Problem Set 2

Due by: May 16, 2018

Answers have to be submitted and prepared by each of you in isolation (though you are encouraged to discuss the problems and class material among you, what you submit should be your own answer)

1. Consider an infinite horizon economy, populated by two types of consumers, $h = 1, 2$. Time is discrete, $t = 1, 2, ..$ and there is a single, non storable consumption good. Endowments of the two consumers at any date t are

$$\omega^1(s_t) = 12 - s_t$$
$$\omega^2(s_t) = 6 + s_t$$

where s_t is a random variable governed by a two state Markov chain taking values $s_t \in \{0, 2\}$ and with time invariant transition probabilities given by $\pi(s_t = 0/s_{t-1} = 0) = 1/2$ and $\pi(s_t = 2/s_{t-1} = 2) = 2/3$ and the probability of the initial state is $\pi(s_1 = 2) = 1/2$. The preferences of consumer h are given by:

$$U(x^h) = \sum_{t=1}^{\infty} \sum_{s^t} \beta^t \pi(s^t) \ln(x_t^h(s^t)), \qquad \beta < 1$$

where $s^t = (s_1, .., s_t)$.

   a. Find the Pareto efficient allocation of this economy corresponding to the welfare weights $\lambda^1 = 1, \lambda^2 = 1$.

   b. Consider the following market structure: a complete set of contingent claims is available for trade at the initial date, prior to the realization o any shock (say $t = 0$). For any date event t, s^t consumers can trade a claim contingent on s^t occurring at date t at a price $p(s^t)$.

      i. Define a competitive equilibrium for this economy.

      ii. Find the competitive equilibrium prices and the consumption allocation.

   c. Consider next the following, alternative market structure: trade occurs sequentially and at each date event t, s^t a complete set given by two one period Arrow securities are available for trade, at the prices $q(s_{t+1} = 0; s^t)$, $q(s_{t+1} = 2; s^t)$.

      i. Write down the consumers' choice problem in this case. Explain why borrowing constraints are needed for the problem to be well defined. Provide one possible specification of the 'natural borrowing constraint' faced by each consumer at each node and argue that under such specification the consumers' problem is well defined and, also, the constraint is never binding at a solution of the consumers' problem. Show that other specifications of the borrowing constraint exist which ensure that the consumers' problem is well defined [*though possibly the resulting equilibrium allocation is not the same*].

      ii. Define a competitive equilibrium in this environment with sequential trades, when consumers face the natural borrowing limit. Find equilibrium prices and allocation.

iii. Suppose an additional asset is introduced, which can be traded at every t, s^t: this is an infinitely lived asset, which is in zero net supply and promises to deliver one unit of the consumption good at each date, whatever the realization of the uncertainty. Let $q(s^t)$ denote the price of this asset at t, s^t. Find the equilibrium price of this asset. Has the introduction of this asset any effect on the equilibrium allocation?

d. Suppose next that, instead of having two Arrow securities at each date event, only one asset, a one period riskless bond is available for trade. What can you say about the properties of the equilibrium allocation and equilibrium prices in this case? And about the efficiency properties of competitive equilibria? [*you are not required to solve for the equilibrium values to answer this question*]

(a) Consider again the same market structure as in c. above, with two Arrow securities available for trade in each period, but now there are also A units of a tree, an infinitely lived real asset, and each agent is initially endowed at $t = 0$ with $A/2$ units of the tree. Each unit of the tree pays a dividend equal to 3 units of the consumption good each period. Suppose here for simplicity that shocks are iid over time: $\pi(s_t = 0) = \pi(s_t = 2) = 1/2$ whatever the past history. In addition, suppose there is limited commitment, so that agents can default on their loans obligations, incurring no other punishment than the loss of the collateral posted. All loans have then to be collateralized: in particular, for each unit shorted of an Arrow security there is the requirement to hold .3 units of the tree as collateral. Suppose $\beta = .9$;find the minimal amount of the tree $\bar{A}$ in the economy which ensures that competitive equilibria with collateral constraints are Pareto efficient.

