Problem Set n. 3 Microeconomics Block I

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Due: by October 4, 2017 at 10am

1. (**duality**) Consider a consumer with preferences represented by $U(x_1, x_2) = 2\,(x_1)^{1/2}\,(x_2)^{1/2}$.

   (a) Find the consumer demand's function $x(p, m)$ and the compensated demand function $x(p, u)$.

   (b) Find the expenditure function $E(p, u)$ and verify it satisfies the property: $\partial E/\partial p_i = x_i(p, u)$, $i = 1; 2$.

2. (**separability**) Consider an additively separable utility function $U(x) = u^1(x_1) + ... + u^L(x_L)$.

   (a) Show that if each $u^i(.)$, $i = 1, .., L$, is strictly concave so is $U(.)$

   (b) Show that all goods are normal (assume $u^i(.)$ is strictly concave).

3. Let $L = 2$ and consider a consumer with the following indirect utility function $V(p, m) = m/(p_1 + 2p_2)$. Find the consumer's demand function $x(p, m)$, his expenditure function $E(p, u)$ and his utility function $U(x)$.

4. (**supply and profit functions**) Consider a firm with technology described by the following production function: $y_1 = (z_1)^\beta + (z_2)^\gamma$ with $\beta, \gamma < 1$. Find the output supply function $y_1(p_1, w)$, the demand function of the two inputs $z(p_1, w)$ and the profit function $\pi(p_1, w)$. Verify then that the profit function is homogeneous (of degree ?). Explain, without doing the computations, what do you need to do to verify that $\pi(p_1, w)$ is convex.

5. (**supply and profit functions again**) Consider a firm with technology described by the production function: $y_1 = \min\{z_1, z_2\}^\gamma$. Find the output supply function $y_1(p_1, w)$, the demand function of the two inputs $z(p_1, w)$ and the profit function $\pi(p_1, w)$. Which restriction do you need to impose on the values of γ for such functions to be well defined?