country O's supply in the first period is either set following strategy (20) to prevent substitution R&D or else following (21) to adjust to substitution R&D. The equilibrium depends on the respective utilities under both strategies. For this trade-off, it is convenient to define total utility under given supply and investment decisions as

$$V^*(n_1^*, n_2^*, b_2) \equiv \max_{T_1^*, T_2^*} \{u_1^* + u_2^*\} \qquad \text{given } n_1^*, n_2^*, b_2 \tag{22}$$

Clearly, if country O concedes to country W's R&D it gets total utility $V^*(n_C^*, N^* - n_C^*, B)$ while preventing R&D renders $V^*(\min\{N^* - n_P^*, N^*/2\}, \max\{n_P^*, N^*/2\}, 0)$. The equilibrium strategy can be read from the sign of the difference of both expressions. The following proposition shows that the optimal decision rule - and hence the equilibrium strategy - depends in a simple way on total reserves N^*.

Proposition 1 *$\exists\ N_0 \in [n_P^*, 2n_P^*]$ so that under full information a subgame perfect Nash Equilibrium exists, is unique, and is described by the strategies*

$$(n_1^*, a_1) = \begin{cases} (n_C^*, A) & \text{if } N^* < N_0 \\ (N^* - n_P^*, 0) & \text{if } N^* > N_0 \end{cases} \tag{23}$$

Proof. Define $\Delta V^*(N^*) \equiv V^*(N^* - n_P^*, n_P^*, 0) - V^*(n_C^*, N^* - n_C^*, B)$. It is sufficient to show first that $\Delta V^*(n_P^*) < 0$, second, that $\Delta V^*(2n_P^*) > 0$, and third, that $N^* - n_P^* < n_C^*$ implies $d\Delta V^*(N^*)/dN^* > 0$ (for $N^* - n_P^* > n_C^*$ implies $b_2 = 0$ in any case). First, observe that

$$\begin{aligned} \Delta V^*(n_P^*) &= V^*(0, n_P^*, 0) - V^*(n_C^*, n_P^* - n_C^*, B) \\ &= V^*(n_P^*, 0, 0) - V^*(n_C^*, n_P^* - n_C^*, B) \\ &= V^*(n_P^*, 0, B) - V^*(n_C^*, n_P^* - n_C^*, B) < 0, \end{aligned}$$

[12] Noticing also that the expression in the first line falls short of $N^*/2$ since under condition (16) $B + 1/4$ exceeds the value of the square root. For $N^* > 0$, however, the term in the third line is strictly larger than $N^*/2$.

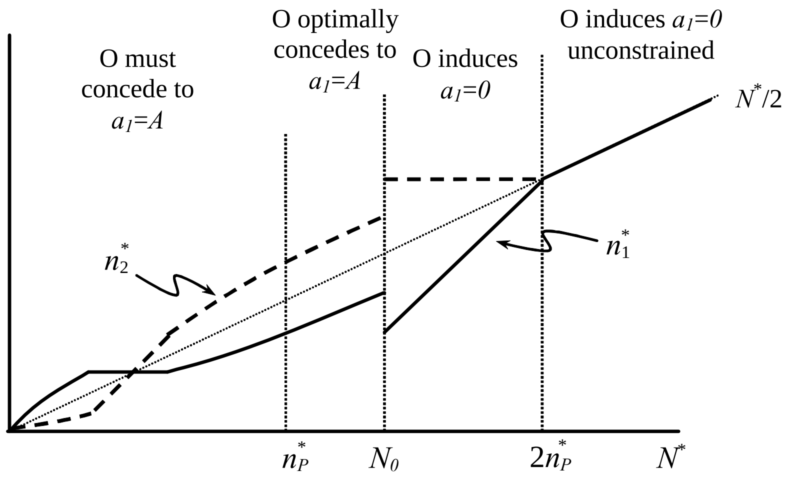

Figure 3: Country O's optimal quantities under full information.

where $n_C^* < N^* = n_P^*$ and (14) were used in the last step. Second,

$$\Delta V^*(2n_P^*) = V^*(n_P^*, n_P^*, 0) - V^*(n_C^*, 2n_P^* - n_C^*, B) > 0,$$

which follows from (15). Third, the Envelope Theorem implies

$$\frac{d\Delta V^*(N^*)}{dN^*} = \left.\frac{du_1^*}{dn_1^*}\right|_{n_1^*=N^*-n_P^*} - \left.\frac{du_1^*}{dn_1^*}\right|_{n_1^*=n_C^*}.$$

Since u_1^* is concave in n_1^* by (14) this expression is positive whenever $N^* - n_P^* < n_C^*$. ■

The result of Proposition 1 is quite intuitive. Country O's two goals are, on the one hand, to smooth supply according to the rule (15) and, on the other, to discourage country W from investing in the substitution technology. Since a minimum supply in the second period ($n_{2,t}^*$) is necessary to reach the second goal, the deviation from the optimal unconstrained path (15) - and hence utility losses - is relatively large whenever the total reserves N^* are low. Thus, the utility losses dominate the gains from preventing investment if N^* falls short of a threshold N_0. In this case, country O adjusts to $a_1 = A$.

Figure 3 illustrates country O's optimal supply n_1^* (solid line) and n_2^* (dashed line) as functions of N^*. There are four different ranges of N^*. First, for $N^* < n_P^*$ country O is not able to prevent country W's investment and country W plays $a_1 = A$. Second, under $N^* \in [n_P^*, N_0]$ country O could possibly prevent W's investment but optimally chooses not to do so. Third, for $N^* \in [N_0, 2n_P^*]$ country O optimally prevents country W's investment under the binding constraint (17); the slope of $n_1^*(N^*)$ is one in this range. Finally, if $N^* > 2n_P^*$, country O's optimal strategy is not constrained. As a reference, Figure 3 includes the unconstrained optimum i.e., the equal allocation over both periods,

$n_1^* = n_2^* = N^*/2$, as a dotted line. Deviations from this strategy reflect either country O's need to react to W's substitution capacity ($b_2 = B$) or, alternatively, its aim to prevent country W's investment. At N_0 where country O is indifferent between preventing and conceding, n_1^* jumps down since

$$V^*(N_0 - n_P^*, n_P^*, 0) = V^*(n_C^*, N_0 - n_C^*, B) < V^*(n_C^*, N_0 - n_C^*, 0)$$

implies $N_0 - n_P^* < n_C^*$. Apart from this discontinuity, supply in both periods is (weakly) increasing in N^*.

Before closing this section it is instructive to contemplate the distortion of supply under prevention of country W's investment. If country O chooses to prevent W's investment, world supply of N is distorted away from the optimal rule ($n_1^* = n_2^*$) towards a more back-loaded supply rule ($n_1^* < n_2^*$). This finding resembles those of earlier work, in which monopolistic supply leads to a partial delay of supply. Hotelling (1931) calls this "retardation of production under monopoly" and Quyen (1988) confirms that "the monopolist is excessively conservationist." These studies predict that the monopolist scarifies supply in early periods, which creates a front-loaded stream of profits and possibly a longer duration of supply period. Stiglitz (1976), however, shows that these results do not stand up to robustness checks, including generalized demand function and extraction costs. The mechanism presented here, instead, is qualitatively different. Supply is partly delayed in order to generate abundant future supply and thus discourage country W's substitution R&D. The causality between future supply, incentives to engage in time-consuming R&D, and optimal supply has a clear orientation on the time-line and suggests that this deviation from the Hotelling rule is quite robust.[13]

With a good idea about the nature of the distortions that country W's R&D opportunities create, one can ask for the gains and losses they induce. Intuitively, country O suffers from the potential increase of competition in the N-market and the distortions this induces to its optimal supply. This intuition is confirmed by verifying

$$V^*\left(n_1^*, N^* - n_1^*, b_2\right) \leq V^*\left(n_1^*, N^* - n_1^*, 0\right) < V^*\left(N^*/2, N^*/2, 0\right)$$

for all $N^* < 2n_P^*$. It might be less intuitive that, for all $N^* \in [N_0, 2n_P^*]$, country W loses from its investment opportunity as well. To see this, check that, under $a_1 = 0$, the optimal export tax (10) and utilities (7) imply $d^2u_t/(dn_t^*)^2\big|_{b_t=0} < 0$ so that u_t is concave in n_t^* and country W's total utility (2) is maximized at $n_1^* = n_2^* = N^*/2$. Consequently, country W's equilibrium utility falls short of the utility it would obtain in a alternative world without R&D opportunities, by the deviations from a smooth supply performed by country O to prevent R&D activity.

