Credit Market Competition and Capital Regulation*

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Abstract

Empirical evidence suggests that banks hold capital in excess of regulatory minimums. This did not prevent the crisis and underlines the importance of understanding bank capital determination. Market discipline is one of the forces that induces banks to hold positive capital. The literature has focused on the liability side. We develop a simple theory based on monitoring to show that discipline from the asset side can also be important. In perfectly competitive markets, banks can find it optimal to use costly capital rather than the interest rate on the loan to commit to monitoring because it allows higher borrower surplus.

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1 Introduction

A common justification for capital regulation for banks is the reduction of bank moral hazard. With high levels of leverage, there is an incentive for banks to take on excessive risk. This incentive is reduced if banks have capital at risk. Given the widely accepted view that equity capital is more costly for banks than other types of funds, the common result in many analyses of bank regulation is that capital adequacy standards are binding as banks attempt to economize on the use of this costly input.

In practice, however, it appears that banks are often willing to hold positive levels of capital well above regulatory minima, and that actual capital holdings tend to vary independently of regulatory changes. For example, comparing actual capital holdings to regulatory requirements in the US, Flannery and Rangan (2008) find that banks’ capital ratios increased substantially in the last decade, with banks holding capital levels that were 75% in excess of the regulatory minima in the early 2000’s. Similar cross-country evidence is provided in Barth et al. (2005) (see Figure 3.8, p. 119). In search of an explanation of the capital buildup in the US throughout the 1980’s, Ashcraft (2001) finds little evidence that changes in banks’ capital structure were related to changes in regulatory requirements. From an international perspective, Barrios and Blanco (2003) argue that Spanish banks’ capital ratios over the period 1985-1991 were primarily driven by the pressure of market forces rather than regulatory constraints. Also, Alfon et al. (2004) report that UK banks increased their capital ratios in the last decade despite a reduction in their individual capital requirements, and operated in the early 2000’s with an average capital buffer of 35-40 percent. Finally, Gropp and Heider (2008) do not detect a first order effect of regulation on banks’ capital holdings.

Despite the fact that throughout the 1990’s and the early part of this decade banks had capital levels well above regulatory minimums, the crisis that started in the summer of 2007 raises the question of whether banks were in fact undercapitalized relative to some ideal social welfare maximizing level. There are clearly many determinants of such an ideal level of capital including many related to crisis factors such as the likelihood of contagion. At present there are no encompassing theories that explain well how much capital banks should hold. It
remains an open question whether, despite having been well above regulatory minimums, banks were nevertheless undercapitalized.

In order to make progress in understanding bank capitalization it is necessary to consider the different determinants. One of the factors deemed important in inducing banks to choose positive amounts of capital is market discipline. Typically such discipline has been considered from the liability side (see, e.g., Calomiris and Kahn (1991), and Flannery and Nikolova (2004) for a survey). The purpose of our paper is to present a theory that demonstrates that inducements to hold capital can also come from the asset side. We show that when credit markets are competitive, market discipline coming from the asset side induces banks to hold positive levels of capital as a way to commit to monitor and attract borrowers.

We develop a simple one-period model of bank lending where firms need external financing to make productive investments. Banks grant loans to firms and monitor them, which helps improve firms’ expected payoff. Given that monitoring is costly and banks have limited liability, banks are subject to a moral hazard problem in the choice of monitoring effort. One way of providing them with greater incentives for monitoring is through the use of equity capital. This forces banks to internalize the costs of their default, thus ameliorating the limited liability problem banks face due to their extensive reliance on deposit-based financing. A second instrument to improve banks’ incentives is embodied in the loan rate. A marginal increase in the loan rate gives banks a greater incentive to monitor in order to receive the higher payoff if the project succeeds and the loan is repaid. Thus, capital and loan rates are alternative ways to improve banks’ monitoring incentives, but entail different costs. Holding capital implies a direct private cost for the banks, whereas increasing the loan rate has a negative impact only for borrowers in terms of a lower return from the investment.

For most of our analysis, we consider the case where banks operate in a perfectly competitive loan market so that borrower surplus is maximized. We first consider the case where there is no deposit insurance. Since depositors do not receive anything if banks’ projects are unsuccessful, they require a premium in non-default states in order to be willing to deposit their funds. By encouraging monitoring, bank capital reduces the premium that needs to be offered to depositors, and thus provides a rationale for holding capital that acts through the bank’s liabilities. In addition, there is also an asset-side incentive to hold capital, since the
market equilibrium entails a combination of capital and loan rate that maximizes borrower surplus. The loan rate is set at the lowest level consistent with bank participation and the remaining incentives for monitoring loans are provided by banks holding positive amounts of capital. Thus, competition in the loan market induces banks to voluntarily hold positive levels of capital as a way to commit to greater monitoring.

We then compare the market solution to the regulatory solution. Although in practice capital regulation is driven by a wide variety of factors such as systemic risk and asset substitution, we focus on the benchmark solution where a regulator chooses the level of capital to maximize social welfare so that we can assess the efficiency of the market solution. We show that when the return on the firm’s project is sufficiently high, the market solution is inefficient as it entails a level of capital above the social welfare maximizing level. The reason is that the market solution maximizes borrower surplus, and borrowers prefer to use as much capital as possible to provide incentives rather than using a higher loan rate. By contrast, the regulator prefers to provide banks with incentives through the loan rate as it is just a transfer and there is no inefficiently high cost as with capital. Thus, when the project return is high, the regulator chooses a lower level of capital than the market because a high loan rate is feasible. As the project return decreases, the market solution becomes constrained efficient in that it provides the correct incentives from a social perspective to hold a positive amount of capital. This is because when the project’s return is relatively low, it is no longer possible to achieve the efficient level of monitoring through high loan rates and incentives are provided by capital in the same way as in the market solution.

We then analyze the case where there is deposit insurance. When deposits are insured, the degree of monitoring no longer affects a bank’s cost of deposits. Still, as in the case without deposit insurance, the market solution entails a positive amount of capital as a result of the competitive pressure in the credit market. For most of the parameter space, the market level of capital is above the socially optimal level or is constrained efficient.

Our basic model can be extended in a number of directions. We first consider alternative market structures to perfect competition in the loan market. When banks have monopoly power and there is no deposit insurance, they use the loan rate as the primary incentive tool. The bank’s incentives are correctly aligned with the objective of maximizing social welfare.
and the market equilibrium is always constrained efficient. By contrast, the presence of fixed deposit insurance introduces the standard moral hazard problem, thus creating a role for capital regulation to improve efficiency. Our main results remain valid with intermediate market structures between monopoly and competition where the surplus is split between banks and borrowers. As a related way for banks to generate surplus, we consider the case where banks have a franchise value from remaining in business. We find that franchise value and capital are substitute ways of providing banks with monitoring incentives.

One of the issues that has been raised during the recent crisis is whether banks had become too much transaction focused and thus neglected relationship banking. To see how capital affects banks’ choices, we study a setting where banks choose between relationship and transactional lending. The former refers to the monitored loan we have considered so far, and the latter to a loan with a fixed lower probability of success but a higher payoff in case of success. We show that capital regulation increases the attractiveness of relationship loans as capital represents a pure cost in the case of transactional lending. Next, we sketch a version of the model where bank monitoring helps alleviate an incentive problem on the side of borrowers as in Holmstrom and Tirole (1997). We argue that the main insight that competition leads banks to greater capitalization as a way to commit to greater monitoring remains valid in this more complex framework. Finally, we briefly discuss the case of fairly priced deposit insurance, where banks pay a premium that reflects their default probability. We show that this leads to the same results as without deposit insurance.

The paper has a number of empirical implications. First, our results are consistent with the fact that banks voluntarily hold higher levels of capital than the regulatory minimum and that changes in capital regulation do not affect banks’ capital structures. Second, the model suggests that greater credit market competition increases capital holdings as it introduces market discipline from the asset side. Third, it predicts that banks that are more involved in monitoring-intensive lending should be more capitalized and that, similarly, firms for which monitoring adds the most value should prefer to borrow from banks with high capital. Fourth, our analysis implies that banks’ capital holdings decrease with fixed deposit insurance coverage. Fifth, it predicts that increased capital requirements imply a shift in banks’ portfolios away from transactional lending towards more relationship lending.
Finally, the model predicts that banks with a lower fraction of outside equity or in countries with stronger shareholder rights should be more capitalized than banks with more dispersed ownership.

