Appendix B – NOT TO BE INCLUDED IN PRINTED VERSION. AVAILABLE ONLINE

Proposition B.1: *When θ is commonly observable, under Assumption 2 the optimal contract still implements actions x and y and is characterized as follows:*

$$\begin{aligned} e^{-a\bar{w}_1} &= \frac{(1-\pi(y,\theta_1))e^{-a(\bar{u}+c_x)} - (1-\pi(x,\theta_1))e^{-a(\bar{u}+c_y)}}{\pi(x,\theta_1)-\pi(y,\theta_1)} \\ e^{-a\underline{w}_1} &= \frac{\pi(x,\theta_1)e^{-a(\bar{u}+c_y)} - \pi(y,\theta_1)e^{-a(\bar{u}+c_x)}}{\pi(x,\theta_1)-\pi(y,\theta_1)} \\ \bar{w}_2 &= \underline{w}_2 = \bar{u}+c_y \end{aligned}$$

Thus th*e agent's expected utility is the same in θ_1 and θ_2.*

Proof of Proposition B.1. Observe first that (IC2) implies that $\bar{w}_1 > \underline{w}_1$. The property $\bar{w}_2 = \underline{w}_2$ can then be easily verified and ensures that (IC5) is always satisfied. Consider then the first order conditions of problem of maximizing the principal's expected revenue subject to (IC2), and (PC):[24]

$$\begin{cases} (i) & -p\pi(x,\theta_1) + \lambda_{IC}(-a\pi(y,\theta_1)e^{-a(\bar{w}_1-c_y)} + a\pi(x,\theta_1)e^{-a(\bar{w}_1-c_x)}) + \\ & \qquad \lambda_{PC}ap\pi(x,\theta_1)e^{-a(\bar{w}_1-c_x)} = 0 \\ & \\ (ii) & -p(1-\pi(x,\theta_1)) + \lambda_{IC}(-a(1-\pi(y,\theta_1))e^{-a(\underline{w}_1-c_y)} + a(1-\pi(x,\theta_1))e^{-a(\underline{w}_1-c_x)} + \\ & \qquad \lambda_{PC}ap(1-\pi(x,\theta_1))e^{-a(\underline{w}_1-c_x)} = 0 \\ & \\ (iii) & -(1-p) + (1-p)\lambda_{PC}ae^{-a(w_2-c_y)} = 0 \end{cases}$$

where $\lambda_{IC}, \lambda_{PC}$ are the Lagrange multipliers attached to constraints (IC2), (PC).

Condition (iii) implies that $\lambda_{PC} = \frac{e^{a(w_2-c_y)}}{a} > 0$ and hence that (PC) is binding. Take now the summation of (i) and (ii) and use the complementary slackness condition (requiring that $\lambda_{IC} \times (IC) = 0$), to obtain:

$$-p + a\lambda_{PC}p[\pi(x,\theta_1)e^{-a(\bar{w}_1-c_x)} + (1-\pi(x,\theta_1))e^{-a(\underline{w}_1-c_x)}] = 0$$

Using the fact that (PC) is binding, this amounts to:

$$-p + a\lambda_{PC}[e^{-a\bar{u}} - (1-p)e^{-a(w_2-c_y)}] = 0$$

[24]Rewrite (IC2) as follows

$$e^{-a\bar{w}_1}[\pi(x,\theta_1)e^{ac_x} - \pi(y,\theta_1)e^{ac_y}] \leq e^{-a\underline{w}_1}[(1-\pi(y,\theta_1))e^{ac_y} - (1-\pi(x,\theta_1))e^{ac_x}]$$

A necessary condition for this to hold and hence for the existence of a solution to the principal's programme is $e^{a\Delta c} \leq \frac{1-\pi(y,\theta_1)}{1-\pi(x,\theta_1)}$.

and finally, $e^{-a(w_2-c_y)} = e^{-a\bar{u}}$. Using again the fact that (PC) binds, we obtain that

$$\pi(x,\theta_1)e^{-a(\bar{w}_1-c_x)} + (1-\pi(x,\theta_1))e^{-a(\underline{w}_1-c_x)} = e^{-a\bar{u}} = e^{-a(w_2-c_y)},$$

thus establishing the fact that at a solution of the above problem the utility of the agent is the same in state θ_1 and θ_2, in contrast with the property established in Proposition 1 for the optimal flexible contract when θ is only privately observed by the agent. □

Proposition B.2: Assume the condition in Assumption 2.ii) is replaced by the following: $\pi(x,\theta_1) > \pi(x,\theta_2) > \pi(y,\theta_1) > \pi(y,\theta_2)$. Then the optimal flexible contract is the same as the optimal contract obtained when θ is observable.