[13] A rigorous analysis of this mechanism in continuous time would be interesting but is beyond the scope of this paper.

Proposition 1 and Figure 3 have provided a description of the full information equilibrium. The nest section turns to the main objective of the paper and analyzes the incentives to overreport under asymmetric information about reserves.

4 Asymmetric Information

This section formalizes country O's incentives to misreport the reserves of the natural resource N. The standard framework for an analysis of the strategic use of private information is the signaling game and its equilibrium concept of Perfect Bayesian Equilibria.

Within this framework, a player with private information can be said to deceive or misinform other players if he conceals his type and benefits from doing so. Applying this concept to the current model, one of country O's types is said to misreport successfully if its signal is not informative about remaining reserves and if, in addition, this specific type enjoys higher utility in the resulting pooling equilibrium than under full information. By definition, successful misreporting and beneficial pooling are the same thing.[14]

To assess whether misreporting can arise in the present model, the equilibrium concept is specified next.

4.1 Equilibrium: Definition

The specification of the game, summarized in Figure 2, shall be briefly repeated. The total amount of the natural resource reserves, N^*, is a random variable, distributed as specified by (1). In stage zero, Nature decides on the realization of N^*, which country O observes but country W does not. The realization of the reserve N^* defines two different types of country O, which are indexed by $\theta = H, L$ and labeled country O_H in the case $N^* = \bar{N}$ and country O_L if $N^* = \xi\bar{N}$ $(\xi < 1)$. In the first stage, country O can signal its type with the first period's supply n_1^* as a signal. In a separating equilibrium, the signal n_1^* differs across types while it equalizes in a pooling equilibrium. In the second stage, country W rationally updates its beliefs and chooses investment $a_1 \in \{0, A\}$. As shown in Section 2.3, export taxes and consumption are no strategic components and follow expressions (10) and (12). Formally, the strategies $(n_{1,H}^e, n_{1,L}^e, a_1^e(n_1^*))$ are said to characterize a Perfect Bayesian Equilibrium if they satisfy the following criteria

- **E(i)** W rationally updates its prior believes given O's strategies,
- **E(ii)** $a_1^e(n_1^*)$ maximizes expected total utility U under W's updated beliefs,
- **E(iii)** for each type $\theta = H, L$, $n_{1,\theta}^e$ maximizes total utility U^*, given W's strategy, prior beliefs, and updating rules.

[14] Strictly speaking, any reporting is discounted as cheap talk by the market and can be regarded as entirely irrelevant in the model. Yet, acknowledging the fact that some types gain from imitating other's this definition of misreporting is a natural one.

The full specification of an equilibrium involves country W's updated beliefs that satisfy requirement E(i). These beliefs are denoted by the function $\mu(.) : [0, \bar{N}] \to [0, 1]$, which represents country W's subjective probabilities that country O is of high type conditional on observing supply n_1^* or

$$\mu(n_1^*) \equiv \mathbb{P}(N^* = \bar{N} \mid n_1^*)$$

The equilibrium strategies of both players are indexed with the superscript ^{e} and are denoted by

$$\left(n_{1,H}^e, n_{1,L}^e\right) \in [0, \bar{N}] \times [0, \xi\bar{N}] \quad \text{and} \quad a_1^e\,(\tilde{n}) : [0, \bar{N}] \to \{0, A\}$$

Country W's equilibrium technology in the second period is labeled $b_{2,\theta}^e \in \{0, B\}$, where θ stands for country O's type $\theta = H, L$. In a pooling equilibrium, a_1 cannot be conditioned on country O's type and $b_{2,H}^e = b_{2,L}^e$ must hold.

For further references, it is useful to denote country O's full information equilibrium strategies (23) under type $\theta = L, H$ as

$$\begin{aligned} n_{1,H}^* &\in [0, \bar{N}] \quad \text{and} \quad a_{1,H}^*(\tilde{n}) : [0, \bar{N}] \to \{0, A\} \\ n_{1,L}^* &\in [0, \xi\bar{N}] \quad \text{and} \quad a_{1,L}^*(\tilde{n}) : [0, \xi\bar{N}] \to \{0, A\} \end{aligned}$$

The variable $b_{2,\theta}^* \in \{0, B\}$ will stand for country W's substitution technology in the second period of the full information equilibrium, given country O's type is θ.

The existence and the characteristic of the signaling game's equilibrium is sensitive to the specification of the receiver's beliefs, including the off-equilibrium beliefs. The minimal requirement that the updating of beliefs be rational (i.e. following the Bayes' rule) leaves a wide range of off-equilibrium beliefs, which implies that equilibria are non-unique in many cases. In the present analysis, however, posterior beliefs μ will be restricted to satisfy the following set of assumptions.

A(i) $\quad n_{1,H}^e \neq n_{1,L}^e \quad \Rightarrow \quad \mu(n_{1,H}^e) = 1$ and $\mu(n_{1,L}^e) = 0$.

A(ii) $\quad n_{1,H}^e = n_{1,L}^e \quad \Rightarrow \quad \mu(n_{1,H}^e) = \pi$.

A(iii) $\quad V^*(n_{1,L}^*, \xi\bar{N} - n_{1,L}^*, b_{2,L}^*) > V^*(n_{1,H}^*, \xi\bar{N} - n_{1,H}^*, b_{2,H}^*) \quad \Rightarrow \quad \mu(n_{1,H}^*) = 1$.

A(iv) $\quad \tilde{n} \in [0, \bar{N}] \qquad \tilde{b}$ outcome of W's optimal $a_1^e(\tilde{n})$ under $\mu(\tilde{n}) = \pi$.

$$\begin{aligned} &V^*(\tilde{n}, \bar{N} - \tilde{n}, \tilde{b}) > V^*(n_{1,H}^e, \bar{N} - n_{1,H}^e, b_{2,H}^e) \\ &V^*(\tilde{n}, \xi\bar{N} - \tilde{n}, \tilde{b}) > V^*(n_{1,L}^*, \xi\bar{N} - n_{1,L}^*, b_{2,L}^*) \\ &\qquad\qquad\qquad\qquad \Rightarrow \qquad \mu(\tilde{n}) = \pi. \end{aligned}$$

A(v) $\quad \tilde{n} \in [0, \bar{N}] \qquad \tilde{b}$ outcome of W's optimal $a_1^e(\tilde{n})$ under $\mu(\tilde{n}) = \pi$.

$$V^*(\tilde{n}, \bar{N} - \tilde{n}, \tilde{b}) < V^*(n_{1,H}^e, \bar{N} - n_{1,H}^e, b_{2,H}^e)$$
$$V^*(\tilde{n}, \xi\bar{N} - \tilde{n}, \tilde{b}) > V^*(n_{1,L}^e, \xi\bar{N} - n_{1,L}^e, b_{2,L}^e)$$
$$\Rightarrow \qquad \mu(\tilde{n}) = 0.$$