Recent research on the role of bank capital has studied a variety of issues. Gale (2003, 2004) and Gale and Özgür (2005) consider the risk sharing function of bank capital and the implications for regulation. They show that less risk averse equity holders share risk with more risk averse depositors. In contrast, in our model agents are risk neutral so risk sharing plays no role in determining banks' capital holdings.

Diamond and Rajan (2000) consider the interaction between the role of capital as a buffer against shocks to asset values and banks' role in the creation of liquidity. Closer to our work, Holmstrom and Tirole (1997) study the role of capital in determining banks' lending capacities and providing incentives to monitor. Other studies such as Hellmann et al. (2000), Repullo (2004) and Morrison and White (2005) analyze the role of capital in reducing risk-taking. In contrast to these papers, our approach studies the circumstances under which the market equilibrium is constrained efficient and the nature of socially optimal capital regulation when it is not.

Possible explanations for capital holding in excess of the regulatory minimum based on dynamic considerations are suggested by Blum and Hellwig (1995), Bolton and Freixas (2006), Peura and Keppo (2006), and Van den Heuvel (2008). In all of these, banks choose a buffer above the regulatory requirement as a way to ensure they do not violate the regulatory constraint. In these models banks’ capital holdings would still be altered by regulatory changes, something rarely observed in the data. Our model provides in a static framework an explanation for why capital holdings may be positive and may not be driven by regulatory changes.

In our model, using capital commits the bank to monitor. With no deposit insurance, this allows the bank to raise deposits more cheaply as depositors’ confidence that they will be repaid increases. On the lending side, the increased commitment to monitor makes a bank with a large amount of capital more attractive to borrowers and thus improves its “product market” opportunities. From this perspective, the use of capital in our model is reminiscent of the literature on the interaction between capital structure and product
market competition, where debt has been identified as having a strategic role in committing
the firm to take actions it might not otherwise find optimal (see, e.g., Brander and Lewis,

Section 2 outlines the model. Section 3 considers banks’ choice of monitoring taking
the loan rates and capital amounts as given. The case where there is no deposit insurance
is analyzed in Section 4, while the case with deposit insurance is investigated in Section
5. Section 6 extends the analysis in various directions. Section 7 contains the empirical
implications of our model. Section 8 concludes.

2 Model

Consider a simple one-period economy with firms and banks. Each firm has access to a risky
investment project and needs external funds to finance it. The banks lend to the firms and
monitor them. For ease of exposition, we assume throughout that each bank lends to one
firm only.

Each firm’s investment project requires 1 unit of funds and yields a total payoff of \( R \)
when successful and 0 when not. The firm raises the funds needed through a bank loan in
exchange for a promised total repayment \( r_L \). The credit market is assumed to be perfectly
competitive, so that the firm appropriates the surplus arising from the investment project.
(We discuss alternative market structures in Section 6.1.)

The bank finances itself with an amount of capital \( k \) at a total cost \( r_E \geq 1 \) per unit, and
an amount of deposits \( 1 - k \) at a total per unit (normalized) opportunity cost of 1. The bank
promises \( r_D \) to depositors. The deposit market is perfectly competitive so that the bank will
always set \( r_D \) at the level required for depositors to recover their opportunity cost of funds
of 1 and be willing to participate. The assumption that \( r_E \geq 1 \) captures the idea that bank
capital is a more expensive form of financing than deposits, as is typically assumed in the
literature.\(^1\)

The function of banks in the economy is to provide monitoring and thus improve firm

\(^1\)See Berger et al. (1995) for a discussion of this issue; and Gorton and Winton (2003), Hellmann et al.
(2000) and Repullo (2004) for a similar assumption. Mehran and Thakor (2006) provide a theoretical and
empirical analysis of the nature and magnitude of the cost of bank equity capital.
performance. The bank chooses an unobservable monitoring effort \( q \) that for simplicity represents the success probability of the firm it finances. Monitoring carries a cost of \( q^2/2 \) for the bank. One way of thinking about this is that the bank observes information about a firm and then uses this to help improve the firm’s performance. Another is that banks and firms have complementary skills. Entrepreneurs have an expertise in running the firm, while banks provide financial expertise and can thus help improve firm expected value.\(^2\) What is important is that greater monitoring is desirable from the borrower’s perspective.

This framework leads to a partial equilibrium analysis focusing on a single bank where the amount of capital \( k \), the loan rate \( r_L \), the deposit rate \( r_D \), and the amount of monitoring \( q \) are determined endogenously. All the variables other than \( q \) are publicly observable. The determination of \( k \) and \( r_L \) depends on the presence of a regulator. We consider two cases: in the first one, which we call the “market case,” both \( k \) and \( r_L \) are determined by the bank, while in the other one, defined as the “regulatory case,” \( k \) is determined by a regulator who maximizes social welfare and \( r_L \) is still set by the bank.

The timing of the model is as follows. In the market case, the bank first selects the level of capital \( k \) and then sets the deposit rate \( r_D \) and the loan rate \( r_L \). The firm chooses whether to take the loan and invest in the risky project. Then the bank chooses the monitoring effort \( q \). The regulatory case works similarly with the only difference that the regulator chooses the level of capital \( k \) initially and then the bank sets \( r_D \) and \( r_L \). Once chosen, \( k, r_D, \) and \( r_L \) are observable to all agents. Figure 1 summarizes the timing of the model.

3 Equilibrium Bank Monitoring

We solve the model by backward induction, and begin with the bank’s optimal choice of monitoring for a given amount of capital \( k \), deposit rate \( r_D \), and loan rate \( r_L \). The bank

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\(^2\)See Besanko and Kanatas (1993), Boot and Greenbaum (1993), Boot and Thakor (2000), Carletti (2004) and Dell’Ariccia and Marquez (2006) for studies with similar monitoring technologies. One justification of this type of assumption is given by Chemmanur and Fulghieri (1994). In their model banks use information acquired about the firm to improve liquidation/continuation decisions and thus increase firm value. Boot and Thakor (2000) suggest a number of other ways bank monitoring can improve firm performance (footnote 9, page 684).
chooses its monitoring effort so as to maximize expected profits as given by

\[
\max_q \Pi = q(r_L - (1 - k)r_D) - kr_E - \frac{1}{2}q^2.
\]  

(1)

The first term, \(q(r_L - (1 - k)r_D)\), represents the expected return to the bank obtained only when the project succeeds net of the repayment to depositors. The second term, \(kr_E\), is the opportunity cost of providing \(k\) units of capital, and the last term is the cost of monitoring.

The solution to this problem yields

\[
q = \min \{r_L - (1 - k)r_D, 1\}
\]

(2)

as the optimal level of monitoring for each bank. Note that, when \(q < 1\), bank monitoring effort is increasing in the loan rate \(r_L\) as well as in the level of capital \(k\) the bank holds, but it decreases in the deposit rate \(r_D\). Thus loan rates and capital are two alternative ways to improve banks’ monitoring incentives.

This framework implies a moral hazard problem in the choice of monitoring when the bank raises a positive amount of deposits. Since monitoring is unobservable, it cannot be determined contractually. Given it is costly to monitor, the bank has a tendency not to monitor properly unless it is provided with incentives to do so.

4 No Deposit Insurance

We now turn to the determination of the amount of capital \(k\), the loan rate \(r_L\), and the deposit rate \(r_D\). We start by analyzing the case where there is no deposit insurance. In this case, the promised repayment must compensate depositors for the risk they face when placing their money in banks that may not repay. This introduces a liability-side disciplining force on bank behavior since banks have to bear the cost of their risk-taking through a higher promised deposit rate. The expected value of the promised payment \(r_D\) must be at least equal to depositors’ opportunity cost of 1. Given the level of capital \(k\) and the loan rate \(r_L\), depositors conjecture a level of monitoring for the bank, \(q\), and set the deposit rate to meet
their opportunity cost. This implies that \( q r_D = 1 \), or that

\[
D = 1, \\
q = 1
\]

(3)

The determination of \( k \) and \( r_L \) depends on the presence of a regulator. We start with the “market” solution in the absence of regulation and we then turn to the “regulatory” solution in which a regulator who maximizes social welfare sets the level of capital.

