Online Appendix

B.1 Equilibrium analysis of spot trade only

The following Proposition characterizes the equilibrium when investors can only trade spot. It also establishes formally the claim made in Section 2.3 that spot trades alone can never allow to support the first best allocation.

Proposition 1. The first best allocation can never be attained when investors trade spot only. There exists a threshold $\bar{a}_{spot}$ such that

1. Low asset quantity: if $a < \bar{a}_{spot}$, then investor 1 sells his entire asset holdings in period 1. The liquidity premium $\mathcal{L}$ is strictly positive.

2. High asset quantity:if $a \geq \bar{a}_{spot}$, then investor 1 sells less $\bar{a}_{spot}$ in period 1. The liquidity premium is $\mathcal{L} = 0$.

Proof. We write down investor 1 consumption in period 3 and 2 and the first order conditions with respect to spot trades in period 1and 2 for both investors:

$$c_3^1(s) = \omega + as, \tag{1}$$

$$c_2^1(s) = \omega + p_2(s)(a_1^1 - a_2^1(s)), \tag{2}$$

$$-p_2(s)v'(c_2^1(s)) + s + \gamma_2^1(s) = 0, \tag{3}$$

$$-p_2(s)u'(c_2^2(s)) + \beta s + \gamma_2^2(s) = 0, \tag{4}$$

$$-p_1 + \mathbb{E}\left[p_2(s)v'(c_2^1(s))\right] + \gamma_1^1 = 0, \tag{5}$$

$$-p_1 + \mathbb{E}\left[p_2(s)u'(c_2^2(s))\right] + \gamma_1^2 = 0, \tag{6}$$

where γ_t^i is the Lagrange multiplier on the no-short sale constraint of investor i in period t. In period 2, the state s is an argument of the multiplier.

We first show that investor 1 must always carry at least some asset into period 3, that is $a_2^1(s) > 0$ or $\gamma_2^1(s) = 0$ for all s. Suppose it is not the case, that is $a_2^1(s) = 0$ for some state s. From equation 1, we obtain $c_2^1(s) > \omega > c_2^2(s)$ so that $v'(c_2^1(s)) < u'(c_2^2(s))$ since $u'(\omega) > v'(\omega)$ by assumption. This is a contradiction with equations (3)-(4) since $\beta < 1$.

We now characterize $p_2(s)$ and $a_2^2(s)$ considering the cases where $\gamma_2^2(s) > 0$ and $\gamma_2^2(s) = 0$ in turn. Let s be such that that $\gamma_2^2(s) > 0$ so that $a_2^2(s) = 0$. Then, we have $c_2^1(s) = \omega - p_2(s)a_1^2$ and from equation (3), $p_2(s)$ is defined implicitly as a function of a_1^2 by $p_2(s)v'(\omega - p_2(s)a_1^2) = s$. In particular, $p_2(s)$ is strictly increasing in s since v is concave. Consider now values of s for which $\gamma_2^2(s) = 0$. Equations (3) and (4) imply that

$$\beta v'(c_2^1(s)) = u'(c_2^2(s))$$

Since $c_2^2(s) = 2\omega - c_2^1(s)$ by the resource constraint, it must be that $c_2^1(s)$ does not depend on s. We let $\hat{c}_2^1$ be this constant. From equation (3), we obtain that $p_2(s)v'(\hat{c}_2^1) - s = 0$. Hence, $p_2(s)$ is again strictly increasing in s. In addition, equation (2) pins down $a_2^2(s)$ since then $\hat{c}_2^1 = \omega - p_2(s)(a_1^2 - a_2^2(s))$.