Assumptions A(i) and A(ii) simply formulate the requirement of Bayesian updating; assumptions A(iii) - A(v), however, constitute non-trivial refinements of the off-equilibrium beliefs. Loosely speaking, among all Bayesian Nash Equilibria assumptions A(iii) - A(v) single out the one that maximizes the high type's payoffs. – A(iii) requires that, if O_L does not gain from imitation of O_H's full information strategy ($n_{1,H}^*$) relative to its own full information strategy ($n_{1,L}^*$), then W, when observing $n_{1,H}^*$, believes in $\theta = H$ with certainty. This assumption implies that O_H plays its full information strategy whenever it does not pay for O_L to pool to $n_{1,H}^*$. Hence, A(iii) establishes the full information equilibrium as the default outcome.[15] Conversely, this implies that a pooling equilibrium only exists if no separating equilibrium including the full information strategies exists. – A(iv) requires that, if O_H gains from a deviation to $\tilde{n}$ relative to an equilibrium outcome $n_{1,H}^e$ provided that $\mu(\tilde{n}) = \pi$ and if, further, O_L prefers to pool to that deviation $\tilde{n}$ rather than to resort to its full information equilibrium, then W, when actually observing strategy $\tilde{n}$, is agnostic about O's type and sticks to its prior beliefs ($\mu(\tilde{n}) = \pi$). This assumption eliminates all equilibria that render the high type less utility than the pooling equilibrium with maximal utility for the high type. – Finally, A(v) requires that, whenever O_H looses from a deviation to $\tilde{n}$ relative to the equilibrium provided that $\mu(\tilde{n}) = \pi$ while O_L gains from a deviation to $\tilde{n}$ relative to its current equilibrium outcome provided that $\mu(\tilde{n}) = \pi$, then W, when observing strategy $\tilde{n}$, believes in $\theta = L$ with certainty ($\mu(\tilde{n}) = 0$). This assumption ties O_L to the equilibrium strategy that is beneficial for O_H.

Making use of the definition and the refinements the equilibrium will be calculated next.

4.2 Equilibrium: Characterization

To determine the equilibrium of the signaling game, country W's optimal decision rule is derived first. The information asymmetries changes country W's situation to the extent that, at the time of making the R&D decision it may face subjective uncertainty about the second period's supply of the natural resource. Consequently, its optimal strategy is now taken on the basis of *expected returns* to substitution R&D, where expectations are

[15] This statement follows from two observations: First, $n_{1,H}^*$ is O_H's unique optimal strategy under full information. Second, asymmetric information adds one more constraint to O_H's optimization program (the incentive compatibility constraint in the case of a separating and the probabilistic equivalent of (17) with prior probabilities π and $1 - \pi$ in the case of a pooling equilibrium) so that in all equilibria of the signaling game O_H obtains weakly less utility than in the full information equilibrium. Thus, whenever O_L looses from pooling to $n_{1,H}^e = n_{1,H}^*$, A(iii) grants that O_H can obtain its full information utility by playing $n_{1,H}^e = n_{1,H}^*$ and implying $a_1^e(n_{1,H}^e) = 0$, which is, by Proposition 1, O_H's unique optimal strategy.

formed using subjective probabilities. More precisely, country W's strategy is based on the probabilistic analog of inequality (17), which, when using the definition of μ, can be written as

$$\begin{aligned} \ln(B) + \tfrac{1}{B} - \mu(n_1^*)\left\{\ln\left(T_2^*(\bar{N} - n_1^*)\right) + \frac{1}{T_2^*(\bar{N} - n_1^*)}\right\} - ... \\ ...(1 - \mu(n_1^*))\left\{\ln\left(T_2^*(\xi\bar{N} - n_1^*)\right) + \frac{1}{T_2^*(\xi\bar{N} - n_1^*)}\right\} \leq A \end{aligned} \tag{24}$$

In (24) $T_2^*(.)$ stands for the optimal export tax under $b_2 = 0$ from (10). Condition (24) determines country W's investment behavior and[16]

$$a_1(n_1^*) = \begin{cases} 0 & \text{if (24) holds} \\ A & \text{else} \end{cases}$$

Unfortunately, country O's optimal strategy does not follow such a handy rule. As in the case of full information, country O gains from depressing the investment in substitution R&D but loses from deviations of its optimal supply rules. When engaging in signaling, country O aims to prevent country W's substitution R&D at the cost of distorted supply. This trade-off between country O's costs and benefits of the signal is central for the computation of the equilibrium. It will prove useful to define the limits on the first period's supply n_1^* which, disregarding information asymmetries, set the bounds of country O's willingness to discourage substitution R&D. Such thresholds must leave country O indifferent between successfully inducing $a_1 = 0$ and conceding to $a_1 = A$. A lower bound, labeled m, is implicitly defined by $m < N^*/2$ and

$$V^*(n_C^*, N^* - n_C^*, B) - V^*(m, N^* - m, 0) = 0 \tag{25}$$

By this definition, m depends on total reserves N^* and some of its properties can be inferred from (25).

Claim 1 *m satisfies the following properties.*

(i) m is well defined and unique for $N^* \in [0, 2n_P^*]$.

(ii) $m < N^* - n_P^*$ if and only if $N^* \in (N_0, 2n_P^*]$.

(iii) $N^*/2 - m > |N^*/2 - n_C^*|$.

(iv) $0 < \frac{dm}{dN^*} < 1$.

(v) m is positive on $N^* \in (0, 2n_P^*]$.

Proof. See appendix. ■

[16] Notice that this seemingly simple decision rule involves the updated beliefs μ. These beliefs must satisfy A(i) - A(v) and hence depend on the payoffs of the types O_θ, which in turn depend on $a_1(n_1^*)$.

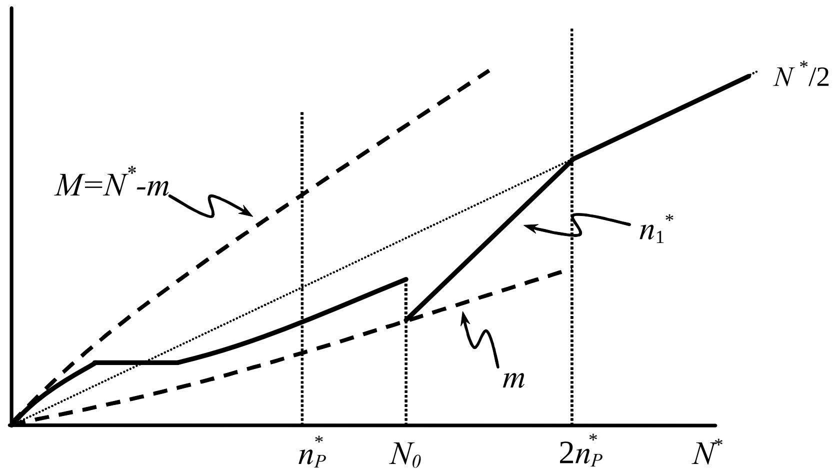

Figure 4: Boundaries of O's net benefits from preventing W's R&D.

By Claim 1 (i), the threshold m is a function of N^* and can be written as $m(N^*)$. Concavity of u^* (see (14)) and $m < N^*/2$ implies that the value $m(N^*)$ constitutes a lower bound on the quantities which country O, endowed with N^*, is willing to supply in the first period to prevent country W from engaging in substitution R&D. Finally, the symmetry of $V^*(n_1^*, n_2^*, 0)$ in the first two arguments implies that the function

$$M(N^*) \equiv N^* - m(N^*) \tag{26}$$

establishes the corresponding upper bound on the quantities n_1^*. Figure 4 illustrates these bounds $m(N^*)$ and $M(N^*)$ as dashed lines, the full information equilibrium n_1^* is represented by the bold line. By Claim 1 (iv) and (v), the functions $m(N^*)$ and $M(N^*)$ are increasing in N^*, lie within the interval $(0, N^*)$, and satisfy $m(N^*) < N^*/2 < M(N^*)$. Country O, endowed with N^*, is willing to supply any $n_1^* \in [m(N^*), M(N^*)]$ in the first period if this prevents substitution R&D in country W (i.e., induces $a_1 = 0$). Notice that, since country O optimally concedes to $a_1 = A$ for $N^* < N_0$, the threshold $m(N^*)$ lies above the line $N^* - n_P^*$ in this range, i.e., $N^* < N_0$ implies $m(N^*) > N^* - n_P^*$. Conversely, for $N^* > N_0$ country O optimally prevents investment in R&D, hence $m(N^*) < N^* - n_P^*$ in this range. The functions $m(N^*)$ and $N^* - n_P^*$ intersect at the value $N^* = N_0$ where country O is indifferent between conceding to $a_1 = A$ and preventing it.