The market solution solves the following problem:

\[
\max_{k, r_L, r_D} BS = q(R - r_L)
\]

subject to

\[
q = \min\{r_L - (1 - k)r_D, 1\},
\]

\[
q r_D = 1
\]

(5)

(6)

\[
\Pi = q(r_L - (1 - k)r_D) - kr_E - \frac{1}{2}q^2 \geq 0,
\]

\[
BS = q(R - r_L) \geq 0,
\]

\[
0 \leq k \leq 1.
\]

(7)

(8)

(9)

The bank chooses \( k, r_L, \) and \( r_D \) to maximize borrower surplus subject to a number of constraints. The first constraint is the monitoring effort chosen by the bank in the final stage after lending is determined. The second constraint is the depositors’ participation constraint discussed above, which holds with equality given that the deposit market is competitive. The third and fourth constraints are the bank’s and the borrower’s participation constraints, respectively. Note that the borrower’s participation constraint boils down to \( r_L \leq R \) if \( q > 0 \). The last constraint is simply a physical constraint on the level of capital.

The solution to this maximization problem yields the following result.

**Proposition 1** In the case of no deposit insurance, the market equilibrium is as follows:

A. For \( R \geq 2 - \frac{1}{2r_E} \), \( k^{BS} = \frac{1}{2r_E}, r_L = 2 - \frac{1}{2r_E}, r_D = 1, q = 1, BS = SW = R - (2 - \frac{1}{2r_E}) \) and \( \Pi = 0 \);

B. For \( R < 2 - \frac{1}{2r_E} \), there is no intermediation.
Proof: See the appendix. □

The results in Proposition 1 highlight that competition in the credit market induces banks to keep a positive level of capital. When projects are sufficiently profitable and intermediation is feasible \((R > 2 - \frac{1}{2r_E})\), banks fully monitor the firms so that \(q = 1\). Banks derive incentives to monitor from a combination of the loan rate and capital. These are substitute ways to provide banks with incentives to monitor but differ in terms of their costs and effects on borrower surplus and bank profits. Borrowers prefer banks to hold high levels of capital as a way to commit to high levels of monitoring. By contrast, since capital is a costly input (i.e., \(r_E \geq 1\)), the bank would prefer to minimize its use and receive incentives through a higher loan rate. While increasing \(r_L\) is good for incentive purposes, its direct effect is to reduce the surplus to the borrowers. Given that with competition the contract maximizes borrower surplus, the equilibrium when there is intermediation entails the maximum level of capital and the lowest level of loan rate consistent with \(q = 1\) and the banks’ participation constraint. In this sense, market discipline is imposed from the asset side as both the loan rate and bank’s capital are used to provide banks with monitoring incentives.\(^3\) In equilibrium, \(k\) decreases with the cost of capital \(r_E\) while the loan rate \(r_L\) increases with \(r_E\). This result implies a negative correlation between capital and the loan rate as a function of the cost of capital in the case of a competitive credit market.\(^4\)

We next analyze the optimal choice of capital when a regulator sets it to maximize social welfare and the loan rate is still determined as part of a market solution that maximizes the surplus of borrowers. This provides a benchmark to assess the efficiency of the market. Formally, a regulator solves the following problem:

\[
\max_k SW = \Pi + BS = q(R - (1 - k)r_D) - kr_E - \frac{1}{2}q^2 \tag{10}
\]

\(^3\)A related issue is studied in Chemmanur and Fulghieri (1994), who analyze how banks can develop a reputation for committing to devote resources to evaluating firms in financial distress and thus make the correct renegotiation versus liquidation decisions. Borrowers who anticipate running into difficulties may therefore prefer to borrow from banks with a reputation for flexibility in dealing with firms in financial distress. Reputation thus serves as a commitment device for banks similarly to capital in our model.

\(^4\)Note that Proposition 1 has the feature that banks either monitor fully \((q = 1)\) or there is no intermediation. This results from our assumption that the monitoring cost equals \(q^2/2\). If we modify it to \(cq^2\) with \(c > 1/2\), then we obtain an extra region with an interior solution for \(q\). This considerably complicates the analysis without providing any additional insights.
subject to the constraints (5)-(7) and (9), and
\[ r_L = \arg \max_r BS = q(R - r) \geq 0. \tag{11} \]

The regulatory problem differs from the market problem in the objective function, which is now social welfare rather than just borrower surplus. The constraints have the same meaning as above, with constraint (11) indicating that the loan rate is still set in the market to maximize borrowers’ surplus. The solution to the maximization problem is given below.

**Proposition 2** In the case of no deposit insurance, the regulatory equilibrium is as follows:

A.1. For \( R \geq 2 \), \( k_{reg} = 0 \), \( r_L = 2 \), \( r_D = 1 \), \( q = 1 \), \( BS = R - 2 \), \( \Pi = 1/2 \), \( SW = R - 3/2 \);

A.2. For \( R_{AB} \leq R < 2 \), \( k_{reg} = 1 - R^2/4 > 0 \), \( r_L = R \), \( r_D = 2R \), \( q = R/2 \), \( BS = 0 \), and
\[
\Pi = SW = \frac{R^2}{8} - (1 - R^2/4)r_E, \quad \text{where} \quad R_{AB} = \frac{4r_E + 2\sqrt{r_E + 2r_E^2 - 6r_E^3 + 4r_E^4}}{r_E + 2r_E^2};
\]

B. For \( 2 - 1/2r_E \leq R < R_{AB} \), \( k_{reg} = 1/2r_E > 0 \), \( r_L = 2 - k_{reg} \), \( r_D = 1 \), \( q = 1 \), \( BS = SW = R - (2 - 1/2r_E) \), and \( \Pi = 0 \);

C. For \( R < 2 - 1/2r_E \), there is no intermediation.

**Proof:** See the appendix. \( \square \)

The proposition is illustrated in Figure 2. The regulatory solution is quite different from the market solution. The reason is that the regulator can choose \( k_{reg} \) but has to take the loan rate \( r_L \) as determined in the market, where it is set to maximize borrower surplus. Given this, the equilibrium loan rate will often not coincide with the loan rate that maximizes social welfare, as borrowers prefer a loan rate that allocates them a greater fraction of the surplus than is socially optimal. Specifically, even though the regulator would prefer to use the loan rate to provide banks with incentives - it is a transfer that does not affect directly the level of social welfare - in its choice of \( k_{reg} \) the regulator has to take into account how the market solution for \( r_L \) affects banks’ incentives to monitor. This can imply a different solution than in the market case. Ideally, the regulator would like to be able to affect the competitive environment. In particular, the regulator would like to reduce competition between banks and have a higher loan rate to obtain the correct incentives. In what follows we assume it is not possible for the regulator to interfere in the market mechanism and thus set the loan
rate. If it was possible then this form of intervention would be superior to simply having higher capital requirements.

In Region A.1 of Proposition 2, projects are so profitable that the equilibrium loan rate \( r_L = 2 \) is sufficient to provide banks with incentives to fully monitor even if they hold no capital. The regulator therefore sets \( k^{reg} = 0 \), the loan rate is set just equal to the level that guarantees \( q = 1 \), and both banks and borrowers earn positive returns.

As the project return \( R \) falls below 2, the loan rate by itself is no longer enough to support full monitoring \( (q = 1) \) without capital. The regulator then has a choice between (a) keeping the capital requirement low and \( r_L \) as high as possible, but recognizing that monitoring may be reduced; or (b) requiring that banks hold more capital so as to maintain complete monitoring. In the first case the regulator sets the level of capital such that the market maximizes borrower surplus by setting \( r_L \) equal to \( R \). Any lower level of \( r_L \) leads to monitoring by the bank that is insufficient to ensure depositors receive their opportunity cost; depositors will then not lend. Any higher level of \( r_L \) violates the borrowers’ participation constraint. This solution is optimal in Region A.2.

In the second case the regulator uses a high level of capital to ensure that banks have the correct incentives to monitor. The market then lowers \( r_L \) so that borrower surplus is made as large as possible. The limit to this process is set by the participation constraint of the banks. In equilibrium \( r_L \) is set so that the banks earn zero profits and borrowers capture the entire surplus. This solution is optimal in Region B. The boundary \( R = R_{AB} \) is where the two types of solution give the same level of social welfare. Finally, as in the market solution, as the project return falls below \( R = 2 - \frac{1}{2r_E} \), we enter Region C where there is no intermediation.

We now turn to the comparison between the market and the regulatory solutions in the case of a competitive credit market. We have the following immediate result.

**Proposition 3** In the case of no deposit insurance:

A. For \( R \geq R_{AB} \) the market solution entails a higher level of capital than the regulatory solution, \( k^{BS} > k^{reg} \);

B. For \( 2 - \frac{1}{2r_E} \leq R < R_{AB} \), the market and the regulatory solutions entail the same level
of capital, \( k^{BS} = k^{reg} \).