We now show that there is a threshold $\hat{s}(a_1^2)$ such that $\gamma_2^2(s) > 0$ for $s < \hat{s}(a_1^2)$ and $\gamma_2^2(s) = 0$ for $s \geq \hat{s}(a_1^2)$. If it is not the case, there exists $(s_1, s_2) \in [\underline{s}, \bar{s}]$ such that $s_1 < s_2$ and $\gamma_2^2(s_1) = 0$ while $\gamma_2^2(s_2) > 0$. From the characterization, in the previous paragraph, this implies that $u'(c_2^2(s_2)) > \beta v'(c_2^1(s_2))$ so that $c_2^1(s_2) > \hat{c}_2^1 = c_2^1(s_1)$. To establish a contradiction, observe that

$$c_2^1(s_2) = \omega - p_2(s_2)a_1^2 = \omega - \frac{s_2}{v'(c_2^1(s_2))}a_1^2 < \omega - \frac{s_2}{v'(\hat{c}_2^1)}a_1^2 < \omega - \frac{s_1}{v'(\hat{c}_2^1)}a_1^2 < c_2^1(s_1)$$

The first inequality follows from $c_2^1(s_2) > \hat{c}_2^1$. The last inequality follows from equation (2) with $p_2(s_1) = s_1/v'(\hat{c}_2^1)$ by definition of s_1. Hence the claimed threshold exists. We thus obtain the following pattern for investor 1 consumption in period 2:

$$c_2^1(s) = \begin{cases} \omega - p_2(s)a_1^2 & s \leq \hat{s}(a_1^2) \\ \hat{c}_2^1 & s > \hat{s}(a_1^2) \end{cases} \tag{7}$$

It is clear from this expression and the definition of $\hat{c}_2^1$ that the first best allocation can never be implemented from spot trades only. Indeed, when $s \leq \hat{s}(a_1^2)$, investors' consumption varies with s while when $s \geq \hat{s}(a_1^2)$ marginal utilities between periods 1 and 2 are not equalized. This proves our claim in the main text.

To finish the characterization of the spot only equilibrium, we are left to pin down a_1^2, the quantity investor 2 initially buys from investor 1. We first show that $\gamma_1^2 = 0$ that is some spot trade occurs in period 1. Indeed, if $\gamma_1^2 > 0$, $a_1^2 = 0$ and from equation

(7), $c_2^1(s) \geq \omega$. By assumption, for all s, we thus have $u'(c_2^2(s)) - v'(c_2^1(s)) > 0$ which is incompatible with $\gamma_1^2 > 0$, using equations (5) and (6) with $\gamma_1^1 = 0$. Hence, we must have $\gamma_1^2 = 0$. Then, from equations (3) and (4):

$$\gamma_1^1 = \mathbb{E}\left\{p_2(s)\left[u'(c_2^2(s)) - v'(c_2^1(s))\right]\right\} \tag{8}$$

$$= \int_{\underline{s}}^{\hat{s}(a_1^2)} \left(p_2(s)(u'(\omega + p_2(s)a_1^2) - s\right) \mathrm{d}F(s) - (1-\beta)v'(\hat{c}_2^1) \int_{\hat{s}(a_1^2)}^{\bar{s}} p_2(s)\mathrm{d}F(s) := K(a_1^2) \tag{9}$$

The total derivative of K with respect to a_1^2 is equal to

$$K'(a_1^2) = \int_{\underline{s}}^{\hat{s}(a_1^2)} \left[\frac{\partial p_2(s)}{\partial a_1^2} u'(\omega + p_2(s)a_1^2) + \frac{\partial a_1^2 p_2(s)}{\partial a_1^2} u''(\omega + p_2(s)a_1^2))p_2(s)\right] \mathrm{d}F(s)$$