With the definitions of m and M and the properties summarized in Claim 1 it is possible to give a first irrelevance result and to formulate specific conditions for the realizations $\xi\bar{N}$ and $\bar{N}$ under which the information asymmetries do not impact the real world economy at all. These conditions are spelled out in the following proposition.

Proposition 2 *Assume that at least one of the following conditions holds* *(i)* $\bar{N} \notin [N_0, 2n_P^*]$

(ii) $M(\xi\bar{N}) < \bar{N} - n_P^*$

then the unique subgame perfect equilibrium in pure strategies is a separating equilibrium characterized by the full information strategies (23).

Proof. By assumption A(iii) it is sufficient to show that O_L's full information strategy $n_{1,L}^*$ dominates pooling to O_H's full information strategy $n_{1,H}^*$.

(i) If $\bar{N} < N_0$ W plays $a_1 = A$ in the full information equilibria under $N^* = \bar{N}$. Thus, for O_L, $n_{1,L}^*$ dominates $n_{1,H}^*$ by construction.

If, instead, $\bar{N} > 2n_P^*$ O_H's full information strategy is $n_{1,H}^* = \bar{N}/2$ by (20). Now distinguish two cases: first, if $b_{2,L}^* = 0$, $n_{1,L}^* = \min\left\{\xi\bar{N} - n_P^*, \xi\bar{N}/2\right\}$ holds by (20). Hence by (14) and symmetry of $V^*(n_1^*, n_2^*, 0)$ is the first two arguments

$$V^*(n_{1,L}^*, \xi\bar{N} - n_{1,L}^*, 0) > V^*(\bar{N}/2, \xi\bar{N} - \bar{N}/2, 0)$$

holds. If, second, $b_{2,L}^* = B$ then

$$V^*(n_{1,L}^*, \xi\bar{N} - n_{1,L}^*, B) > V^*(\xi\bar{N} - n_P^*, n_P^*, 0) > V^*(\bar{N}/2, \xi\bar{N} - \bar{N}/2, 0)$$

holds. Thus, $n_{1,L}^*$ dominates pooling to $n_{1,H}^*$ in this last case, too.

(ii) By (i) one can focus on $\bar{N} \in [N_0, 2n_P^*]$. Condition (ii) and definition (26) imply $m(\xi\bar{N}) > \xi\bar{N} - (\bar{N} - n_P^*) > \xi\bar{N} - n_P^*$ and hence $b_{2,L}^* = B$. Thus, by construction of M and m

$$\begin{aligned} V^*(n_{1,L}^*, \xi\bar{N} - n_{1,L}^*, B) &= V^*(m(\xi\bar{N}), \xi\bar{N} - m(\xi\bar{N}), 0) \\ &= V^*(M(\xi\bar{N}), \xi\bar{N} - M(\xi\bar{N}), 0) \\ &> V^*(\bar{N} - n_P^*, \xi\bar{N} - (\bar{N} - n_P^*), 0) \end{aligned}$$

holds and proves the statement. ■

The first part of the proposition, related to condition (i), reflects, that for very large $\bar{N}$, the low type's pooling strategy is more costly than inducing $a_1 = 0$ directly, i.e., under revelation of its type. Similarly, for small $\bar{N}$ ($\bar{N} < N_0$) even the high type optimally concedes to $a_1 = A$ and there is no gain for O_L that compensates for the cost of pooling.

Figure 5 illustrates the result of Proposition 2 related to condition (ii). Whenever $\xi\bar{N}$ is small and lies below the value $M^{-1}(\bar{N} - n_P^*)$, the figure shows that the high type's full information strategy $\bar{N} - n_P^*$ lies outside the interval $[m(\xi\bar{N}), M((\xi\bar{N}))]$, which comprises all signals O_L is willing to set in order to induce $a_1 = 0$. Consequently, the low type's

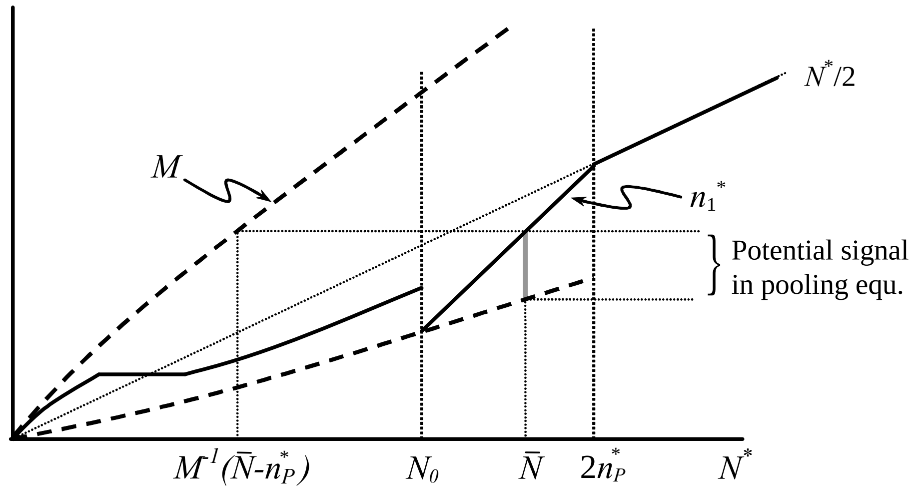

Figure 5: O_L's incentives to imitate O_H and equilibrium signals.

pooling to the strategy $\bar{N} - n_P^*$ leads to strictly less utility than its optimal strategy under full identification of its type. Hence, the full information equilibrium prevails. In all cases, the two types resort to the respective full information strategies.

Proposition 2 has excluded the existence of pooling equilibria for some parameter range. Thus, the attention rests on the intermediate range of resources and the remainder of the section will focus on the cases where the conditions

$$\bar{N} \in (N_0, 2n_P^*) \tag{27}$$

and

$$\xi \in [M^{-1}(\bar{N} - n_P^*)/\bar{N}, 1) \tag{28}$$

are satisfied. Conditions (27) and (28) assure that type O_L gains from imitating O_H's full information strategy $n_{1,H}^*$ if that discourages substitution R&D. Yet, under such pooling attempts, country W adapts its beliefs so that $n_{1,H}^*$ is not an equilibrium signal. Instead, the natural candidate for the signal of a pooling equilibrium is the quantity that solves (24) with equality under prior beliefs $\mu \equiv \pi$. Let this value be denoted by n_P^e, defined as the implicit solution of

$$\begin{aligned} \ln(B) + \tfrac{1}{B} - \pi \left\{ \ln\left(T_2^*(\bar{N} - n_P^e)\right) + \frac{1}{T_2^*(\bar{N} - n_P^e)} \right\} - \ldots \\ \ldots(1-\pi) \left\{ \ln\left(T_2^*(\xi\bar{N} - n_P^e)\right) + \frac{1}{T_2^*(\xi\bar{N} - n_P^e)} \right\} = A \end{aligned} \tag{29}$$