Proof of Proposition B.2

To establish the result it suffices to show that the optimal flexible contract obtained in Proposition B.1 remains feasible when θ is unobservable and $\pi(x,\theta_1) > \pi(x,\theta_2) > \pi(y,\theta_1) > \pi(y,\theta_2)$ holds, that is, it satisfies all the remaining constraints, (IC1, IC3, IC4, IC6).

It follows from the proof of Proposition B.1 that the optimal contract when θ is observable satisfies (IC2) and the two following conditions (corresponding to (PC) when $\bar{w}_2 = \underline{w}_2$ and $u(\theta_1) = u(\theta_2)$):

(a) $\pi(x,\theta_1)e^{-a(\bar{w}_1-c_x)} + (1-\pi(x,\theta_1))e^{-a(\underline{w}_1-c_x)} = e^{-a\bar{u}}$

(b) $e^{-a(w_2-c_y)} = e^{-a\bar{u}}$

(IC1), given (a) and (b), amounts to $e^{-a\bar{u}} \le e^{-a\bar{u}}e^{a\Delta c}$, which is satisfied given that $\Delta c > 0$.

(IC3), given (a) and (b), amounts to $e^{-a\bar{u}} \le e^{-a\bar{u}}$, and hence also holds.

(IC4), given (b), amounts to $-e^{-a\bar{u}} \ge -\pi(y,\theta_2)e^{-a(\bar{w}_1-c_y)} - (1-\pi(y,\theta_2))e^{-a(\underline{w}_1-c_y)}$. Using condition (a), this inequality can be equivalently written as $-\pi(x,\theta_1)e^{-a(\bar{w}_1-c_x)} - (1-\pi(x,\theta_1))e^{-a(\underline{w}_1-c_x)} \ge -\pi(y,\theta_2)e^{-a(\bar{w}_1-c_y)} - (1-\pi(y,\theta_2))e^{-a(\underline{w}_1-c_y)}$. Since $\pi(y,\theta_1) > \pi(y,\theta_2)$ and $\bar{w}_1 > \underline{w}_1$, we have $-\pi(y,\theta_1)e^{-a(\bar{w}_1-c_y)} - (1-\pi(y,\theta_1))e^{-a(\underline{w}_1-c_y)} > -\pi(y,\theta_2)e^{-a(\bar{w}_1-c_y)} - (1-\pi(y,\theta_2))e^{-a(\underline{w}_1-c_y)}$, and hence, using (IC2), we get $-\pi(x,\theta_1)e^{-a(\bar{w}_1-c_x)} - (1-\pi(x,\theta_1))e^{-a(\underline{w}_1-c_x)} \ge -\pi(y,\theta_2)e^{-a(\bar{w}_1-c_y)} - (1-\pi(y,\theta_2))e^{-a(\underline{w}_1-c_y)}$. Given the argument above, this ensures that (IC4) is satisfied. Notice that this result does not hold if, instead of the condition, $\pi(x,\theta_1) > \pi(x,\theta_2) > \pi(y,\theta_1) > \pi(y,\theta_2)$, we impose Assumption 2.ii).

(IC6), given (a) and (b), amounts to $-e^{-a\bar{u}} = -\pi(x,\theta_1)e^{-a(\bar{w}_1-c_x)} - (1-\pi(x,\theta_1))e^{-a(\underline{w}_1-c_x)} > -\pi(x,\theta_2)e^{-a(\bar{w}_1-c_x)} - (1-\pi(x,\theta_2))e^{-a(\underline{w}_1-c_x)}$, which is satisfied given that $\pi(x,\theta_1) > \pi(x,\theta_2)$ and $\bar{w}_1 > \underline{w}_1$.

