Problem Set n. 1 **Microeconomics Block I**

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Due: by September 21, 2017 at 10am

1. Prove the Claim on p. 3 of the slides.

2. **(testable implications of rationality)** Suppose there are two consumption goods and a consumer with income m facing prices $p = (p_1, p_2)$ in the market. Suppose this consumer makes the following consumption choices c^k for the different levels of income m_k and prices p^k, $k = i., ii., iii., iv.$ in the table below [*you can assume that the optimal choice for any pair p^k, m_k is unique*]:

	p^k	income m^k	c^k
i.	(1,2)	5	(3,1)
ii.	(2,1)	5	(1,3)
iii.	(1,3)	6	(1,5/3)
iv.	(1,1)	4	(x_1, x_2)

   (a) Show, with the help of a figure, that choices i. and ii. above are consistent with the rationality of the consumer's choice

   (b) Show, with the help of a figure, that choices i. and iii. above are inconsistent with the rationality of the consumer's choice

   (c) Draw in a figure the region of values of (x_1, x_2) such that choices i., ii., and iv. above are consistent with the rationality of the consumer's choice

3. (**WARP**) Let X be the set of alternatives and suppose it contains 4 elements. Find a collection $\{B_i\}_{i=1}^I$ of I subsets of X such that, if you are given the choice made for each B_i and such choices satisfy WARP, you can uniquely determine the preference relationship of the decision maker which rationalizes these choices. Verify such preference relationship is rational. [*you must find a collection of subsets which allows you to recover the preference relationship and then to show there is only one such preference relationship*]

4. (**homothetic preferences**)

   (a) Prove Claim A.3 in the slides**:** Under A.3, $x(p, \alpha m) = \alpha x(p, m)$ for all $m, p \gg 0, \alpha > 0$ (you can assume also $A.1'$ holds).

5. Prove the following propositions by the corresponding techniques.

   (a) (Direct proof) Let $a, b, c \in \boldsymbol{N}$ and let a be divisible by b and b be divisible by c. Then a is divisible by c.

   (b) (Proof by contradiction Reductio ad absurdum) $\boldsymbol{R}_{++}$ has no smallest element. (Recall that $\boldsymbol{R}_{++} := \{x : x > 0,\ x \in \boldsymbol{R}\}$)

   (c) (Proof by contraposition) Let $a \in \boldsymbol{N}$. If a^2 is odd, then so is a.

   (d) (proof by counterexample) Exercise 1