where $T_2^*(.)$ stands for the second period's export tax (10) under $b_2 = 0$. It is quickly verified that the expression on the left of (29) is decreasing in export taxes and, hence, by (10), increasing in n_P^e. Further, (10) implies that the term in the first slanted brackets is larger than the term in the second slanted brackets and thus, the whole expression on the left is decreasing in π. Further, it is quick to check that the expression on the left on (29) is decreasing in ξ. Consequently, by the implicit function theorem, n_P^e is increasing in ξ and π. Finally, at $\pi = 1$ condition (29) coincides with (17) in which case $n_P^e = \bar{N} - n_P^*$ while at $\pi = 0$ (29) leads to $n_P^e = \xi\bar{N} - n_P^*$. These properties of n_P^e are summarized by

$$\frac{d}{d\xi} n_P^e > 0 \tag{30}$$

$$\frac{d}{d\pi} n_P^e > 0 \tag{31}$$

$$\lim_{\pi \to 1} n_P^e = \bar{N} - n_P^* \tag{32}$$

$$\lim_{\pi \to 0} n_P^e = \xi\bar{N} - n_P^* \tag{33}$$

The gap between n_P^e and n_P^* reflects that country W reacts to the pooling of type O_L by adapting expectations and, relative to the full information equilibrium under $N^* = \bar{N}$, a downward revision of expected future supply. To compensate for this drop of expected future supply, the O_H must further increase the second period supply in order to discourage country W's R&D activity; hence $n_P^e < \bar{N} - n_P^*$ holds.

In addition to country W, type O_H also reacts to O_L's pooling attempts, and may choose not to discourage substitution R&D any more. In this case, O_L's incentives to pool cease to exist. This introduces an additional condition to be satisfied in a pooling equilibrium: the relevant signal $n_{1,H}^e = n_{1,L}^e$ must be element of the set $[m(\bar{N}), M(\bar{N})]$. Since conditions (31) and (32) imply $n_P^e < \bar{N} - n_P^*$ and since $\bar{N} - n_P^* < \bar{N}/2$ by (27), the relevant constraint is thus

$$n_P^e \geq m(\bar{N}) \tag{34}$$

Since n_P^e is a function of π and ξ, condition (34) implicitly defines a constraint on the parameters ξ and π. In particular, the equation $n_P^e = m(\bar{N})$ defines a schedule on the (ξ, π)-plane which, by virtue of properties (30) and (31), represents a decreasing function $\underline{\pi}(\xi)$ that marks the limits for a pooling equilibrium to exist. For values of $\pi < \underline{\pi}(\xi)$, condition (34) is violated and type O_H does not induce $a_1 = 0$ but optimally concedes to $a_1 = A$, in which case O_L lacks incentives to imitate O_H.

These observations suggest that – in addition to the necessary conditions (27) and (28) – the requirement (34) is necessary for a pooling equilibrium to exists. The following proposition identifies conditions (27), (28), and (34) as jointly sufficient, granting that the two-stage signaling game has a pooling equilibrium in pure strategies that is – modulo country W's off-equilibrium beliefs μ and strategies – unique.

Proposition 3 *If (27), (28), and (34) hold, a subgame perfect Bayesian Nash Equilibrium*

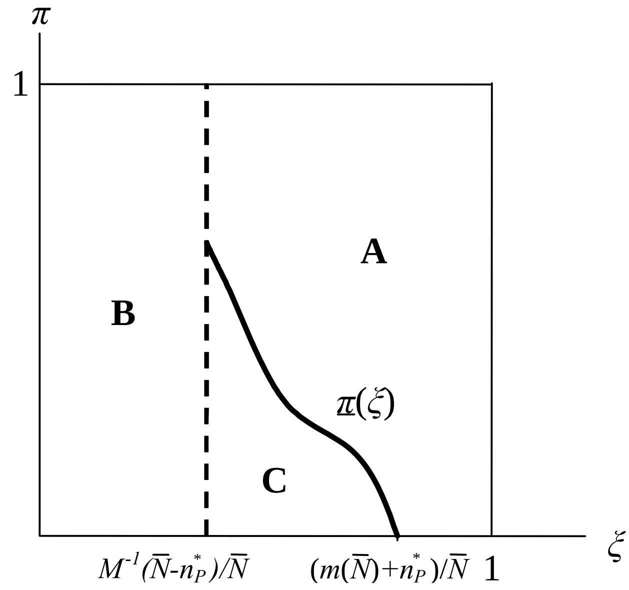

Figure 6: Three different types of equilibria.

in pure strategies exists and includes the strategies

$$(n^e_{1,H}, n^e_{1,L}) = (n^e_P, n^e_P) \quad \text{and} \quad a^e_1(n^e_{1,H}) = a^e_1(n^e_{1,L}) = 0 \tag{35}$$

Proof. See Appendix. ■

Figure 6 illustrates the two key conditions (28) and (34) that delimit the range for the parameters ξ and π which pooling equilibria prevail. Condition (28) sets a minimum that ξ needs to exceed, represented by the dashed vertical line in the figure. Condition (34) defines a minimum $\underline{\pi}(\xi)$ that the ex ante probability π must exceed to grant (34). The function $\underline{\pi}(\xi)$ is marked as a bold line. Both conditions are satisfied for parameters within the area **A**. Notice with (31) and (33) that for $\xi > \left[m(\bar{N}) + n^*_P\right]/\bar{N}$, the value n^e_P exceeds $m(\bar{N})$ for *any* probability $\pi \in [0, 1]$, in which case the requirements on π are empty and hence the bold line hits the ξ-axis at the value $\left[m(\bar{N}) + n^*_P\right]/\bar{N}$.[17]

To the left of the dashed line, in area **B**, condition (28) is violated. Hence Proposition 2 (ii) applies and the unique equilibrium in pure strategies are those replicating the full information equilibrium ($n^*_{1,\theta}$ and $a^*_1(n^*_{1,\theta})$ for $\theta = H, L$, respectively).

Finally, if (28) holds but (34) is violated (area **C** in Figure 6), type O_H optimally chooses not to induce $a_1 = 0$ under O_L's pooling attempts. Consequently, O_L lacks incentives to pool and the equilibrium strategies can be shown to follow the supply rules (21) for $N^* = \xi\bar{N}, \bar{N}$, respectively: a separating equilibrium prevails, with O_H deviating from its

[17] For $\bar{N} \gtrsim N_0$ it is quick to check that this value falls short of one.

full information strategies.

Finally, going back to Figure 5 reveals one important and somewhat surprising feature of the pooling equilibria characterized in Proposition 3. The potential signal that the low type must incur in order to imitate the high type is restricted to lie on the grey vertical line in the figure. For the adequate size of low reserves ($\xi \bar{N}$) this implies that the signal of the pooling equilibrium (n_P^e) lies strictly closer to the unconstrained optimal supply ($\xi \bar{N}/2$) of the low type. In this case, the low type does not only gain from prevention of country W's substitution R&D but also from the decrease in the distortions that occur in presence of substitution R&D. Thus, the costs of overreporting can be said to be negative.

Summarizing, one can formulate the following property of the pooling equilibrium.

Corollary to Proposition 3. *Assume that* $\xi/2 \in (m(\bar{N})/\bar{N}, (1 - n_P^*/\bar{N}))$ *holds. Then there are* π_1, π_2 *with* $\underline{\pi}(\xi) < \pi_1 < \pi_2 < 1$ *so that for all* $\pi \in [\pi_1, \pi_2]$ *the cost of signalling defined as*

$$C \equiv U^*(n_L^*)|_{a_1=0} - U^*(n_L^e)|_{a_1=0}$$

is negative.