When $s \leq \hat{s}(a_1^2)$, $p_2(s)v'(\omega - p_2(s)a_1^2) = s$ so that $p_2(s)$ is strictly decreasing in a_1^2 while $a_1^2 p_2(s)$ is strictly increasing in a_1^2 on $[\underline{s}, \hat{s}(a_1^2)]$. Since u is strictly concave, this proves that $K' < 0$. In addition, observe that given a_1^2, $\hat{s}(a_1^2)$ is the minimal state where $\omega - p_2(s)a_1^2 = \hat{c}_2^1$ for $p_2(s) = s/v'(\hat{c}_2^1)$ and thus that $\hat{s}(a_1^2)$ is decreasing in a_1^2. Hence as a_1^2 goes to 0, $\hat{s}(a_1^2)$ becomes larger than $\underline{s}$. As a_1^2 goes to ∞, $\hat{s}(a_1^2)$ becomes smaller than $\underline{s}$. Then, observe that

$$K(0) = \mathbb{E}[p_2(s)](u'(\omega) - v'(\omega)) > 0$$
$$\lim_{x\to\infty} K(x) = -(1-\beta)v'(\hat{c}_2^1)\mathbb{E}[p_2(s)] < 0$$

Hence, since K is strictly decreasing and continuous, by the intermediate value theorem, there exists a unique value $\bar{a}_{spot} > 0$ such that $K(\bar{a}_{spot}) = 0$. Two cases are then possible. Either $a \geq \bar{a}_{spot}$ and $\gamma_1^1 = 0$ and $a_1^2 = \bar{a}_{spot}$or $a < \bar{a}_{spot}$ and $\gamma_1^1 > 0$ that is $a_1^2 = a$. This concludes the proof. □

B.2 Equilibrium uniqueness in Proposition 1

We now prove the uniqueness part of the result in Proposition 1 when $\theta_1 > 0$ and $s^* > \underline{s}$. We must show that the pattern of trades is unique and in particular that investors do not trade spot or a repo sold by investor 2. We first analyse and dismiss two cases. In Case 1, agents would only trade spot. In Case 2, agents would only trade spot and a repo f_{21}. Then, we show that in any equilibrium, investors only trade a repo $f_{12} \in \mathcal{F}_{12}(\boldsymbol{p}_2)$. This establishes uniqueness since we showed in Proposition 1 that the equilibrium is unique based on this conjecture.

Case 1: Only spot trades

We characterized the equilibrium when agents only trade spot in Section B.1. Using the first order conditions (5) and (6), we find that investors do not want to trade a repo $\tilde{f}_{12} \in \mathcal{F}_{12}(\boldsymbol{p}_2)$ if and only if

$$\mathbb{E}\left[\left(\tilde{f}_{12}(s) - p_2(s)\right)\left(u'(c_2^2(s)) - v'(c_2^1(s))\right)\right] \geq 0 \tag{10}$$

We have shown that it cannot be the case that $u'(c_2^2(s)) - v'(c_2^1(s)) = 0$ for all s. If there exists a positive measure subset $\mathcal{S}_0$ such that $u'(c_2^2(s)) - v'(c_2^1(s)) > 0$, then condition (10) is not compatible with contract $\tilde{f}_{12}$ such that $\tilde{f}_{12}(s) = 0$ on $\mathcal{S}_0$ and $\tilde{f}_{12}(s) = p_2(s)$ otherwise. If there exists a positive measure subset $\mathcal{S}_0$ such that $u'(c_2^2(s)) - v'(c_2^1(s)) < 0$, then condition 4 is not compatible with contract $\tilde{f}_{12}$ such that $\tilde{f}_{12}(s) = \frac{p_2(s)}{1-\theta_1}$ on $\mathcal{S}_0$ and $\tilde{f}_{12}(s) = p_2(s)$ otherwise since $\theta_1 > 0$. This proves that there is no equilibrium where agents only trade spot.

Case 2: Only spot trades and a repo contract $f_{21} \in \mathcal{F}_{21}(\boldsymbol{p}_2)$

When investors trade a repo $f_{21} \in \mathcal{F}_{21}(\boldsymbol{p}_2)$, the first-order conditions with respect to l^{12} and b^{21} are respectively:

$$-q_{21} + \mathbb{E}\left[f_{21}(s)v'(c_2^1(s))\right] = 0 \tag{11}$$

$$q_{21} - \mathbb{E}\left[f_{21}(s)u'(c_2^2(s))\right] - \gamma_1^2 = 0 \tag{12}$$