We have thus shown that, when $\pi(x,\theta_1) > \pi(x,\theta_2) > \pi(y,\theta_1) > \pi(y,\theta_2)$, the optimal contract for the case where θ is observable is a feasible contract also when θ is unobservable. Hence it is the optimal contract also in that case. □

Figures

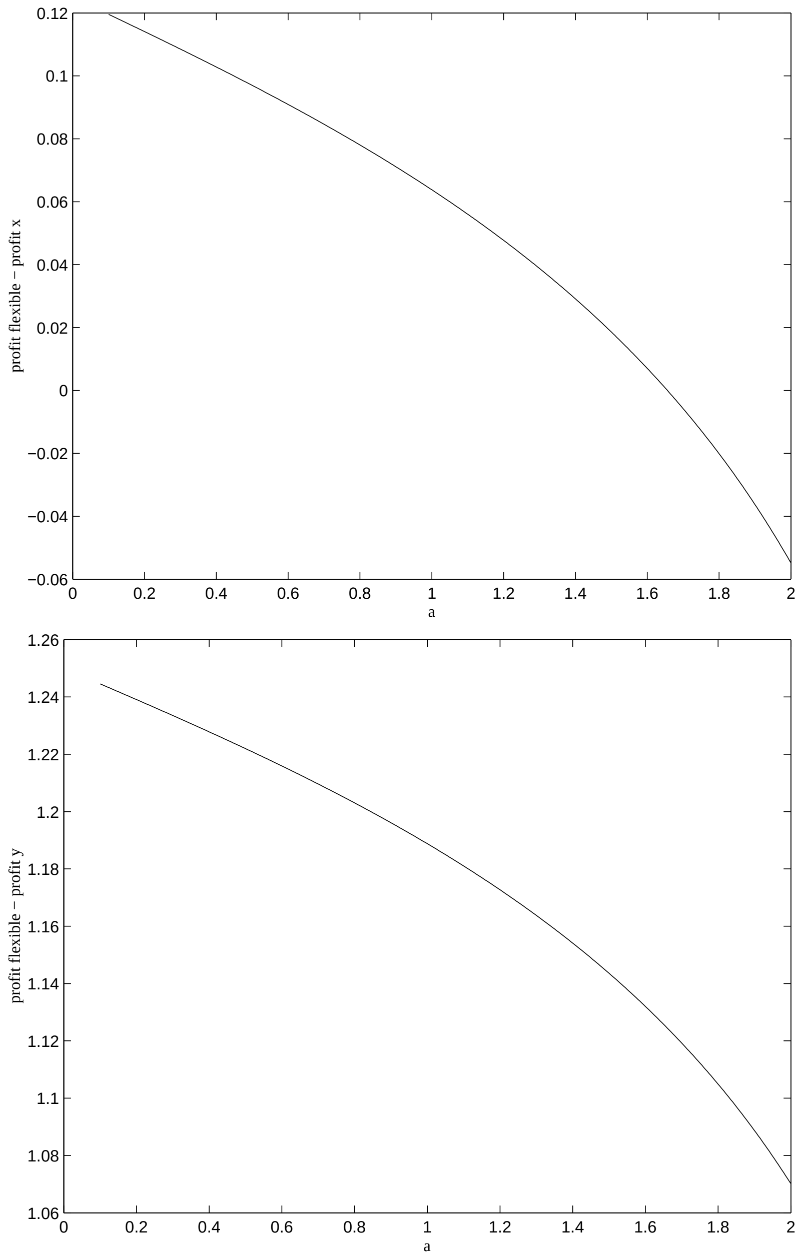

Figure 1: Profit differential between the flexible and rigid contracts as a function of risk aversion