Figure 2 illustrates the proposition (note that Region A comprises A.1 and A.2 from Proposition 2). The results show that the market solution is inefficient as it induces banks to hold inefficiently high levels of capital when the return of the project is sufficiently high. The basic intuition is that whereas the regulator prefers to economize on the use of costly capital and provide incentives through the loan rate, the market prefers to use capital as long as this is consistent with banks’ participation constraint. This implies that banks always break even in the market solution (\( \Pi = 0 \)), while they make positive profits in the regulatory solution in Regions A.1 and A.2 of Proposition 2. As the project return falls below \( R_{AB} \) and banks break even in the regulatory solution, the market solution coincides with the regulatory one and the market equilibrium is constrained efficient.

These results show that competition in the credit markets induces banks to make use of capital despite it being a costly form of finance. A social welfare maximizing regulator may find it optimal not to impose such high levels of capital and rather fix capital to the level that induces banks to maximize the use of the loan rate as an incentive tool. When this occurs, the market solution involves a higher level of capital than the regulatory solution. Otherwise the market and the regulatory solutions coincide so that the market solution is constrained efficient. One important feature of our analysis is that the regulator cannot set the loan rate. If it could do so, it would always set \( r_L \) equal to the project return so as to minimize the need for costly capital.

It is important to note that our analysis has focused on one aspect of regulation. In practice there are many other considerations driving capital regulation and minimum capital requirements. These include asset substitution where banks have an incentive to reduce safe investments and increase risky ones and systemic risk. These problems tend to be more severe the more competitive is the environment. The effect of these considerations would likely be to increase regulatory levels of capital.
5 Deposit Insurance

The standard argument concerning deposit insurance is that it makes funds more easily available to banks and this accentuates the banks’ moral hazard problem. Capital regulation is then required to offset the increased moral hazard problem. The purpose of this section is to investigate this argument in the context of our model.

We start by considering how a perfectly competitive market operates when there is deposit insurance and no capital regulation. As before, the market sets $k$ and $r_L$ to maximize borrower surplus, taking into account the subsequent monitoring choice and the fact that the bank has to make non-negative profits. In contrast to the previous section, the government now guarantees deposits in that it pays $r_D$ to the depositors if the bank goes bankrupt. We assume the cost of this deposit insurance is paid from revenues raised by non-distortionary lump sum taxes. The amount that banks promise to pay depositors is therefore just $r_D = 1$.

Solving the maximization problem (4) without the constraint (6) and setting $r_D = 1$ gives the following result.

**Proposition 4** In the case of deposit insurance, the market equilibrium always involves $r_L < R$ so $BS > 0$, and $\Pi = 0$. The level of capital, loan rate and monitoring are as follows:

A. For $R \geq R_{AB}$, $k^{BS} = \frac{1}{2r_E}, r_L = 2 - \frac{1}{2r_E}, q = 1$ and $BS = SW = R - (2 - \frac{1}{2r_E})$;

B. For $R < R_{AB}$, $k^{BS} = \left(\frac{\sqrt{2r_E} - \sqrt{2r_E - 3(R - 1)}}{3}\right)^2 < \frac{1}{2r_E}$, $r_L = 1 - k^{BS} + \sqrt{2r_E k^{BS}}, q = \sqrt{2r_E k^{BS}} < 1$, $BS = q(R - 1 + k^{BS} - q)$ and $SW = qR - q^2 - (1 - k^{BS}) \geq 0$ for $R \gtrless \min\{R_{AB}, \hat{R}\}$, where $\hat{R}$ solves $SW(\hat{R}) = q\hat{R} - q^2 - (1 - k^{BS}) = 0$.

The boundary $R_{AB}$ is defined as $R_{AB} = \frac{3}{2} - \frac{3}{8r_E} + \frac{r_E}{2}$ for $r_E < \frac{3}{2}$ and $R_{AB} = 3 - \frac{3}{2r_E}$ for $r_E \geq \frac{3}{2}$.

**Proof:** See the appendix. □

The results in Proposition 4 again highlight the incentive mechanisms for bank monitoring that are used in a competitive credit market. As usual, borrowers prefer that banks charge lower interest rates and hold large amounts of capital, whereas banks prefer to minimize the use of capital and receive incentives through a higher loan rate. Given that the market solution maximizes borrower surplus, the equilibrium involves the maximum amount...
of capital consistent with banks’ participation constraint and provides a loan rate up to the point where the (marginal) positive incentive effect of a higher loan rate equals its negative direct effect on borrower surplus. Thus, in addition to capital, the loan rate is still used to provide monitoring incentives - and thus market discipline - from the asset side. However, the market solution may now entail lower levels of monitoring and capital relative to the case without deposit insurance.

The proposition is illustrated in Figure 3. In both regions the zero-profit constraint for banks binds. If it did not it would always be possible to increase $BS$ by lowering $r_L$ and increasing $k$ while holding $q$ constant. The exact amounts of monitoring and capital in equilibrium depend on the project return $R$ and on the cost of capital $r_E$. In Region A project returns are high relative to $r_E$ so it is worth setting a high $r_L$ and $k$ to ensure full monitoring. As the returns fall in Region B, both $r_L$ and $k$ are reduced and $q < 1$.

One interesting feature of the equilibrium is that, differently from the case without deposit insurance, intermediation is now always feasible. The reason is that since the cost of raising deposits is fixed at $r_D = 1$ but they are repaid by the bank only in the case of project success, it is always possible to create positive borrower surplus and satisfy the zero profit constraint. However, social welfare is negative for low enough $R$ because of the cost of repaying depositors when the bank fails. This means that there would be no intermediation in this region if the institution insuring depositors refused to provide the insurance.

Following the same structure as before, we now analyze the optimal choice of capital from a social welfare perspective when loan rates are set as part of a market solution to maximize borrower surplus. The solution to this gives the following result.

**Proposition 5** In the case of deposit insurance, the regulatory equilibrium is as follows:

A. $k^{reg} = 0$, $r_L = \frac{R+1}{2}$, $q = 1$, $BS > 0$, $\Pi > 0$, and $SW > 0$;

B. $k^{reg} = 3 - R$, $r_L = R - 1$, $q = 1$, $BS > 0$, $\Pi > 0$, and $SW > 0$;

C. $k^{reg} = R + 1 - 4(r_E - 1)$, $r_L = 2(r_E - 1)$, $q = R - 2(r_E - 1) < 1$, $BS > 0$, $\Pi > 0$, and $SW > 0$;

D. $k^{reg} = 0$, $r_L = \frac{R}{2}$, $q = \frac{R-1}{2} < 1$, $BS > 0$, $\Pi > 0$, and $SW > 0$;

E. $k^{reg} = \frac{1}{2r_E}$, $r_L = 2 - \frac{1}{2r_E}$, $q = 1$, $BS = SW > 0$, $\Pi = 0$;
F. There is no intermediation because SW < 0.

The boundaries defining regions A through F are shown in Figure 4 and, together with the expressions for BS, Π, and SW, are defined in the appendix.

**Proof:** See the appendix. □

The proposition is illustrated in Figure 4. As usual, both capital and the loan rate are used to provide monitoring incentives, and their exact amounts depend on the return of the project \( R \) and the cost of equity \( r_E \). In Region A, \( R \) is sufficiently large that it is possible for the regulator to set \( k^{reg} = 0 \) and still have full monitoring, with incentives being provided by the loan rate \( r_L \). Both profits and borrower surplus are positive in this region. For lower \( R \), in Region B, borrowers prefer to reduce \( r_L \), thus providing lower incentives through the interest rate. Since \( r_E \) is relatively low, the regulator chooses a positive level of capital, \( k^{reg} > 0 \), to provide the remaining incentives to monitor. In Region C, the regulator uses less capital since \( r_E \) is higher, and it is no longer optimal to provide full incentives to monitor, so that \( q < 1 \). In Region D capital is too expensive to be worth using to provide incentives to monitor and imperfect incentives are provided through \( r_L \) alone. In Region E the regulator uses capital to make up for low incentives provided by a low value of \( r_L \). In Region F there is no intermediation since social welfare is negative. The regulator could prevent intermediation by eliminating the provision of deposit insurance or by setting \( k^{reg} \) sufficiently high that banks’ participation constraint is violated.