The other first-order conditions with respect to spot trades are given by equation (3) to (6). By the same argument used in the proof of Proposition 1, the condition that investors do not trade another contract $\tilde{f}_{21} \in \mathcal{F}_{21}(\boldsymbol{p}_2)$ writes:

$$\mathbb{E}\left[\left(\tilde{f}_{21}(s) - f_{21}(s)\right)\left(u'(c_2^2(s)) - v'(c_2^1(s))\right)\right] \geq 0 \tag{13}$$

The condition that investors do not trade a contract $\tilde{f}_{12} \in \mathcal{F}_{12}(\boldsymbol{p}_2)$ writes:

$$\mathbb{E}\left[\left(p_2(s) - f_{21}(s) - \tilde{f}_{12}(s)\right)\left(u'(c_2^2(s)) - v'(c_2^1(s))\right)\right] \geq 0 \tag{14}$$

We first show that $u'(c_2^2(s)) \leq v'(c_2^1(s))$ for all s. Suppose it is not the case on a subset $\mathcal{S}_0$ with positive measure. Then, condition (13) can hold for all repos $\tilde{f}_{21}$ if and only if $f_{12}(s) = 0$ for $s \in \mathcal{S}_0$. But then, condition (14) is incompatible with repo contract $\tilde{f}_{12}$ where $\tilde{f}_{12}(s) = p_2(s)/(1-\theta_1) > p_2(s)$ for $s \in \mathcal{S}_0$ and $\tilde{f}_{12}(s) = p_2(s)$ otherwise. This proves that $u'(c_2^2(s)) \leq v'(c_2^1(s))$ for all s.

We now show that this inequality must hold as an equality. Suppose it is not the case on a subset $\mathcal{S}_0$ with positive measure. Then, condition (13) imposes that $f_{21}(s) = p_2(s)/(1-\theta_2)$ for $s \in \mathcal{S}_0$. It then follows from (11) and (12) that $\gamma_1^2 > 0$ and thus that $b^{21} = a_1^2$. We now establish a contraction. Using the budget constraint (12), we have

$$c_2^1(s) = \omega + (a_1^1 - a_2^1(s))p_2(s) + a_1^2 \frac{p_2(s)}{1-\theta_2} = \omega - (a_1^2 - a_2^2(s))p_2(s) + a_1^2 \frac{p_2(s)}{1-\theta_2} \geq \omega$$

where the second equality follows from spot market clearing in periods 1 and 2. By assumption $v'(\omega) < u'(\omega)$. Hence, $c_2^1(s) \geq \omega$ is a contradiction with $u'(c_2^2(s)) - v'(c_2^1(s)) < 0$. The only possibility is that $u'(c_2^2(s)) = v'(c_2^1(s))$, for all s. This implies in particular that $p_2(s) = s/v'(c_{2,*}^1)$ from equation (4). But then, we have

$$c_2^1(s) = \omega - (a_1^2 - a_2^2(s))p_2(s) + b^{21} f_{21}(s) \geq \omega - a_1^2 p_2(s) \geq \omega - a p_2(s)$$

Hence, the first best allocation can be attained in state $\underline{s}$ only if $c_{2,*}^1 \geq \omega - a\underline{s}/v'(c_{2,*}^1)$. By definition of s^* in (19), this violates our assumption that $s^* > \underline{s}$.

Case 3: Spot trades, repo contracts $f_{12} \in \mathcal{F}_{12}(\boldsymbol{p}_2)$ and $f_{21} \in \mathcal{F}_{21}(\boldsymbol{p}_2)$

We now analyze the last possibility where in addition to a repo contract f_{12}, agents can trade spot and a repo contract f_{21}. We show that these last two trades do not arise in equilibrium. The first order conditions with respect to spot trades and repo trades of contract f_{12} are given in the proof of Proposition 1 by equation (33) to (38). Investor 1 consumption in period 2 and state s is given by:

$$c_2^1(s) = \omega + (a_1^1 - a_2^1(s))p_2(s) - b^{12} f_{12}(s) + b^{21} f_{21}(s) \tag{15}$$

If $b^{21} > 0$, the first order conditions (11) and (12) must also hold. The conditions for investors not to trade another contract $\tilde{f}_{12} \in \mathcal{F}_{12}(\boldsymbol{p}_2)$ and another contract $\tilde{f}_{21} \in \mathcal{F}_{21}(\boldsymbol{p}_2)$ are given by equations (40) and (42) respectively.

