Problem Set n. 5 Microeconomics Block I

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Due: by October 25, 2017 at 7pm

1. Consider an economy in which there are two consumers $h = 1, 2$ and two commodities, $l = 1, 2$. There is a representative firm, using commodity 1 to produce commodity 2. The firm's production activities create a negative externality (industrial pollution) which reduces consumer 1's welfare. The two consumers have the same endowment $(2, 0)$ and preferences

$$\begin{aligned} U^1(x_1^1, x_2^1; y_2) &= x_1^1 x_2^1 - \bar{y}_2 \\ U^2(x_1^2, x_2^2) &= x_1^2 + x_2^2 \end{aligned}$$

where $\bar{y}_2$ is the total output level of the representative firm (taken as given by the consumer in his decisions). The firm's technology is described by the production function

$$y_2 = 4(z_1)^{1/2} \text{ for all } z_1 \geq 0$$

Consumer 2 owns the firm and receives all the profits. In equilibrium, the firm chooses inputs and outputs to maximize profits, taking prices as given. Consumers maximize utility subject to a budget constraint, taking as given the prices of both goods, the firm's profits (recall the firm's profits enter as income of the consumers who are owners of the firm), and its production decision.

   (a) Find the equilibrium allocation and equilibrium prices.

   (b) Show that the equilibrium allocation is not Pareto efficient and explain why.

2. Consider an economy composed by two countries, which are identical (have the same number and type of consumers and commodities). Each country taken in isolation has 3 competitive equilibria.

   (a) Argue that the number of equilibria in the overall economy in a situation of autarky, that is when the two countries cannot trade among them, is 3^2.

   (b) Suppose next the two countries can trade among them, that is there is free trade. Show that the number of equilibria of the overall economy in this case is 3.

3. For each of the following cases, give an example of an economy with H individuals with and L commodities (you can pick $H = L = 2$ and use an Edgeworth box representation to describe the economy) exhibiting that property:

   (a) There exists a Pareto suboptimal competitive equilibrium allocation

   (b) No competitive equilibrium exists.

   (c) There is a continuum of competitive equilibria.

2. Consider an exchange economy with H consumers and L goods. Suppose that this economy admits a no trade equilibrium (that is $x^h = \omega^h$ constitutes a competitive equilibrium allocation). Show that if utility functions are strictly quasi-concave (preferences are strictly convex) and satisfy local non satiation, this is in fact the only competitive equilibrium allocation.

3. Suppose there are two dates, $t = 0$ and $t = 1$ where a single (perishable) consumption good ($L = 1$) is exchanged and consumed. There are two consumers ($H = 2$), the first one with endowment $\omega^1 = (4, 0)$ (where the first element denotes the endowment at date 0 and the second one the endowment at date 1) while the endowment of individual 2 is $\omega^2 = (0; g)$ for some $g > 0$. The intertemporal utility function of the two individuals is given by

$$U^h(x_0; x_1) = \ln x_0 + \beta^h \ln x_1 \qquad \text{for some } \beta^h > 0;\ h = 1, 2$$

   (a) Suppose there is a perfect capital market: individuals can save or borrow against future income by buying or selling units of a riskless bond (promising one unit of income at date 1) at the price q_1. What is the relationship between q_1 and the interest rate on the riskless bond? Compute competitive allocations and the equilibrium value of the interest rate r as a function of g.

   (b) Is the competitive allocation Pareto optimal, and if so (or not), why? Does the rate of interest (always, that is for all g) coincide with the rate of time preference of individuals ? or for some special values of g, β^h?

   The economy is modified to allow for uncertainty, affecting the endowment at date 1 of the second consumer which is now random and take value g_1, g_2 with probability $\pi(1), \pi(2)$. The endowment of consumer 1 is still equal to zero at date 1 for every realization of the uncertainty. The consumers have intertemporal VNM utility functions which are immediate generalizations of the ones above (set for simplicity here $\beta^h = 1$ for all h):

$$U^h(x_0, x_1(1), x_1(2)) = \ln x_0 + \sum_{s=1}^{2} \pi(s) (\ln x_1(s)) \qquad h = 1, 2$$

   The uncertainty is realized at the beginning of date 1. At date 0 a market opens where the consumers can trade a complete market in elementary (Arrow) securities, that is, at a price $q(1)$ the consumer can trade a claim paying one unit iff state 1 realizes, and at a price $q(2)$ a claim paying one unit iff state 2 realizes.

   (c) State the optimization problem of an individual and derive the first-order conditions for an individual optimum.

   (d) Define a competitive equilibrium in this environment and derive competitive equilibrium prices and allocations. Is the competitive allocation Pareto optimal, and if so (or not), why? Do prices of Arrow securities $q(1), q(2)$ always coincide with the probabilities π (that is, for all values of g_1, g_2)? Or only in some special cases (for some special values of g_1, g_2)?