Proof. At $\pi = 1$ $n_P^e = \bar{N} - n_P^*$ holds, implying $n_P^e > \xi \bar{N}/2$ by assumption; at $\pi = \underline{\pi}(\xi)$ $n_P^e = m(\bar{N}) < \xi \bar{N}/2$. Since n_P^e is increasing and continuous in π there is a π_0 so that $n_P^e = \xi \bar{N}/2$. Again, by continuity of $n_P^e(\pi)$ there are π_1, π_2 with the stated properties so that $\left|\xi \bar{N}/2 - n_P^e\right| < \left|\xi \bar{N}/2 - n_L^*\right|$ for $\pi \in [\pi_1, \pi_2]$. This proves the claim. ■

The intuition for this somewhat surprising result is the following.[18] First, observe that in absence of R&D activity, country O's first best supply follows (15) and is constant over time. Country W's option to engage in R&D, however, makes country O deviate from this first best supply either to prevent R&D by back-loading supply (i.e., by increasing future supply) or else by adjusting to the R&D activity. In either case, potential R&D activity introduces a deviation from the first best supply. Now, under private information, overreporting requires an additional deviation of supply. If this last deviation cancels with the earlier one, it mitigates the original welfare losses, in which case the costs of overreporting are said to be negative.

4.3 Equilibrium: Discussion

At the beginning of the present section, type O_θ was said to misreport its reserves if its equilibrium action $n_{1,\theta}^*$ did not reveal its type and his utility in the resulting pooling equilibrium was higher than in the respective full information equilibrium. Compared to its full information equilibrium, the low type benefits from imitating the high (granted by

[18] Notice that $\bar{N} - n_P^* > m(\bar{N})$ for $\bar{N} > N_0$ so that the condition on ξ defines a non-empty set.

condition (28)). Conversely, the high type must loose from pooling (conditions (27), (31) and (32) imply $n_P^e < \bar{N} - n_P^* < \bar{N}/2$). Thus, by definition, the low type O_L overreports its reserves in the equilibria characterized in Proposition 3. Underreporting, however, does not occur.

Without formally extending the model, the next paragraphs will briefly discuss the conditions for the results above, their restrictiveness, and the modelling framework under which they have been derived.

Conditions of Pooling Equilibria. The conditions Proposition 3, and hence for credible overreporting can be summarized to two simple and intuitive requirements. First, the lower bound of condition (27) requires that, under full information, the high type prevents country W's R&D activity. In other words, under high realizations of reserves N^* country W refrains from substitution R&D under full information. Otherwise, substitution R&D took place irrespective of the realization N^* and the goal to prevent substitution R&D could not be realized by overreporting. Thus, overreporting would loose its objective. Second, the costs arising from the signal must be limited for the players. Thus, the upper bound of (27) and condition (28) imply that the low type's costs of overreporting do not exceed his gains. If these conditions are violated, the necessary signal introduces excessive distortions of supply and is too costly. Similarly, if the high type's distortions from the pooling equilibrium are too costly, i.e., if condition (34) is violated, then the pooling equilibrium ceases to exist since in that case the high type retreats by adjusting to R&D activity.

The fact that credible overreporting emerges from two of the most standard assumptions of the economics of natural resources is quite noteworthy already. The corollary of the proposition – the fact the costs of substantial overreporting do not need to be positive – is even more striking. One could naïvely expect that the cost overreporting are proportional to the extent to which reserves are actually overstated, implying that "bigger lies bear bigger costs". Yet, this intuition is wrong. Instead, an overreporting exporter may actually gain not only from the implication of overreporting but from also from signaling (or "lying") itself. Thus, the general principle, according to which the low type trades off the gains from pooling against the costs of the signal, does not apply in the present model. Consequently, in the present qualitative exercise, the costs of the signal do not impose a strong restriction on the existence of the pooling equilibrium.

It is important to notice that the high type cannot signal its type by increasing the first period's supply because it is constrained by the incentive compatibility constraint (24). Any increase in the first period's supply would reduce the remaining reserves, decrease the second period's supply and hence increase country W's incentives to engage in substitution R&D.

Modelling framework. The analysis above relies on a set of convenient assumptions, which make the model tractable. These assumptions will be briefly evaluated concerning their role and essentiality.

First, good N is supplied monopolistically. This assumption is relevant for the mechanism because it enables the exporter of good N to control the intertemporal distribution of aggregate reserves. It allows the exporter to discourage the importer's substitution R&D, which is critical for the mechanism to bite.

Second, following the literature, substitution R&D is assumed to be a binary choice[19]. This is a convenient simplification but not a crucial assumption. A natural generalization would introduce the productivity B of the substitution technology in (4) as a continuous function of total R&D expenditure $a_1 \in \mathbb{R}_+$. Intuitively, under full information, the equilibrium $B(a_1)$ is then a decreasing function of the second period's supply n_2^* and of total reserves N^*. Hence, the scenario preserves the incentives to overreport reserves in order to discourage *marginal* substitution R&D and reduce the intensity of competition in period two. In a pooling equilibrium, then, the signal can still be arbitrarily close to the low type's first best supply $n_t^* = \xi \bar{N}/2$, rendering negative signalling costs for the low type.

Third, only one country imports good N. Country W's gains from substitution R&D accrue via reductions in export taxes while domestic production is zero for a wide parameter range.[20] Consequently, no private firm can recoup the investment $a_1 = A$, which must be financed publicly. If there are many small countries importing N, this generates free-riding incentives among N-importers regarding R&D expenditures. Yet, one should expect that the general mechanism outlined in this paper remains intact in a world where the technology B, once invented, delivers a flow of good N. Similarly, the argument applies if consumers, anticipating future prices, choose between alternative durable equipment thus affecting aggregate demand for N. In these cases, the returns to substitution R&D are appropriable by private firms or consumers and, thus, the assumption of a two-country world appears to be non-essential.

Fourth, only the importer of good N may engage in substitution R&D by assumption. This assumption can be justified by a comparative advantage in the R&D sector. Moreover, even under equal R&D opportunities country W benefits more from substitution R&D since it does not only expand its production possibility frontier but also trims country O's export taxes. Hence, it is more likely to incur the R&D costs.

Fifth, any aggregate uncertainty about resources and the R&D outcomes has been eliminated from the model. Introducing such additional uncertainties, the trade-off between country W's cost and benefits of substitution R&D and country O's prevention of R&D is

[19] See, *e.g.*, Dasgupta and Heal (1974), Deshmukh and Pliska (1983), Quyen (1988), and Barrett (2006)

[20] The range for N^* is $(3B + \sqrt{B} - 4, N_0)$ under full information (see the proof of (21) in the Appendix).

based on expected utilities (affecting conditions (17), (22), and (24)). It is unlikely that this can overturn the qualitative results.

Finally, the inclusion of export taxes T_t^* in the model may appear needless as, quite obviously, the N-exporter's incentives to discourage substitution R&D are present even if price discrimination between domestic and export markets is not possible. One reason to offer is realism, as standard models of world oil supply assume monopolistic or oligopolistic supply on the world market. More importantly, however, the export taxes generally affect the equilibrium of the signalling game under asymmetric information. Thus, country W gains from R&D not only by adding N-output world supply in the second period, but rather by reducing country W export taxes T_2^*. These export taxes increase in n_2^* (see (8)), which could, in general, mean that country W's engagement in substitution R&D is not monotonous in total reserves (in particular, $a_1(N^*)$ decreasing in N^* for some rage). Such a reversal of R&D activity due to the effects of export tariffs, could, in turn, imply N-exporters engage is under- instead of overreporting. A Second and related effect arise when country O's losses due to substitution R&D are increasing in its second period's supply n_2^*, as the constraint $T_2^* p_2 \leq 1/B$ is more restrictive for larger is n_2^*. The gains from discouraging substitution R&D are thus smaller the lower total reserves N^*. This second effect, working again through export taxes, could potentially limit the incentive to overreporting for exporters with low realizations of reserves and thus break the main argument of the paper. In view of these effects that tend to curb incentives to overreporting, the net effect of export taxes in the model is hard to assess *ex ante* and incorporating them can be seen as a conservative modelling strategy. Implicitly, the present paper's results include the observation that none of the effects mentioned dominates its main mechanism.