We first prove that if (40) and (42) hold, then we must have $u'(c_2^2(s)) \geq v'(c_2^1(s))$ for all s. Equation (40) holds for any $\tilde{f}_{12} \in \mathcal{F}_{12}(\boldsymbol{p}_2)$ if and only if $f_{12}(s) = 0$ whenever $u'(c_2^2(s)) < v'(c_2^1(s))$ and $f_{12}(s) = p_2(s)/(1-\theta_1)$ whenever $u'(c_2^2(s)) > v'(c_2^1(s))$. Suppose then that there exists a subset $\mathcal{S}_0$ of positive measure such that $u'(c_2^2(s)) < v'(c_2^1(s))$ for all $s \in \mathcal{S}_0$. This implies that $\gamma_1^2 > 0$ and thus that $b^{21} = a_1^2$. If $b^{21} = 0$ so that no repo contract f_{21} is traded, using equation(15), we have that for all $s \in \mathcal{S}_0$:

$$c_2^1(s) = \omega + a_2^2(s)p_2(s) > \omega > c_{2,*}^1$$

which is a contradiction with $u'(c_2^2(s)) < v'(c_2^1(s))$ by definition of $c_{2,*}^1$. If $b^{21} > 0$ so that a repo contract f_{21} is traded, using equations (11), (12) , condition (14) can be rewritten

$$\mathbb{E}\left[\left(\tilde{f}_{21}(s) - f_{21}(s)\right)\left(u'(c_2^2(s)) - v'(c_2^1(s))\right)\right] \geq 0$$

which imposes that $f_{21}(s) = p_2(s)/(1-\theta_2)$ for $s \in \mathcal{S}_0$. However, using again equation (15), we have that for all $s \in \mathcal{S}_0$:

$$c_2^1(s) = \omega - (a_2^2 - a_2^2(s))p_2(s) + a_1^2 \frac{p_2(s)}{1-\theta_2} > \omega > c_{2,*}^1$$

which is again a contradiction with $u'(c_2^2(s)) < v'(c_2^1(s))$ for $s \in \mathcal{S}_0$. Hence, we showed that $u'(c_2^2(s)) \geq v'(c_2^1(s))$ for all $s \in [\underline{s}, \bar{s}]$.

We now prove that no contract $f_{21} \in \mathcal{F}_{21}(\boldsymbol{p}_2)$ is traded. We show first that $\gamma_1^1 > 0$. Suppose to the contrary that $\gamma_1^1 = 0$. It implies that $u'(c_2^2(s)) = v'(c_2^1(s))$ so that the first-best allocation is attained in all states. However, using equation (15), we obtain that

$$c_2^1(\underline{s}) \geq \omega - a\frac{p_2(\underline{s})}{1-\theta_1}$$

so that $c_2^1(\underline{s}) = c_{2,*}^1$ is possible only if $\underline{s} > s^*$. Since we assumed $s^* > \underline{s}$ here, it must be that $\gamma_1^1 > 0$. This proves that there exists a subset $\mathcal{S}_0$ of $[\underline{s}, \bar{s}]$ of positive measure where $u'(c_2^2(.)) > v'(c_2^1(.))$ as otherwise from equations (34) and (36), γ_1^1 would be 0. Then, by an argument made above, we have that $f_{12}(s) = p_2(s)/(1-\theta_2)$ and $f_{21}(s) = 0$ for $s \in \mathcal{S}_0$. Then, from equations (33) to (36) that

