Microeconomics, Block I
Mock Exam

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*[**Instructions.** ... You are allowed to bring to the exam a pocket calculator. (Please turn off your mobile phone.) Exchanging information or communicating with other people, as well as any other form of cheating, implies the immediate disqualification of your exam. The weight attributed to each question for the final grade is indicated below (total points are 100 plus 2 bonus points, the threshold for passing is 50 points). Good luck!]*

A) Consumer and Producer Theory

1. (**21**) Consider a firm producing commodity one using commodities 2 and 3 as inputs according to the following production function $y_1 = (z_2 + z_3)^\beta$, for $\beta > 0$.

   (a) (**8**) Suppose $w_2 > w_3$ and $\beta < 1$. Find the firm's conditional factor demand (solution of the cost minimization problem) $z_i(w_2, w_3, y_1)$, $i = 2, 3$ as well as the firm's unconditional factor demand (solution of the profit maximization problem) $z_i(w_2, w_3, p_1)$, $i = 2, 3$.

   (b) (**5**) Suppose again $w_2 > w_3$ but now $\beta > 1$. Does the solution of the cost minimization problem vary in this case (with respect to the one in (a))? and the one of profit maximization problem? Comment your findings.

   (c) (**8**) Derive the cost function for this firm (again supposing $w_2 > w_3$). Verify that $\partial C/\partial w_3 = z_3(w_1, w_2, y_1)$. Derive the expression of $\partial z_3/\partial w_3$ in this case.

2. (**15**) Assume $U(x)$ is continuous.

   (a) (**10**) Show that the solution $x(p, m)$ of the consumer's choice problem:

   $$\max_{x \in R^L_+} U(x)$$
   $$\text{s.t. } p \cdot x \leq m$$

   is (i) homogeneous of degree 0 in p, m and (ii) satisfies WARP.

   (b) (**5**) show that, if the consumer faces the additional constraint requiring that the expenditure in the first two goods cannot exceed some exogenous value $\bar{K}$, $p_1 x_1 + p_2 x_2 \leq \bar{K}$, the homogeneity property (i) no longer holds for all p, m.

B) General Equilibrium

1. (**46**) Consider a pure exchange economy with two (types of) consumers, A and B, and two commodities, 1 and 2. Agent A has an initial endowment $\omega^A = (8, 0)$ and B has endowment $\omega^B = (2, 4)$.

   (a) **(12)** Suppose that agents' preferences are $U^A(x_1^A, x_2^A) = 2 \ln x_1^A + \ln x_2^A$ for A and $U^B(x_1^B, x_2^B) = \ln x_1^B + 2 \ln x_2^B$ for B. Find the set of all the Pareto efficient allocations for this economy.

   (b) (**8**) Show that the initial endowment allocation ω^A, ω^B is not Pareto efficient [*hint: find a possible direction in which the distribution of total resources at ω^A, ω^B could be modified so as to improve both agents' utilities*]

   (c) **(16)** Find a competitive equilibrium (prices and allocation). Verify that the equilibrium allocation is Pareto efficient.

   (d) (**10**) Consider the effects of an increase in the endowment of commodity 1 of type B agents. Their endowment is now $\omega'^B = (2 + 1, 4)$. What is the effect of this increase in $B's$ endowment on the equilibrium relative price p_2/p_1? Will agent 1 benefit or loose as result of this change? Explain the result you obtained.

2. (**20**) Consider a pure exchange economy under uncertainty with two types of consumer (A and B), a single commodity ($L = 1$) and $S = 2$ states of nature. The two consumers have VNM preferences $\mathbb{E}_{\pi^A} u^A(x)$, $\mathbb{E}_{\pi^B} u^B(x)$, with identical beliefs $\pi^A(s) = \pi^B(s) = 1/2$, s=1,2. Also assume that agent 1 is risk neutral ($u^A(x) = x$) while agent 2 is risk averse (that is $\partial^2 u^B / \partial x^2 < 0$).
   The type A consumer has then endowments equal to 10 units in state 1 and 2 units in state 2 while type B has endowments equal to 2 in state 1 and to 6 in state 2.

   (a) (**15**) Find a competitive equilibrium (prices and allocation) when a complete set of Arrow securities is available for trade in this economy (that is one security paying one unit if and only if state $s = 1$ occurs, and a second security paying one unit if and only if state 2 occurs). [*note that you do not need to know the specific form of $u^B(x)$ to answer this question*].

   (b) (**5**) Are the prices and allocation you obtained in (a) still an equilibrium if $\pi^A(1) > \pi^B(1) = 1/2$, that is the two agents have now different beliefs? [*you should say whether any agent has an incentive to deviate/change the level of his trades. If so, which agent, and in which direction.*]