4.4 The Crude Oil Market

Motivated by rising concerns about supply security, this paper has raised the questions why, how, and under what conditions natural resource reserves can be overreported. It shows that exporting countries indeed have motives to overreport and that they can credibly do so under rational expectations. The necessary conditions for overreporting to occur are intuitive: substitution R&D must respond significantly to expected future supply and the costs of the required signal need to be limited. This subsection tries to answer the remaining question regarding the motivating example of the oil market: can the alleged overreporting of today's oil market be refuted?

First, the model's key assumptions are to be checked: a strong reaction of substitution R&D to expected and monopolistic supply. Concerning the first condition, evidence suggests that substitution R&D indeed responds to shortages of the market. Figure 1 illustrates the relation between non-oil energy R&D in IEA member countries and world oil prices for the period 1973-2006.[21] Moreover, current prices strongly correlate with price forecasts in the relevant period (see Lynch (2002) and IMF (2003)), which implies a comovement between expected future supply and R&D activity. Of course, a simple correlation does not imply causality. Yet, hard evidence shows that energy saving R&D is indeed responsive to supply shortages (see Newell et al (1999) and Popp (2002)). Thus, the first of the necessary conditions seems to be satisfied. The second requirement of non-competitive oil markets seems obvious. Contrary to conventional wisdom, however, empirical literature is inconclusive about OPEC's actual market power. Some quantitative studies indicate that in the years following the counter-oil shock in 1986, OPEC countries failed to behave as a cartel and over-supplied the world market instead of under-supplying it (Almoguera and Herrera (2007) and Lin (2007)). Other empirical studies such as Griffin (1985) and Smith (2003), report, however, substantial coordination and cartel discipline of OPEC members and a significant shortage of contemporaneous supply. In the latter case, a rough and tentative application of the model cannot refute that OPEC member states overreport their crude oil reserves.

Finally, Proposition 3 applies only if the respective costs of the signal, induced by the required supply deviations, are limited. A thorough quantitative assessment of the likelihood of overreporting must involve these costs. Within a first qualitative application of the theory, however, this requirement does not serve as a meaningful criterion since there is no positive lower bound on these costs of signaling.

In sum, the possibility of overreporting in today's oil market cannot be easily refuted by applying the present paper's predictions qualitatively. Thus, the last and – from the viewpoint of policymaking – the most urgent of the initial questions remains unanswered.[22] This observation calls for a thorough quantitative research of the issue, which thereby will answer the question whether the interpretation of OPEC is to be extended to a cartel of not only supply but also of information.

5 Conclusion

Concerns are rising about the supply security of crude oil. In addition to geological and political risks, some experts are pointing at overreporting as one – possibly significant – source of uncertainty. This paper has provided a simple but suggestive framework for the analysis of the incentive to overreport. The main elements of the theory are, first, market

[21] Strictly speaking, expected future supply is the determinant of substitution R&D and contemporaneous supply is irrelevant. The logical gap is bridged when the current price is the best predictor of future prices.

[22] Notice that credible overreporting – as defined above – can be refuted when the relevant conditions are violated. Conversely, however, misreporting cannot be proven before private information is revealed. Under credible overreporting the probability that reserves are high (π is the present model) must be positive.

power of the oil supplier, second, the possibility to engage in R&D for technologies that substitute oil, and third, private information about its remaining reserves. It has been shown that, within this framework, the only incentive to overreport can be attributed to the aim of exporters to discourage importers' R&D for substitution technologies. Surprisingly, an exporter with low reserves can pretend high reserves at zero or even negative costs. Finally, conditional on the reported realizations of reserves, supply is partly delayed under successful overreporting. In a tentative application of the main results to today's crude oil market overreporting cannot be dismissed.

In the discourse on supply security of crude oil overreporting of reserves is only one of many aspects and thus needs to be discussed in a broader picture. In absence of uncertainty the economics of exhaustible resources sketch a comforting image: the continued exhaustion of a natural resources raises the returns to oil-saving substitution technologies, which are eventually generated by intensifying research (see Davidson (1978), Deshmukh and Pliska (1983) and Tsur and Zemel (2003)). In this process forward-looking firms anticipate future profits and, motivated by consumer's willingness to pay for steady consumption flows, grant a smooth transition between a resource- and a substitution-based regime. This picture, however, changes when oil reserves are uncertain and information shocks cause ex-post inefficiencies. Hence, one must focus on the sources and magnitudes of uncertainty. Traditionally, geological and political unknowns are viewed as the major sources of uncertainty. Today, advanced exploration technology allows accurate assessments of the size of oil fields and tough surprises due to technological drawbacks seem unlikely (see e.g. Cuddington and Moss (2001)). Thus, man-made uncertainty appears to be the main source of worries. Within that category, political instability is usually focused on with a special emphasis on the geopolitical situation of the Middle East (see, e.g., IEA (2005)). Yet, if overreporting turned out to happen as reported, then the resulting supply shocks could easily dominate those stemming from the political field. In sum, overreporting may deserve some more attention after all.

A Appendix

Proof of (21). For $n_2 = 0$ use u_t^* from (13) to compute with the help of the envelope theorem

$$\frac{du_1^*}{dn_1^*} = \frac{T_1^* + 1}{(n_1^* + 2)^2} + \frac{1}{n_1^* + 2} = \frac{1}{(T_1^*)^2}$$

where (10) with $b_1 = 0$ was used in the second step. Use (10) with $b_2 = B$ and (13) to write $u_2^* = \ln(bT_2^*) + y_2^* + 1 - 1/B$ so that $du_2^*/dT_2^* = 1/T_2^*$. With $dT_2^*/dn_2^* = 1/B$ and $dn_1^*/dn_2^* = -1$ optimality requires

$$(T_1^*)^2 = BT_2^*$$

With (10) and $n_1^* + n_2^* = N^*$ rewrite this as $(\sqrt{9/4 + n_1^*} - 1/2)^2 = N^* - n_1^* + 2 - B$ or

$$2n_1^* + \tfrac{1}{2} - N^* + B = \sqrt{n_1^* + \tfrac{9}{4}}$$

Taking squares on both sides and solving for n_1^* leads to

$$n_1^* = 1/2 \left[N^* - B - 1/4 \pm \sqrt{(N^* - B)/2 + 2 + 1/16}\right]$$

The negative root is ruled out with the condition $N^* = B - 1 \Rightarrow n_1^* = 0$. The relevant condition for $n_2 = 0$ to hold is $n_2^* > 2(B - 1)$, which is equivalent to $N^* > N_o$ where solves

$$N_o - n_1^* = 2(B - 1) = 1/2 \left[N_o + B + 1/4 - \sqrt{(N_o - B)/2 + 2 + 1/16}\right]$$

or $N_o = 3B + \sqrt{B} - 4$. This proves the fist line of (21).