2. Consider an infinite horizon, pure exchange economy populated by two (types of) individuals with identical preferences

$$E\sum_{t=0}^{\infty}\beta^t u(x_t) = E\sum_{t=0}^{\infty}\beta^t (Bx_t - \frac{x_t^2}{2}), \qquad \text{with } \beta = 0.8$$

Each individual of type $h = 1$ receives 4 units of the (single) consumption good at any given date $t = 0, 1, ..$ when the state realized is $s_t = 1$ and 8 units in state $s_t = 2$, while individual of type $h = 2$ receives 12 units in state $s_t = 1$ and 8 units in state $s_t = 2$. The probability of $s_t = 1$ is 0.5 in any given period and the realization of the state is iid across periods. Thus the per capita endowment is always 8 in every period. Suppose a complete set of contingent claims is available for trade at date 0, prior to the realization of the uncertainty in that period: for any date event t, s^t consumers can trade a claim contingent on s^t occurring at date $t = 0, ..$ at a price $p(s^t)$.

(a) Suppose consumers are able to fully commit to the obligations undertaken with their trades in the market. Find a competitive equilibrium in this case.

(b) Next, suppose consumers are not able to commit, as every individual is free to renegue on the obligations undertaken with his trades at $t = 0$ (that is, he can refuse to deliver the consumption good he promised to deliver), but when he does so he is excluded from al future transactions and must then live in autarky forever after.

A. Find the equilibrium consumption allocation in the special case where the consumption of any individual in any period t can only depend on the current realization s_t, not on past realizations.

B. Can the equilibrium allocation found above (in A.) be improved - still satisfying feasibility and the limited commitment constraints - if we allow for history dependent consumption levels (that is the consumption in any given period can depend on the whole history of endowment realizations of the individual up to that point in time)?

C. Explain how the optimal consumption plan varies when the discount factor β goes to one. Explain how the optimal consumption plan varies when β goes to zero.

D. Explain how the equilibrium consumption found in A. above varies if type 1 consumers have endowment 2 and 10 in the two states, while type 2 consumers have endowments 14 and 6.

3. Illustrate the main differences between the default models of Dubey, Geanakoplos, Shubik (2005) and Kehoe, Levine (2001). Explain why default may occur in equilibrium in the first model but never in the latter. Compare the efficiency properties of competitive equilibria in the two models (In particular, can we have Pareto ranked equilibria - that is two equilibria in one of which all agents have a higher utility than in the second equilibrium - in the first model? and in the second one? explain why).

4. Consider an economy populated by a continuum of identical farmers. Each farmer has an endowment of one unit of labor; there is a single consumption good (and consumption only takes place after the farmers' production activity). He has access to two risky production technologies: technology P requires an input of $l = 1$ units of labor and yields 25 units of the consumption good with probability 0.7 and 9 unit of the good with probability 0.3. Technology U requires no input of labor (thus $l = 0$) and yields 9 units (resp. 25) units of the consumption good with probability 0.7 (resp. 0.3). The productivity shocks are i.i.d. across the population. Each farmer has Von-Neumann Morgenstern preferences over his consumption in his two productivity states and his leisure $(1 - l) : E[\ln(x)] + v(1 - l)$ where $v(.)$ is an increasing function of the amount of leisure (note that the choice of leisure can be either 0 or 1). Assume that $0.7 \ln(25) + 0.2 \ln(9) > \ln(13.8) + v(1)$.

   (a) Consider first the case where the farmers' choice of the technology is commonly observable: find the symmetric Pareto optimal allocations of this economy. Present a structure of markets which allow the decentralization of this Pareto optimum as a competitive equilibrium.

   (b) Suppose next the technology choice is each farmer's private information. Find the (now incentive constrained) Pareto optimal allocation in this case, and present a structure of markets which allow its decentralization as a competitive equilibrium.

   (c) Argue that full information is needed over the trades made by farmers in the existing markets for the equilibrium described in (b). If on the other hand no information is available over the trades made by farmers, how would the structure of feasible markets change? Write down the agents' choice problem in this case and find then a competitive equilibrium. Compare it to the one found in b.