$$\gamma_1^2 = \mathbb{E}\left[\left(f_{12}(s) - p_2(s)\right)\left(u'(c_2^2(s)) - v'(c_2^1(s))\right)\right] > 0$$

since there exists a subset $\mathcal{S}_0$ where $f_{12}(s) = p_2(s)/(1-\theta_1) > p_2(s)$ and $u'(c_2^2(s)) > v'(c_2^1(s))$. Suppose now that a contract f_{21} is traded. Using equations (11) and (12), since either $u'(c_2^2(s)) = v'(c_2^1(s))$ holds or $u'(c_2^2(s)) > v'(c_2^1(s))$ and $f_{21}(s) = 0$ hold, then $\gamma_1^2 = 0$, a contradiction. This proves that no contract f_{21} is traded and hence that $b^{21} = l^{12} = 0$.

We are then left to show that investors do not trade spot either. Since $b^{21} = 0$ and constraint (14) for investor 2 is binding, we have $a_1^2 = 0$ and hence $b^{12} = a_1^1 = a$. This means that no asset is traded spot. Since investors only trade a repo contract $f_{12} \in \mathcal{F}_{12}(\boldsymbol{p}_2)$, the equilibrium is unique and as characterized in Proposition 1.

B.3 Chain of Repos with $\nu_2 > 0$.

We prove that an equilibrium with a chain of repos, similar to that of Proposition (6), also exists when ν_2 is positive but not too large.

Claim 1. There exists $\bar{\delta}_B(\nu_2) > \underline{\delta_B}(\nu_2) > \delta$ such that the equilibrium features intermediation with a chain of repos if and only if $\delta_B \in [\underline{\delta_B}(\nu_2), \bar{\delta}_B(\nu_2)]$ and

$$\nu_B \geq \frac{1}{1-\nu_2(1-\theta_B)}\left[\frac{(1-\nu_2)(1-\theta_B)}{1-\theta_1} + \nu_2\theta_B\right] \tag{16}$$

Investors 1 sells all the asset in a repo f_{1B} to B with

$$f_{1B}(s) = \frac{s}{1-\theta_1} \qquad \forall s \in [\underline{s}, \bar{s}] \tag{17}$$

Investor B sells part of the asset in a repo f_{B2} to 2 with

$$f_{\mathrm{B}2}(s) = \begin{cases} \frac{p_2(s)}{1-\theta_B} & \text{if } s < s_{B2}^*(\nu_2) \\ \frac{p_2(s_{B2}^*)}{1-\theta_B} + \nu_2(p_2(s) - p_2(s_{B2}^*)) & \text{if } s \geq s_{B2}^*(\nu_2) \end{cases}$$

for some $s_{B2}^*(\nu_2) \in [\underline{s}, \bar{s}]$ and the remaining part in a spot sale to investor 1.

Proof. The first order conditions are those we derived in the proof of Proposition (6) except for that of investor 2 with respect to repo trades of contract f_{B2} which becomes

$$-q_{B2} + \mathbb{E}[f_{B2}(s)u'(c_2^2(s))] + \nu_2\gamma_1^2 = 0 \tag{18}$$

With $\nu_2 > 0$, the equivalent of condition (62) becomes

$$a = (1 - \nu_B)b^{1B} + (1 - \nu_2)b^{B2}$$

Let us denote $b := b^{B2}(1 - \nu_2)$ which can be described as the amount of asset used in the transaction between investors B and 2. This defines again a range $[0, \nu_B a]$ of possible values for b. We showed in the poof of Proposition 6 that $p_2(s) = s/\delta$ since investor 1 holds the asset into period 3. Let us now define $s^*_{B2}(b)$ implicitly as:

$$u'\left(\omega + \frac{b s^*_{B2}(b)}{(1-\nu_2)\delta}\left[\frac{1}{1-\theta_B} - \nu_2\right]\right) = \delta_B,$$