We next compare the market and regulatory solutions. The comparison between the values of \( k^{BS} \) and \( k^{reg} \) leads to the following result.

**Proposition 6** With deposit insurance the comparison between the market and the regulatory solutions is as follows:

A. \( k^{BS} > k^{reg} \);

B. \( k^{BS} = k^{reg} \);

C. \( k^{BS} < k^{reg} \).

D. No intermediation with regulation.

**Proof:** See the appendix. □
The proposition is illustrated in Figure 5. The main result of the proposition is that even with deposit insurance the market solution entails a positive level of capital because of competition in the credit market. As in the case without deposit insurance, the market solution entails too much capital (Region A) or is constrained efficient (Region B). The inefficient use of capital arises because, as before, borrowers are always better off with lower \( r_L \) and higher capital as long as this is consistent with banks’ participation constraint. The regulator, on the other hand, prefers to use a lower level of capital and provides incentives through a higher interest rate, particularly when the project return \( R \) is high. As \( R \) falls, the market solution becomes constrained efficient. Differently from the case without deposit insurance, there is now also a small area (Region C) where the market solution entails a lower level of capital than the regulatory solution. As the return \( R \) falls even further, the regulatory solution is no longer viable as the regulator prefers not to have intermediation when this implies negative social welfare.

Overall, the main conclusions of Section 4 remain valid when there is deposit insurance. The basic tendency when credit markets are competitive is for banks to hold a positive level of capital, sometimes even above the level that maximizes social welfare. However, deposit insurance blunts monitoring incentives and thus more capital must be used to provide incentives. This can be easily seen by comparing Propositions 3 and 6. The presence of deposit insurance implies an upward shifting in the boundaries defining the regions. For example, the boundary for \( k^{BS} > k^{reg} \) now lies entirely above the line \( R = 2 \), whereas without deposit insurance it lies below \( R = 2 \). Similarly, the boundary for the region where \( k^{BS} = k^{reg} \) in Figure 5 for the case of deposit insurance lies above the same boundary in Figure 2 for the case without deposit insurance.

6 Extensions

In this section we extend our basic model in various directions. First, we analyze alternative market structures to perfect competition. Second, in relation to banks’ ability to generate rents, we study the case where banks have a franchise value from continuing to operate. Third, we analyze a classic asset substitution problem where banks can choose loans with a
lower probability of success but with a higher payoff in the case of success. This extension can be used to obtain insight on the role of capital in the context of relationship versus transactional lending. Fourth, we consider an alternative framework where the borrower exerts effort and monitoring helps alleviate the resulting entrepreneurial moral hazard. Finally, we discuss the case of fairly priced deposit insurance.

6.1 Alternative market structures

The analysis above has been conducted assuming that credit markets are competitive. Here, we relax this assumption and consider the case where a bank operates as a monopolist and can therefore appropriate the surplus generated by the investment projects.\footnote{See Allen et al. (2008) for a full analysis of the monopoly case.}

When a bank is a monopolist, the contract it offers borrowers is set to maximize the bank’s profit, \( \Pi = q(r_L - (1 - k)r_D) - kr_E - \frac{1}{2}q^2 \), subject to the constraints (5) and (9) as well as (8) to guarantee that borrowers are willing to participate. As with competition, there are again the two cases of no deposit insurance and deposit insurance. In both it can be shown that the bank maximizes its surplus by setting the highest interest rate possible, \( r_L = \bar{r} \), which gives it a relatively high incentive to monitor.

In the case of no deposit insurance, the constraint (6), that is \( qr_D = 1 \), must again be satisfied. Since the bank internalizes fully the entire benefit from monitoring as well as the cost associated with non-repayment of depositors, it will always have the appropriate incentives to monitor efficiently. The liability-side discipline exerted by depositors induces banks to keep a positive amount of capital in situations where it is needed. There is therefore no scope for capital regulation to improve welfare, as social welfare maximization coincides with the maximization of bank profits and the market solution is always constrained efficient.

By contrast, a monopolist bank would never hold any capital in the market solution when there is deposit insurance. Since the deposit rate becomes independent of the level of bank monitoring, the bank has no incentive to use capital to commit to monitor. Given this, the presence of deposit insurance may give a role to capital regulation as a way of providing the bank with incentives to monitor and of reducing the disbursement of the deposit insurance fund as in, for example, Hellmann et al. (2000), Repullo (2004) and Morrison and White.
The solution that maximizes social welfare (given a market-determined loan rate) requires that banks hold a positive level of capital whenever the social benefit in terms of increased monitoring incentives and lower costs for the deposit insurance fund outweigh the cost of raising capital. Capital regulation is therefore a second best solution to the distortion introduced by deposit insurance when credit markets are monopolistic. This is entirely due to the presence of deposit insurance which allows the bank to take advantage of the implicit subsidy it provides.

It is worth noting that, as with perfect competition, intermediation is sometimes not feasible under monopoly. With no deposit insurance, there is no intermediation when project payoffs are sufficiently low. The boundary for the no intermediation region in the market solution, \( R < 2 - \frac{1}{2r_E} \), coincides not only with that for the regulatory solution but also with that under competition. Essentially, with no deposit insurance there is no intermediation whenever it is socially inefficient, both under monopoly as well as under competition.

The case with deposit insurance is somewhat different. Since neither banks nor borrowers bear the cost of repaying depositors when projects fail, financing is always available in the market solution. By contrast, there is no intermediation with regulation when project payoffs are sufficiently low. Now, however, there is a difference between the monopoly and the competition case discussed above. With deposit insurance in the monopoly case, intermediation is sometimes feasible even when not feasible without deposit insurance or under competition. This occurs when the project return \( R \) is between \( \sqrt{3} \) and \( 2 - \frac{1}{2r_E} \), or equivalently, when the cost of capital \( r_E \) is sufficiently high relative to the project’s payoff \( R \). The reason for this is that when capital is relatively costly, deposit insurance may be a more economical way of offering repayment to depositors than forcing banks to raise more capital in order to commit to monitor more. There may therefore be projects that are sufficiently profitable (i.e., for which \( R > \sqrt{3} \)) that are worth financing when deposit insurance is in place and banks are monopolists, but would not be worth financing (i.e., projects for which \( R < 2 - \frac{1}{2r_E} \)) in the absence of deposit insurance or when markets are competitive. In this sense, deposit insurance may increase social welfare and expand the possibilities for intermediation with monopolistic credit markets.\(^6\) This result is related to Morrison and

\(^6\)Of course, this positive result on the role of deposit insurance relies also on the fact that we have assumed
White (2006) in that deposit insurance helps correct a market failure and expands markets.

The analysis so far has focused on the extreme cases of perfect competition and monopoly. With perfect competition, borrower surplus is maximized and capital is used in the market solution to provide incentives for banks to monitor. Because capital is costly, competition can lead to inefficiencies. In the monopoly case the contract maximizes the bank’s profit and the bank gets the surplus. The high surplus provides banks with incentives to monitor efficiently with little or no capital. With intermediate market structures surplus is split between banks and borrowers, with each obtaining a positive expected return. The effects identified above will remain in such cases. In particular, the more surplus that banks obtain the less capital they will use. The more surplus borrowers obtain the greater will be the tendency for banks to use capital. These arguments also suggest that when capital regulation is too costly or ineffective as may be the case in our model in competitive credit markets, the regulator could seek to reduce interbank competition so as to provide banks with incentives to monitor through a higher loan rate instead of higher capital.

### 6.2 Bank franchise value

Together with the market structure, much discussion of bank behavior has focused on the role of franchise value as a possible way to reduce risk-taking (see, e.g., Keeley, 1990). Franchise value acts as an additional instrument providing a commitment to monitor. The intuition is simply that a greater franchise value means that the bank has a larger incentive to remain viable and in business, which leads it to dedicate more resources to monitor its borrowers so as to increase the success probability of its loans. As a consequence, the optimal level of capital needed to provide monitoring incentives is lower than without franchise value.