Consider now $N^* < N_o$ as long as O exports N (i.e. $c_{n,2}^* < n_2^*$) (13) applies and $n_2^* + n_2 + 2 = 2B$ imply $du_2^*/dT_2^* = 1/B$ so that optimality requires $(T_1^*)^2 = B$ or $n_1^* = B^2 + B - 2$. The relevant conditions for $n_2 > 0$ and $c_{n,2}^* < n_2^*$ to hold is

$$n_2^* = N^* - \left(B^2 + B - 2\right) \in ((B - 1), 2(B - 1))$$

or $N^* \in (2B - (\sqrt{4B + 5} + 1)/2, 3B + \sqrt{B} - 4)$. Finally, if $N^* < 2B - \left(\sqrt{4B + 5} + 1\right)/2$ optimality requires $c_{n,1}^* = c_{n,2}^* = n_2^*$ or $n_1^* = N/2 + 1/8\sqrt{8N + 25} - 5/8$. ∎

Proof of Claim 1. First, define the expression on the left of the identity (25) by $\Gamma(N^*, m)$. Now, by the definition (22) of $V^*(n_1^*, n_2^*, 0)$ and concavity of u_t^* (14) the partial derivative $\partial_m\Gamma$ is negative

$$\partial_m\Gamma = -\left[V_1^*(m, N^* - m, 0) - V_2^*(m, N^* - m, 0)\right] < 0 \tag{36}$$

for $m \in (0, N^*/2)$. (Subscripts stand for partial derivatives.)

(i) Check with (22) that

$$\begin{aligned} \Gamma(N^*, 0) &= V^*\left(n_C^*, N^* - n_C^*, B\right) - V^*\left(0, N^*, 0\right) \\ &= V^*\left(n_C^*, N^* - n_C^*, B\right) - V^*\left(N^*, 0, B\right) > 0 \end{aligned}$$

Further, $\Gamma(N^*, N^*/2) < 0$ holds by optimality (15) so that there is a solution to (25) with $m < N^*/2$. By $\partial_m \Gamma < 0$ this solution is unique.

(ii) The definition (22) of N_0 implies that $\Gamma(N^*, N^* - n_P^*) > 0$ if and only if $N^* \in (N_0, 2n_P^*]$ and the claim follows with $\Gamma(N_0, N_0 - n_P^*) = 0$ and (36).

(iii) $V^*(n_C^*, N^* - n_C^*, 0) > V^*(n_C^*, N^* - n_C^*, B)$ and (25) imply

$$V^*(n_C^*, N^* - n_C^*, 0) > V^*(m, N^* - m, 0)$$

By the concavity of u_t^* (14) and $m < N^*/2$ this shows the statement.

(iv) Compute

$$\begin{aligned} \partial_{N^*} \Gamma &= V_2^*(n_C^*, N^* - n_C^*, B) - V_2^*(m, N^* - m, 0) \\ &= V_1^*(n_C^*, N^* - n_C^*, B) - V_1^*(N^* - m, m, 0) \\ &= V_1^*(n_C^*, N^* - n_C^*, 0) - V_1^*(N^* - m, m, 0) \end{aligned}$$

The second equality holds by optimality of n_C^* and symmetry; the third since u_1^* is independent of b_2. The last expression is positive by (iii) and concavity of u_1^*, i.e. (14). Thus, with (36) the derivative

$$\frac{dm}{dN^*} = -\frac{\partial_{N^*} \Gamma}{\partial_m \Gamma} = \frac{V_1^*(n_C^*, N^* - n_C^*, 0) - V_2^*(m, N^* - m, 0)}{V_1^*(m, N^* - m, 0) - V_2^*(m, N^* - m, 0)}$$

is positive. As numerator and denominator are positive and $V_1^*(n_C^*, N^* - n_C^*, 0) < V_1^*(m, N^* - m, 0)$ holds by (iii), this shows $dm/dN^* < 1$.

(v) Follows from $\lim_{N^* \to 0} \Gamma(N^*, 0) = 0$ and (iv). ■

Proof of Proposition3. The proof consists of two parts: (i) Under (28) and (34) the strategies (35) and belief μ with A(i) - A(v) characterize an equilibrium. (ii) Under (28), (34), and A(i) - A(v) no other equilibria exist.

(i) E(i) - E(iii) are to be established.

E(i) $n_{1,H}^* = n_{1,L}^* = n_P^e$ and Bayesian updating requires $\mu(n_P^e) = \pi$.

E(ii) By $\mu(n_P^e) = \pi$ (24) holds for n_P^e and $a_1^e(n_P^e) = 0$ follows.

E(iii) Maximization of O_H. If O_H deviates to $\tilde{n} < n_P^e$, W's optimal off-equilibrium strategy induces either $\tilde{b} = 0$ or $\tilde{b} = B$. In both cases

$$V^*(\tilde{n}, \bar{N} - \tilde{n}, \tilde{b}) < V^*(n_P^e, \bar{N} - n_P^e, 0)$$

holds since $\tilde{n} < n_P^e < \bar{N}/2$.

If O_H deviates to $\tilde{n} \in (n_P^e, \bar{N} - n_P^*]$, condition (27) implies $n_P^e < \tilde{n} < \bar{N}/2$ and (28), (34), and A(iv) lead to $\mu(\tilde{n}) = \pi$ so that, finally, (24) is violated and W plays $a_1^e(\tilde{n}) = A$. If, instead, O_H deviates to $\tilde{n} > \bar{N} - n_P^*$ W's optimal strategy is $a_1^e(\tilde{n}) = A$ regardless of its beliefs. Thus, all deviations $\tilde{n} > n_P^e$ imply $\tilde{b} = B$. But

$$V^*(\tilde{n}, \bar{N} - \tilde{n}, B) < V^*(n_C^*(\bar{N}), \bar{N} - n_C^*(\bar{N}), B) < V^*(n_P^e, \bar{N} - n_P^e, 0)$$

(the last inequality follows by (27) and (34)).so that O_H's optimal strategy is $n_{1,H}^* = n_P^e$.

Maximization of O_L. If O_L deviates to $\tilde{n} \in (n_P^e, \bar{N} - n_P^*]$, condition (27) implies $n_P^e < \tilde{n} < \bar{N}/2$ and (28), (34), and A(iv) lead to $\mu(\tilde{n}) = \pi$ so that, finally, (24) is violated and W plays $a_1^e(\tilde{n}) = A$. If, instead, O_L deviates to $\tilde{n} > \bar{N} - n_P^*$ W's optimal strategy is $a_1^e(\tilde{n}) = A$ regardless of its beliefs. Thus, all deviations $\tilde{n} > n_P^e$ imply $\tilde{b} = B$, but

$$V^*(\tilde{n}, \xi\bar{N} - \tilde{n}, B) \leq V^*(n_{1,L}^*, \xi\bar{N} - n_{1,L}^*, b_{2,L}^*) < V^*(n_P^e, \xi\bar{N} - n_P^e, 0) \quad (37)$$

holds. The second inequality holds since either $b_{2,L}^* = B$ and $m(\xi\bar{N}) < m(\bar{N}) < n_P^e$ by Claim 1 (iv) and condition (34), or else $b_{2,L}^* = 0$ and (31) and (33) imply $n_P^e > \xi\bar{N} - n_P^*$ while (27), (31), and (32) imply $n_P^e < \bar{N} - n_P^* < n_P^*$.

If O_L deviates to $\tilde{n} < n_P^e$ with $\left|\xi\bar{N}/2 - \tilde{n}\right| < \left|\xi\bar{N}/2 - n_P^e\right|$, then $\tilde{n} < n_P^e < \bar{N}/2$ and A(v) imply $\mu(\tilde{n}) = 0$. Thus, (37) applies again. If O_L deviates to $\tilde{n} < n_P^e$ with $\left|\xi\bar{N}/2 - \tilde{n}\right| \geq \left|\xi\bar{N}/2 - n_P^e\right|$ O_L's total utility decreases under the deviation. Hence, O_L optimal strategy is $n_{1,L}^* = n_P^e$.

(ii) Assume there is an equilibrium with $n_{1,H}^e \neq n_P^e$. By A(iv) O_H's deviation to $\tilde{n} = n_P^e$ induces $a_1^e(\tilde{n}) = 0$ by construction of n_P^e. This deviation gives O_H higher payoffs. Hence $n_{1,H}^e = n_P^e$ in any equilibrium. By (i) this implies that $n_{1,P}^e = n_P^e$ and proves the claim. ∎

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