Using the results from Proposition 4, we know that investors B and 2 will trade repo contract $f_{B2}(b)$ defined implicitly as a function of the amount b used in the transaction:

$$f_{\mathrm{B2}}(b, \nu_2, s) = \begin{cases} \frac{p_2(s)}{1-\theta_B} & \text{if } s < s^*_{B2}(b) \\ \frac{s^*_{B2}(b)}{\delta(1-\theta_B)} + \frac{\nu_2(s - s^*_{B2}(b))}{\delta} & \text{if } s \geq s^*_{B2}(b) \end{cases} \tag{19}$$

We may also define investor 2 consumption as a function of b

$$c_2^2(b, s) = \omega + \frac{b}{1-\nu_2}\left(f_{B2}(b, \nu_2 s) - \nu_2 p_2(s)\right)$$

where $f(b, \nu_2, s)$ is defined in (19). Since, investors 1 and B are risk-neutral, the same argument used in Proposition 6 applies here to derive the repo contract sold by investor 1 to investor B:

$$f_{1B}(s) = \frac{p_2(s)}{1-\theta_1} \qquad \forall s \in [\underline{s}, \bar{s}] \tag{20}$$

We now pin down the equilibrium value of b. From equations (49) and (55) to (57), we obtain:

$$\gamma_1^B = \frac{(\delta_B - \delta)\theta_1}{(1-\nu_B)(1-\theta_1)}\mathbb{E}[p_2(s)]$$

From equations (58) and (18), we obtain:

$$\gamma_1^B = \frac{1 - (1-\theta_B)\nu_2}{(1-\nu_2)}\int_{\underline{s}}^{s^*_{B2}(b)}\left[u'\left(c_2^2(b,s)\right) - \delta_B\right]\frac{p_2(s)}{1-\theta_B}\mathrm{d}G(s)$$

Equalizing these two expressions for γ_1^B derived above, we obtain:

$$\frac{1-(1-\theta_B)\nu_2}{(1-\theta_B)(1-\nu_2)}\int_{\underline{s}}^{s^*_{B2}(b)}\left[u'\left(c_2^2(b,s)\right)-\delta_B\right]\mathrm{p}_2(\mathrm{s})\mathrm{d}G(s)=\frac{(\delta_B-\delta)\theta_1}{(1-\nu_B)(1-\theta_1)}\mathbb{E}[p_2(s)]$$

Since the mapping

$$b\rightarrow\int_{\underline{s}}^{s^*_{B2}(b)}\left[u'\left(c_2^2(b,s)\right)-\delta_B\right]\mathrm{p}_2(\mathrm{s})\mathrm{d}G(s)$$

is strictly decreasing in b, the equality above pins down a unique value for b. Our equilibrium conjecture can only be true if $b\in[0,\nu_B a]$. Plugging the lower bound for b into expression (19) yields the following condition:

$$\frac{1-(1-\theta_B)\nu_2}{1-\nu_2}\frac{u'(\omega)-\delta_B}{1-\theta_B}\geq\frac{(\delta_B-\delta)\theta_1}{(1-\nu_B)(1-\theta_1)}\tag{21}$$

This inequality is equivalent to

$$\delta_B\leq\bar{\delta}_B(\nu_2):=\frac{\frac{\theta_1}{(1-\nu_B)(1-\theta_1)}\delta+\frac{[1-(1-\theta_B)\nu_2]u'(\omega)}{(1-\theta_B)(1-\nu_2)}}{\frac{\theta_1}{(1-\nu_B)(1-\theta_1)}+\frac{[1-(1-\theta_B)\nu_2]u'(\omega)}{(1-\theta_B)(1-\nu_2)}}$$