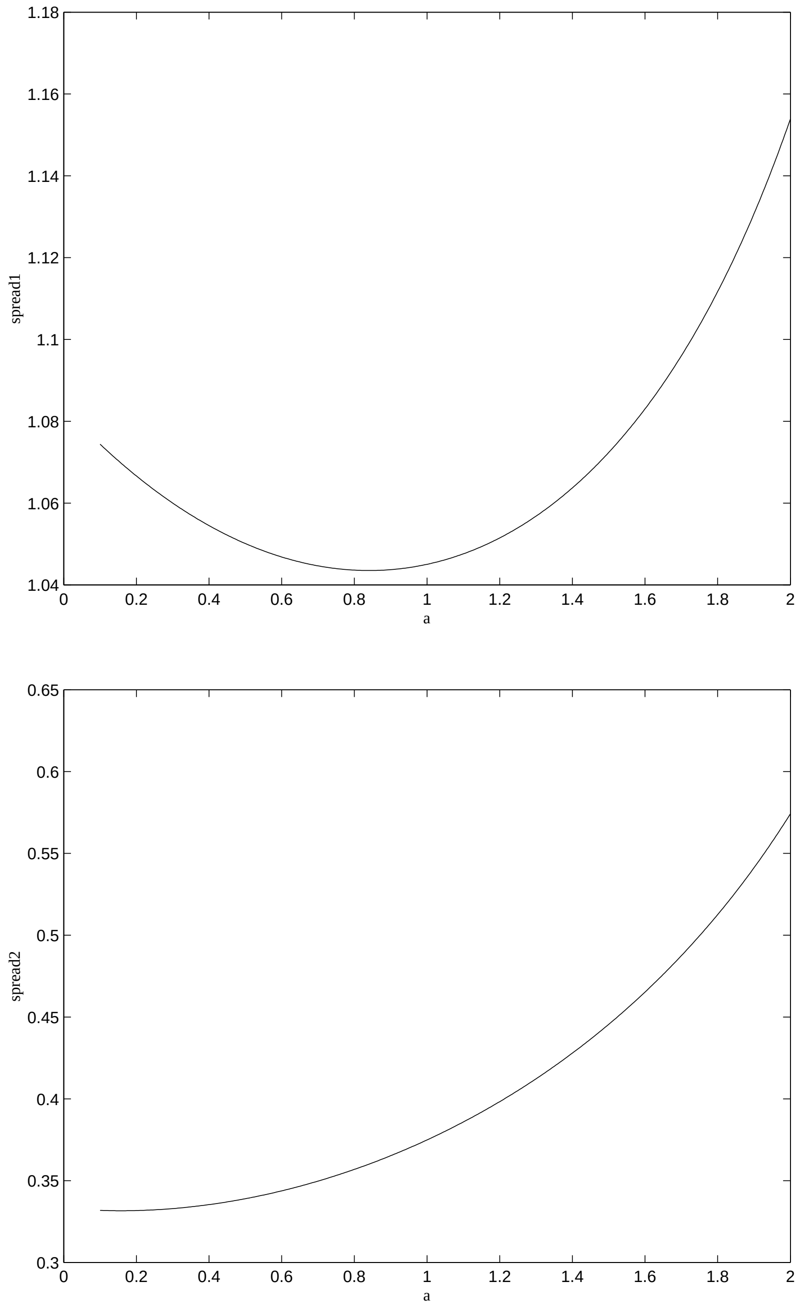

Figure 2: Wage differentials at the optimal flexible contract as a function of risk aversion

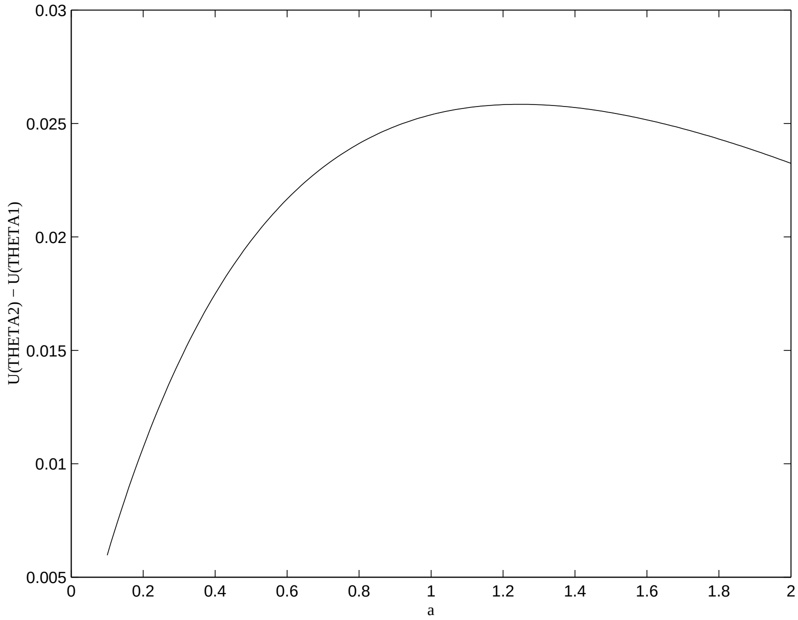

Figure 3: Utility differential $u(\theta_2) - u(\theta_1)$ at the optimal flexible contract as a function of a