

## Problem Set n. 5

## Microeconomics Block I

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Name \_\_\_\_\_

*Due: by October 18, 2017 at 6pm*

1. Consider an economy with two individuals and two consumption goods. Agent A has endowments  $\omega^A = (5, 2)$  and agent B's endowment is  $\omega^B = (0, 1)$ .
  - (a) Suppose that A's preferences are described by  $u^A = 2 \ln x_1 + \ln x_2$ , while B's preferences are described by  $u^B = \ln x_1 + \ln x_2$ .
    1. Determine the set of Pareto efficient allocations.
    2. Find a competitive equilibrium for such economy.
  - (b) Suppose next that A's preferences are described by  $u^A = x_1$ , while B's preferences are described by  $u^B = 3x_2$ . Find the set of Pareto efficient allocations in this case. Does a competitive equilibrium exist in this case? Explain.
2. Consider a pure exchange economy with two commodities, 1 and 2, and two consumers, A and B, with endowments:

$$\omega^A = (1, k), k > 0$$

$$\omega^B = (2, 2)$$

and preferences represented by the following utility functions

$$u^A(x_2^A, x_1^A) = x_1^A$$
$$u^B(x_2^B, x_1^B) = x_1^B + \frac{1}{\beta} (x_2^B)^\beta, \beta < 1$$

- (a) Find a competitive equilibrium for the economy described above, deriving an expression of equilibrium prices and allocations as a function of the endowment parameter  $k$  and the preference parameter  $\beta$ .
  - (b) Show that for some values of  $\beta$  the equilibrium level of the utility of agent A is decreasing in  $k$ . In that case agent 1 may have an incentive to destroy part of his initial resources. If this does not happen at equilibrium, do we have a contradiction with the rationality of consumers in the economy?
3. There is one (representative) consumer and one (representative) firm in the economy. The consumer's preferences are represented by the utility function  $u(x_1, x_2) = x_1 + \ln x_2$ . Good 1 is "leisure" and initially the consumer owns one unit of good 1, and nothing of good 2. The firm is a price taker and its production function is  $f(z) = kz$ , where  $k > 0$  is a constant; that is, the producer produces  $kz$  units of good 2 from  $z$  units of good 1 (i.e., with one unit of "labor"  $k$  units of good 2 are produced.) All the profits are distributed to the consumer. Good 1 is the numéraire; i.e.,  $p_1 = 1$  in the following, and write  $p$  instead of  $p_2$  for simplicity.

- (a) What does the constant  $k$  represent?
  - (b) Argue that the firm cannot make profits (nor losses) in equilibrium.
  - (c) Derive the (consumers') demand curve for good 2.
  - (d) Derive the (firm's) supply curve for good 2.
  - (e) As  $k$  changes, how does the equilibrium price  $p$  change? and the output level (production of good 2)? Interpret.
4. There are two countries, A and B. There are two (types of) consumers, 1 and 2, in country A and there is one (type of) consumer, 3, in country B. Price of good 2 is set to be 1 throughout, and write  $p$  instead of  $p_1$ . Utility and endowments of consumers are summarized below:

	utility	endowment
1	$\ln x_1 + x_2$	$(6, 2)$
2	$\ln x_1 + 3x_2$	$(3, 3)$
3	$\ln x_1 + x_2$	$(4, 12)$

- (a) First assume that there is no international trade. Find the equilibrium in each country.
- (b) Suppose consumers can trade internationally. Find the equilibrium.
- (c) Does free world trade make everybody better off? Is this consistent with the first fundamental theorem of welfare economics? Discuss.