Observe in particular that $\bar{\delta}_B(\nu_2)\leq u'(\omega)$. Plugging now the upper bound for b into expression (19), we obtain:

$$\frac{1-(1-\theta_B)\nu_2}{1-\nu_2}\int_{\underline{s}}^{s^*_{B2}(\nu_B a)}\frac{u'\left(c_2^2(\nu_B a,s)\right)-\delta_B}{1-\theta_B}s\mathrm{d}F(s)\leq\frac{(\delta_B-\delta)\theta_1}{(1-\nu_B)(1-\theta_1)}\tag{22}$$

which is equivalent to

$$\delta\geq\underline{\delta}_B(\nu_2)=\frac{\frac{\theta_1}{(1-\nu_B)(1-\theta_1)}\delta+\frac{1-(1-\theta_B)\nu_2}{1-\nu_2}\int_{\underline{s}}^{s^*_{B2}(\nu_B a)}\frac{u'\left(c_2^2(\nu_B a,s)\right)}{1-\theta_B}s\mathrm{d}F(s)}{\frac{\theta_1}{(1-\nu_B)(1-\theta_1)}+\frac{1-(1-\theta_B)\nu_2}{1-\nu_2}\int_{\underline{s}}^{s^*_{B2}(\nu_B a)}\frac{1}{1-\theta_B}s\mathrm{d}F(s)}$$

with $\underline{\delta}_B(\nu_2)\geq\delta$. Since $\delta_B\leq\bar{\delta}_B(\nu_2)$and $\delta_B\geq\underline{\delta}_B(\nu_2)$ are respectively equivalent to (21) and (22), it is easy to see from these expressions that $\underline{\delta}_B(\nu_2)\leq\bar{\delta}_B(\nu_2)$.

We are left to show that there is no contract in $\mathcal{F}_{12}(\boldsymbol{p}_2)$ that investor 1 desires to sell to investor 2. Letting $\tilde{f}_{12}$ be a generic contract in $\mathcal{F}_{12}(\boldsymbol{p}_2)$, the first condition writes:

$$\delta\mathbb{E}[\tilde{f}_{12}(s)] + \gamma_1^1 \geq \mathbb{E}\left[\tilde{f}_{12}(s)u'(c_2^2(s))\right] + \nu_2\gamma_1^2$$

which, substituting for γ_1^1 and γ_1^2 thanks to equations (61) and (18), becomes:

$$\mathbb{E}\left[\left(\tilde{f}_{12}(s) - p_2(s)\right)\left(u'(c_2^2(s)) - \delta\right)\right] \leq \mathbb{E}\left[\left(f_{B2}(s) - p_2(s)\right)\left(u'(c_2^2(s)) - \delta_B\right)\right]$$

This inequality holds for all $\tilde{f}_{12} \in \mathcal{F}_{12}(\boldsymbol{p}_2)$ if it holds for the repo contract f_{12} with payoff $f_{12}(s) = \frac{p_2(s)}{1-\theta_1}$. Plugging this expression in the inequality above and rearranging terms we obtain:

$$
\begin{aligned}
& 0 \leq \mathbb{E}\left[\left(f_{B2}(s) - f_{12}(s)\right)\left(u'(c_2^2(s)) - \delta_B\right)\right] - \mathbb{E}\left[\left(f_{12}(s) - p_2(s)\right)\left(\delta_B - \delta\right)\right] \\
\Leftrightarrow \quad & 0 \leq \left[\frac{1}{1-\theta_B} - \frac{1}{1-\theta_1}\right] \int_{\underline{s}}^{s_{B2}^*(b^{B2})} \left[u'\left(c_2^2(b^{B2}, s)\right) - \delta_B\right] p_2(s)\mathrm{d}G(s) - \frac{\theta_1(\delta_B - \delta)}{1-\theta_1}\mathbb{E}[p_2(s)] \\
\Leftrightarrow \quad & 0 \leq \frac{1-\nu_2}{1-(1-\theta_B)\nu_2}\left[1 - \frac{1-\theta_B}{1-\theta_1}\right]\gamma_1^B - \gamma_1^B(1-\nu_B) \\
\Leftrightarrow \quad & 0 \leq \nu_B - \frac{1}{1-\nu_2(1-\theta_B)}\left[\frac{(1-\nu_2)(1-\theta_B)}{1-\theta_1}\right]
\end{aligned}
$$

where the last inequality is Condition (16). This concludes the proof. □