Problem Set n. 4 Microeconomics Block I

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Due: by October 11, 2016 at 6pm

1. (**linear activity production set**) Consider the following production set

$$Y = \left\{ y \in \mathbb{R}^L : y = \sum_{k=1}^{K} t_k a_k \text{ for some } t_k \in \mathbb{R}_+,\ k = 1,..,K \right\}$$

where for each $k = 1,..,K$, a_k is a fixed vector in $\mathbb{R}^L$.

   (a) Show that Y exhibits constant returns to scale

   (b) Show that if for some p we have $p{\cdot}a_k > 0$ for some k, then the maximal profit is infinite, while if $p{\cdot}a_k \leq 0$ for all k the maximal profit is zero and the supply correspondence is given by $y(p) = \left\{ y \in \mathbb{R}^L : y = \sum_{k=1}^{K} t_k a_k \text{ for } t_k \geq 0,\ t_k = 0 \text{ if } p \cdot a_k < 0 \right\}$.

   (c) Suppose that $L = 2$, $K = 3$, and $a_1 = (-2, 1)$, $a_2 = (-3, 4)$, $a_3 = (1, -5)$. Draw the set Y in this case. If the price vector is $p = (1, 1)$ what is the level(s) of the optimal net supply? And for $p = (4, 1)$? And if $p = (2, 1)$?

2. (**production under financial constraints**) Consider a competitive firm with a concave production function $y_1 = f(z_1, ..., z_{L-1})$. The price of the output is $p_1 = 1$ and the price of input l is w_l so the total cost of inputs is $\sum_{l=1}^{L-1} w_l z_l$. The firm is under a financial constraint which implies that there is a maximum amount $\bar{C}$ it can spend for purchase of factors of production.

   (a) Write the firm's profit maximization problem.

   (b) Show that for all $f(.)$ strictly concave the financial constraint does not bind for at least some prices p, w if $\bar{C}$ is large enough.

   (c) Show that the firm's profit is non decreasing in $\bar{C}$ (as the financial constraint gets less severe, the profit increases, at least weakly).

3. Prove that the cost function $C(w, y)$ exhibits the following properties:

   (a) $C(\alpha w, y) = \alpha C(w, y)$

   (b) $C(w, y)$ is convex in y if the firm's production function is concave

4. Consider a firm producing a good as output using three goods as inputs according to the following production function:

$$y = f(z_1, z_2, z_3) = (z_1 + z_2)^{1/2}\ z_3^{1/2}$$

   (a) Does the technology described by this function exhibit increasing, constant or decreasing returns to scale?

(b) Let w_1, w_2, w_3 denote the prices of the three inputs. Consider the firm's problem of minimizing costs so as to attain a given level of production $y > 0$. Describe (i) first the conditions under which the solution of this problem obtains at $z_1 = 0$, then (ii) those under which the solution obtains at $z_2 = 0$, and last (iii) when it obtains at $z_3 = 0$.

(c) On the basis of your findings in the answer to the previous question, find the conditional factor demand and the firm's cost function.

5. Prove the two following claims in the slides:

(a) If consumers have identical homethetic preferences a representative consumer exists, that is we can find a utility function $\mathbf{U} : \mathbf{R}_+^L \to \mathbf{R}$ and income level $\mathbf{m}$ such that $x(p, (m^h)_{h=1}^H) = x(p, \mathbf{m}; \mathbf{U})$ for all $p, (m^h)_{h=1}^H$ (as on p. 49 slides)

Illustrate the validity of the result for the case where there are two commodities ($L = 2$) and two consumers with identical preferences given by $U(x_1, x_2) = 5 \ln x_1 + \ln x_2$ and income m^1, m^2.[*hint: you can derive the individual demand of the two consumers, then the aggregate demand* $x(p, (m^h)_{h=1}^2)$, *and find then the preferences* $\mathbf{U}$ *of the representative consumer such that* $x(p, \mathbf{m}; \mathbf{U}) = x(p, (m^h)_{h=1}^2)$]

(b) (p. 53 slides) A representative producer always exists.