We endogenize the franchise value by characterizing the equilibrium of the dynamic model that is just a repeated version of our model. If a bank stays solvent it is able to continue to the next period. If it defaults it goes out of business. Introducing a discount factor of $\delta$ and a time index $t$ for each period, the franchise value at date $t$, denoted by $FV_t$, is given by the

---

that deposit insurance is funded through general revenues raised by non-distortionary taxes. If distortionary taxes were used, then the effective cost of deposit insurance would be higher.
current profits and the discounted value of the franchise value at date \( t + 1 \) so

\[
FV_t = \Pi_t + q_t \delta FV_{t+1} = q_t(r_{Lt} - (1 - k_t)r_{Dt}) - kr_E - \frac{1}{2}q_t^2 + q_t \delta FV_{t+1}.
\]

The maximization of \( FV_t \) leads to a monitoring effort at time \( t, q_t \), equal to

\[
q_t = \min \{r_{Lt} - (1 - k_t)r_{Dt} + \delta FV_{t+1}, 1\}.
\]

This implies that, for interior solutions, monitoring depends positively on the current returns from monitoring as well as on the future expected rents as, for example, in Boot and Greenbaum (1993). Given the problem is the same in each period, the optimal solution must be the same each period and thus \( FV_t = FV_{t+1} = FV \). Taking the interior solution for \( q \) and eliminating the \( t \) indexing, we can then express \( FV \) as

\[
FV = q(r_L - (1 - k)r_D) - kr_E - \frac{1}{2}q^2 + q\delta FV,
\]

from which

\[
FV = \frac{1}{1 - q\delta} \left( q(r_L - (1 - k)r_D) - kr_E - \frac{1}{2}q^2 \right) = \frac{1}{1 - q\delta} \Pi.
\]

From this, it can be seen that the franchise value depends positively on the bank’s static profit \( \Pi \) and equals zero whenever \( \Pi = 0 \). Thus, the role of the franchise value in reducing risk-taking depends crucially on the market structure of the credit market in that bank profits will usually be higher in monopolistic markets than in competitive markets. It may also depend on the presence or absence of capital regulation since, as shown above, optimal capital regulation may entail setting a capital requirement that provides banks with rents, even when the market is competitive.

### 6.3 Relationship and transactional lending

We have assumed throughout that banks can only finance projects that benefit from monitoring. In that context, we have shown that capital plays a role as a commitment device for banks to monitor and thus attract borrowers. We now modify this basic framework and, similarly to Boot and Thakor (2000), we consider the case where banks can choose between investing in a project which is identical to the one studied so far, and an alternative project
with a fixed probability $p_T$ of returning a payoff $R_T$. We will refer to the first kind of loan as a “relationship” loan since it benefits from the interaction with the bank, and the latter loan as a “transactional” loan. The crucial difference is that bank monitoring affects only the success probability of the relationship loan, given as before by $q$. As a consequence, the bank’s capital holdings will now affect the relative attractiveness of the two projects and capital regulation will play the additional role of affecting the distribution of bank funds across projects.

Assume that $p_T < q(0) < 1$, $R_T > R$, and $p_T R_T < q(0) R$, where $q(k)$ is the level of monitoring for a relationship loan when the bank has capital $k$. The transactional project has a lower probability of success than a relationship loan even with no capital ($k = 0$), a higher payoff in case of success, but a lower expected payoff. These assumptions introduce the possibility of a classic asset substitution problem. Banks may prefer to make transactional loans even though relationship loans are more valuable socially. Capital regulation can help to correct this market failure.

To analyze the bank’s choice in more detail, consider, for example, the case of monopoly banking where banks set the loan rate to obtain all the returns from the projects and have expected profits equal to

$$
\Pi_R = q(R - (1 - k)r_D) - kr_E - \frac{1}{2}q^2,
$$

$$
\Pi_T = p_T(R_T - (1 - k)r_D) - kr_E,
$$

from the relationship and the transactional loans, respectively. We first note that $\frac{\partial \Pi_T}{\partial k} = p_T r_D - r_E < 0$ so that capital decreases the attractiveness of the transactional loan and the bank would not want to hold any capital when investing in this project. This implies that capital regulation has the additional role of affecting the distribution of funds towards socially valuable investment projects. In situations where the asset substitution problem leads to an inefficiency, a minimum capital requirement can be used to rule out transactional lending and ensure relationship lending. Such a requirement will need to be higher the higher are $R_T$ and $r_D$. Once this capital regulation is in place, the factors considered in the basic model concerning relationship lending will come into play. Capital is further used to provide
monitoring incentives, and the qualitative results of our basic model remain valid.

6.4 The monitoring technology

So far we have assumed that bank monitoring directly determines the probability of success of the investment project. This captures the idea of bank monitoring being desirable for borrowers, and it simplifies the analysis in that the borrower does not exert any effort. Holmstrom and Tirole (1997) use a different framework where bank monitoring reduces borrowers’ private benefits. We adapt their approach so that monitoring influences the project success probability only indirectly. Specifically, assume that the firm invests in a project which, as before, yields a total payoff of $R$ when successful and 0 when not. The probability of success depends now on the effort of the borrower. In particular, the borrower chooses an unobservable effort $e \in [0, 1]$ that determines the probability of success of the project and carries a cost of $e^2/2$. The borrower also enjoys a (nonpecuniary) private benefit $(1 - e)B > 0$, which is maximized when he exerts no effort. One way of interpreting the cost $-eB$ is that putting in effort reduces the amount of time the borrower can spend pursuing privately beneficial activities, or enjoying the perks of being in charge of the project. Bank monitoring helps alleviate moral hazard in this framework. In particular, the bank chooses a monitoring effort $q$ which reduces the private benefit of the borrower to $(1 - e)B(1 - q)$ and entails a cost of $q^2/2$. We can think of bank monitoring as taking the form of using accounting and other controls to reduce the borrower’s private effort, or to reduce his ability to consume perks. Monitoring is chosen before the borrower’s effort.

Given this set up, for given $k$, $r_D$, and $r_L$, the borrower chooses his effort to maximize

$$BS = e(R - r_L) + (1 - e)B(1 - q) - \frac{1}{2}e^2$$

so that

$$e = \min \{(R - r_L) - B(1 - q), 1\}.$$  

The bank chooses $q$ to maximize

$$\Pi = e(r_L - (1 - k)r_D) - kr_E - \frac{1}{2}q^2$$
which yields

\[ q = \min \left\{ (r_L - (1 - k)r_D)B, 1 \right\}. \]

It can be seen that in this version of the model the borrower’s effort decreases with the loan rate \( r_L \) and the private benefit \( B \) while it increases with the project return \( R \) and the monitoring effort \( q \). Bank monitoring in turn increases in the loan rate \( r_L \), the level of capital \( k \) and the private benefit \( B \). Thus, as before, bank monitoring positively affects the success probability of the project as it reduces borrower’s moral hazard. The difference is that, as in Boyd and De Nicolo (2005), in setting the loan rate \( r_L \) the bank will now have to consider also the negative effect that this has on the borrower’s effort so that in equilibrium its level generally will be lower than the one found in our basic model. This implies also different levels of capital and of monitoring in equilibrium relative to those in our basic model, but it does not affect the qualitative results as long as the loan rate \( r_L \) is still used to provide banks with incentives to monitor and this is desirable from the borrower’s perspective. A sufficient condition to guarantee this is that the private benefit \( B \) is greater than one, as this implies that the indirect positive effect through a higher \( q \) of an increase in \( r_L \) dominates the negative direct effect on the entrepreneur’s effort \( e \).

### 6.5 Fairly priced deposit insurance

We have shown above that deposit insurance accentuates banks’ moral hazard problem in monitoring as deposit rates become insensitive to the risk of banks’ assets. In doing this, we have assumed that the cost of deposit insurance is paid from revenues raised by non-distortionary lump sum taxes and it is therefore independent of banks’ risk and capital. We now consider the case of fairly priced deposit insurance, that is the case where banks pay a deposit insurance premium that reflects their default probability. We denote this cost as \( C \) and, as is common in the literature (see, for example, Chan et al., 1992), we assume that the bank pays it in advance.\(^7\) This implies that the bank needs to raise a total of \( 1 + C \) units of funds to finance the loan to the borrower as well as to pay the insurance premium. As usual, \( k \) of these funds represent capital, and \( 1 - k + C \) is deposits. Given that the deposit

\(^7\)It can be shown that the same results hold if the bank pays the deposit insurance premium ex post from the revenue derived from its loan.
rate is still \( r_D = 1 \) as deposits are fully insured, bank profits can be written as

\[
\Pi = q (r_L - (1 - k + C)) - kr_E - \frac{1}{2} q^2,
\]  

(12)

reflecting the fact that all deposits, \( 1 - k + C \), are only repaid by the bank when its project is successful.

