Problem Set n. 2 Microeconomics Block I

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Due: by 8pm September 27, 2017

1. Show that the following properties of the terms S and v stated on p. 22 of the slides hold:

   (a) $v \cdot p = 1$ and

   (b) S is symmetric, negative semi-definite and such that $p^T S = 0$.

2. Consider a consumer with utility function $U(x_1, x_2) = 10\,(x_1)^{1/2} + (x_2)^{1/2}$ and income m.

   (a) find the consumer's demand functions $x_i(p_1, p_2, m)$, $i = 1, 2$.

   (b) find the substitution term in the Slutsky equation for $\partial x_1/\partial p_2$.

   (c) Say whether x_1 and x_2 are gross substitutes or gross complements.

3. Let $L = 2$.

   (a) Show that the following pair of functions, defined for all $m, p_1, p_2 > 0$:

   $$\begin{aligned} x_1 &= \left(\alpha m^2 + \beta\,(p_2)^2\right)/\gamma\,(p_1)^2 \\ x_2 &= \delta m + \eta p_1 + \xi p_2 \end{aligned}$$

   cannot be (that is, for no value of the parameters $\alpha, \beta, \gamma, \delta, \eta, \xi$) the demand functions for the two commodities of a consumer with preferences satisfying A.1 (local non satiation). What if A.1 is not imposed?

   (b) Consider next the following other pair of functions:

   $$\begin{aligned} x_1 &= \frac{\alpha m}{\beta p_1 + \gamma p_2} \\ x_2 &= \frac{\delta m}{\eta p_1 + \xi p_2} \end{aligned}$$

   Show that these can be the demand functions for the two commodities of a consumer with preferences satisfying A.1 (local non satiation) and $A.2'$ (strict convexity). Which restrictions on the parameters $\alpha, \beta, \gamma, \delta, \eta, \xi$ suffice for this?

4. Consider a consumer with preferences represented by $U(x_1, x_2) = \min\{x_1, x_2\}$.

   (a) Verify the preferences are convex

   (b) Find the consumer demand's function $x(p, m)$ and the indirect utility function $V(p, m)$

   (c) Verify Roy's identity.

(d) Find the consumer's compensated demand $x(p, u)$ and expenditure function $E(p, u)$.

5. Consider the case where $L = 2$ and preferences are represented by a quasi-linear utility function $U(x_1, x_2) = x_1 + v(x_2)$ with $v' > 0, v'' < 0$. Show that the demand for good 2 does not depend on income [*you can ignore the possibility of solutions on the boundary of the consumption set*]. What can you say on the properties of the indirect utility function $V(p, m)$, the compensated demand $x(p, u)$ and the expenditure function $E(p, u)$ [*in addition to the properties we derived in class for* $V(.), x(.), E(.)$ *for the case of general* $U(x_1, x_2)$]?