In order to be fairly priced, the premium \( C \) has to be equal to the expected future disbursement of the deposit insurance fund. This equals \( 1 - k + C \) upon default by the bank, which in expectation is \( (1 - q)(1 - k + C) \). Setting the payment equal to the expected disbursement and solving for \( C \) we obtain

\[
C = \frac{(1 - q)(1 - k)}{q}.
\]

Substituting this into (12) and simplifying gives the same expression for the bank’s profit, \( \Pi = q (r_L - (1 - k)\frac{1}{q}) - kr_E - \frac{1}{2} q^2 \), as in the case of no deposit insurance. This implies that the case of fairly priced deposit insurance delivers the same results as the case with no deposit insurance. The intuition is that the fairly priced deposit insurance premium plays the same role as the deposit rate in the case of no deposit insurance in terms of providing “liability-side” discipline.

7 **Empirical predictions**

The main insight of the paper is to show that competition in the credit market provides an incentive for banks to use capital as a way to commit to greater monitoring. This is consistent with the fact that banks voluntarily hold high levels of capital even above the regulatory levels and that changes in capital regulation do not affect banks’ capital structures as found by Flannery and Rangan (2008), Ashcraft (2001), Barrios and Blanco (2003) and Alfon et al. (2004).

According to our model, banks’ levels of capital vary with the degree of competition. As explained above, with market structures intermediate between perfect competition and monopoly, surplus is split between banks and borrowers, with each obtaining a positive expected return. The more surplus that firms obtain, the more capital banks will use. This
suggests the empirical prediction that the more competitive is the banking sector, the greater will capital holdings be. This prediction finds empirical support in the result of Cihak and Schaeck (2008) that European banks hold higher capital ratios when operating in a more competitive environment.

The mechanism that capital improves banks’ incentives to monitor leads to some cross-sectional implications concerning banks’ capital holdings and firms’ source of borrowing. In particular, it suggests that banks engaged in monitoring-intensive lending should be more capitalized than other banks. To the extent that small banks are more involved in more monitored lending to small and medium firms, the model predicts that small banks should be better capitalized than larger banks, in line with the empirical findings in Alfon et al. (2004), Ayuso et al. (2004) and Gropp and Heider (2008).

Concerning firms’ choice of financing, our model predicts that firms for which monitoring adds the most value should prefer to borrow from banks with high capital. Billett et al. (1995) find that lender “identity,” in the sense of the lender’s credit rating, is an important determinant of the market’s reaction to the announcement of a loan. To the extent that capitalization improves a lender’s rating and reputation, these results are in line with the predictions of our model.

Concerning the introduction of fixed rate deposit insurance, the model predicts that banks’ capital will fall. As a result their monitoring efforts will be reduced and risk will increase. This result is consistent with the finding in most empirical studies considering whether deposit insurance increases the riskiness of banks (see, for example, Ioannidou and Penas, 2009).

Our analysis also has some implications concerning banks’ portfolio choice. As shown earlier, capital decreases the attractiveness of transactional loans while increasing that of relationship loans. Thus, anything that induces banks to hold more capital affects the allocation of banks’ funds. This implies that increased competition or increased capital requirements should be associated with a shift in banks’ portfolios away from transactional lending towards more relationship lending. Similarly, the presence of capital markets reduces banks’ incentives to hold capital, as found empirically by Cihak and Schaeck (2008), and consequently should be associated with higher transactional lending. These predictions are
consistent with the theoretical results in Boot and Thakor (2000) that stronger interbank competition and weaker capital market competition should induce more relationship lending.

Our model also has implications for the penetration of banks into foreign markets. Among other things, information asymmetries developed through long term relationships have been identified as possible barriers to entry. This leads banks to focus their entry toward market segments less subject to private information (see Dell’Ariccia and Marquez, 2004, and Marquez, 2002, and the evidence in Clarke et al., 2001, and Martinez-Peria and Mody, 2004). These results point to the need for entrant banks to have a competitive edge. Capital provides banks with an advantage in attracting borrowers as it allows them to commit to monitor. Our analysis thus predicts that it will be well-capitalized banks that enter foreign markets.

In our model there is no agency problem either within the bank or the firm. Introducing an agency problem within the bank may reduce the role of capital as a way to commit to greater monitoring as in Besanko and Kanatas (1996), where raising equity dilutes current managers’ stake in the firm and thus reduces their incentives to exert effort. This suggests that our analysis applies to banks where the agency problem is limited because of a small share of outside equity or to countries where the interests of insider and outsider investors are aligned through a range of contractual provisions. In this respect, our model predicts that banks with a lower fraction of outside equity or in countries with stronger shareholder rights should be more capitalized than banks with more dispersed ownership. This prediction is consistent with the empirical finding in Cihak and Schaeck (2008) of a positive relationship between shareholder rights and banks’ capital holdings. Finally, introducing an effort requirement in the borrowing firm as in Boyd and De Nicolo (2005) reduces the attractiveness of the loan rate as an incentive tool and increases the importance of capital. To the extent that small and medium firms are more subject to an effort requirement, banks lending to them will use more capital. This is consistent with the evidence in Alfon et al. (2004), Ayuso et al. (2004) and Gropp and Heider (2008) mentioned above.
8 Concluding Remarks

In this paper we have developed a theory of capital that is consistent with the observation that banks may hold levels of capital even above the levels required by regulation. Our approach is based on the idea that both the loan rate charged by the bank and capital provide incentives to monitor, and that competition in the credit market may operate as market discipline from the asset side of banks’s balance sheets. We adopt the standard assumption in the literature that capital is more costly than other sources of funds. In the case of no deposit insurance, a competitive market structure provides incentives for banks to use a positive level of capital. The reason is that borrowers prefer lower interest rates and higher capital as they do not bear the cost of the capital. When there is deposit insurance banks’ incentives to monitor are reduced, but the market solution may still entail too much capital. However, banks now use too little capital for a small range of parameters.

There are many interesting directions for future research. First, the fact that banks hold levels of capital above the regulatory minimum does not necessarily imply that they are well capitalized. Although in our model excess capital implies levels above those maximizing social welfare, in more complex models reflecting other important aspects of capital requirements banks may still be undercapitalized despite them holding capital well above the regulatory minimum. This is a particularly important line of research given the current crisis in the financial system and the discussion of whether banks were indeed adequately capitalized. For example, we disregard sources of systemic risk. In our model banks are subject only to idiosyncratic individual risk because of the possibility that their loans do not repay. The failure of one bank does not have any spillover on the other banks. If it did, there would be an additional role for capital regulation. The market solution would not internalize this contagion risk when setting the level of capital banks should hold. By contrast, a social welfare maximizing regulator would internalize this risk and would therefore require banks to hold higher levels of capital than the ones obtained in our model.

Second, in our model we assume that all banks are the same. Boot and Marinč (2006) consider heterogeneous banks with a fixed cost of monitoring operating in markets with different degrees of competition. Incorporating these elements into our framework could be
an interesting direction to pursue in the future.

We have focused on regulatory capital that maximizes social welfare. A number of other approaches are possible. For example, in many instances it seems that actual regulatory capital levels have been set based on historically observed levels. Basel II represents another type of approach where regulatory capital is derived from the criterion of covering the bank’s losses 99.9% of the time. The discrete version of the model we have developed is not appropriate for analyzing this type of criterion. Instead, a version with a continuous distribution of returns is necessary. Developing this extension of our model is another interesting topic for future research. This would also help shed light on the question as to whether banks can be undercapitalized even when they hold capital in excess of the regulatory minimum.
A Proofs

Full details of the algebra in the proofs are given in Allen et al. (2008).

Proof of Proposition 1: Substituting (6) into (5) when \( q < 1 \) and solving for the equilibrium value of monitoring, we obtain two solutions as given by

\[
q = \frac{1}{2} \left( r_L \pm \sqrt{r_L^2 - 4(1-k)} \right).
\]

The relevant solution is the positive root, as it can be shown that both banks and borrowers are better off with the higher level of monitoring. This implies that

\[
q = \min \left\{ \frac{1}{2} \left( r_L + \sqrt{r_L^2 - 4(1-k)} \right), 1 \right\}.
\]  

(13)

Assuming \( q < 1 \), we substitute for \( q \) in the expression for borrower surplus to obtain

\[
BS = q(R - r_L) = \frac{1}{2} \left( r_L + \sqrt{r_L^2 - 4(1-k)} \right) (R - r_L).
\]

Clearly we need \( r_L \geq 2\sqrt{1-k} \) for an equilibrium to exist.

We now turn to the determination of \( r_L \) and \( k \). It can be shown that \( \Pi > 0 \) is never optimal. For each of the four possible combinations of \( q \leq 1 \) and \( k \leq 1 \), it is either possible to increase \( BS \) by changing \( q, k \) and \( r_L \) appropriately (\( q, k < 1; q = 1, k < 1; q = k = 1 \)) or the bank’s participation constraint is violated (\( q < 1, k = 1 \)).

Given \( \Pi = 0 \), consider now a candidate solution for the optimum with \( q = 1 \). From \( \Pi = r_L - \frac{3}{2} + k(1-r_E) = 0 \), we obtain \( r_L = \frac{3}{2} + k(r_E - 1) \). For \( r_L \) to be optimal for borrowers, \( k \) must be the lowest value consistent with \( q = 1 \). Substituting the expression for \( r_L \) into (13), setting this equal to one and solving for \( k \) gives \( k = \frac{1}{2r_E} \). With this value for \( k \) the expression for \( r_L \) gives \( r_L = 2 - \frac{1}{2r_E} \). Note that, given our candidate solution has \( q = 1 \), no other solution can increase \( BS \) while satisfying the bank’s participation constraint. For \( k > \frac{1}{2r_E} \), \( r_L > 2 - \frac{1}{2r_E} \), but \( q \) does not increase beyond 1, thus lowering \( BS \). For \( k < \frac{1}{2r_E} \), satisfying the bank’s participation constraint with equality requires reducing \( r_L \). This lowers \( q \) to below 1, violating the assumption that \( q = 1 \) at the optimum. Note further that for \( q = 1 \), \( BS = R - \left( 2 - \frac{1}{2r_E} \right) \), which is clearly greater than zero only for \( R > 2 - \frac{1}{2r_E} \).

It remains to be shown that at the optimum \( q = 1 \) must hold. To see this, substitute the expression for \( q < 1 \) from (13) into \( \Pi(q) \). Solving simultaneously for \( k \) and \( r_L \) gives

\[
k = \frac{q^2}{2r_E}, r_L = q + \frac{1}{q} - \frac{q}{2r_E}.
\]
We can now substitute these expressions into the problem of maximizing borrower surplus with the maximization now taken with respect to $q$: $\max_q BS = q \left( R - q - \frac{1}{q} + \frac{q}{2r_E} \right) = qR - q^2 - 1 + \frac{q^2}{2r_E}$. The derivative yields
\[
\frac{\partial BS}{\partial q} = R - 2q + \frac{q}{r_E},
\]
with the second derivative being negative so that $BS$ is concave in $q$. Note now that $\frac{\partial BS}{\partial q} \bigg|_{q=0} = R > 0$, so that clearly $q > 0$ is optimal. Setting (14) equal to zero and solving for $q$, we obtain $q^* = \frac{R}{2 - \frac{1}{r_E}}$. From this we see that for $R > 2 - \frac{1}{r_E} > 2 - \frac{1}{r_E}$, $q^* > 1$, so that the solution must have $q = 1$. Moreover, from above we know that for $q = 1$, $BS = SW \geq 0$ for $R \geq 2 - \frac{1}{2r_E}$, so that clearly $q > 0$ is optimal. Setting (14) equal to zero and solving for $q$, we obtain $q^* = \frac{R}{2 - \frac{1}{r_E}}$. From this we see that for $R > 2 - \frac{1}{2r_E} > 2 - \frac{1}{2r_E}$, no intermediation is possible. This gives part A of the proposition.

Finally, consider the case where $R < 2 - \frac{1}{2r_E}$. For $2 - \frac{1}{2r_E} > R \geq 2 - \frac{1}{r_E}$, $q = 1$ and $BS < 0$. For $R < 2 - \frac{1}{r_E}$, $q < 1$. Substituting the optimal value of $q$ into $BS$ it can be shown that $BS = q^2 \left( 1 - \frac{1}{2r_E} \right) - 1 < 0$ since $q = R/(2 - \frac{1}{r_E}) < 1$. Thus for $R < 2 - \frac{1}{2r_E}$, no intermediation is possible. This gives part B of the proposition.

**Proof of Proposition 2:** As before, the equilibrium value of monitoring $q$ is given by (13) and $r_L \geq 2\sqrt{1-k}$ is needed for an equilibrium to exist when $q < 1$. Assuming that $r_L$ is large enough, we can show that the unique solution that satisfies (11) is
\[
\hat{r}_L \equiv \frac{R}{2} + \frac{2(1-k)}{R}.
\]
For $r_L \to 2\sqrt{1-k}, \partial BS/\partial r_L > 0$. Substituting $r_L = \hat{r}_L + \varepsilon$ into $\partial BS/\partial r_L$ and evaluating for $\varepsilon > 0$ by differentiating with respect to $\varepsilon$, it can be shown that $\partial BS/\partial r_L < 0$ for $r_L > \hat{r}_L$. It follows from this that $BS(r_L)$ is a concave function in the relevant range.

Note also that for $r_L \geq \tau_L \equiv 2 - k \geq 2\sqrt{1-k}$, it follows from (13) that $q = 1$ and for $r_L < \tau_L$, $q < 1$.

We now divide the analysis into two cases: (1) $R \geq 2$; and (2) $R < 2$.

Case 1: $R \geq 2$. Now $\hat{r}_L > \tau_L$ for $R > 2$. To see this note that $\hat{r}_L = \tau_L$ at $R = 2$ and $\frac{\partial(\tau_L - \tau_L)}{\partial R} = \frac{1}{2} - 2(1-k)/R^2 > 0$ for $R > 2$. Given the concavity of $BS(r_L)$ it follows that $\frac{\partial BS}{\partial r_L} \bigg|_{r_L} > 0$ for $R > 2$. This implies that borrowers always demand a loan rate equal to $r_L = \tau_L = 2 - k$ so that $q = 1$ as long as this satisfies the bank’s participation constraint,
\[ \Pi = \frac{1}{2} - k r_E \geq 0, \] which it does for \( k \leq \frac{1}{2 r_E} \). For \( k > \frac{1}{2 r_E} \) such that the bank’s participation constraint binds, we need to set \( r_L \) to satisfy \( \Pi (r_L | k) = 0 \).

Assuming the bank’s participation constraint is satisfied, we can now turn to the problem in the first stage to determine \( k \). Since \( q = 1 \), the problem simplifies to

\[ \max_k SW = R - \frac{3}{2} + k (1 - r_E). \]

The first order condition yields \( \partial SW / \partial k = 1 - r_E < 0 \), so that \( k = 0 \) is optimal. We check that this solution does in fact satisfy the bank’s participation constraint, as \( \Pi = qr_L - (1 - k) - k r_E - \frac{1}{2} q^2 = 2 - k - (1 - k) - k r_E - \frac{1}{2} = \frac{1}{2} > 0 \). Therefore, \( k = 0 \), \( q = 1 \), and \( r_L = 2 \) is a candidate solution for \( R \geq 2 \). That it is also the optimal solution can be seen from noting that higher values of \( k \) cannot increase \( q \) further, so that any solution with \( k > \frac{1}{2 r_E} \) and \( r_L \) determined from \( \Pi (r_L | k) = 0 \) when the bank’s participation constraint binds must necessarily lead to lower \( SW \).

Case 2: \( R < 2 \). We know that a minimum condition for an equilibrium to exist is that \( r_L \geq 2 \sqrt{1 - k} \). Solving for \( k \), this is equivalent to requiring \( k \geq 1 - \frac{R^2}{4} \). For \( r_L = R \), this implies \( k_{\text{min}} = 1 - \frac{R^2}{4} \) as an absolute lower bound on the level of capital that is consistent with equilibrium.

Using the expressions for \( \hat{r}_L \) and \( r_L \), substituting for \( k_{\text{min}} \) and rearranging gives \( \hat{r}_L - r_L = -(1 - \frac{R^2}{4})^2 < 0 \). It follows that for \( R < 2 \), \( \hat{r}_L < r_L \) and from (13) that \( q(\hat{r}_L) < 1 \).

Define now \( r_B^L \) as the loan rate that satisfies the bank’s participation constraint with equality, that is \( \Pi (r_B^L | k) = 0 \). Also, note that \( \frac{\partial \Pi}{\partial r_L} = (r_L - q) \frac{\partial q}{\partial r_L} + q > 0 \). If, for a given \( k \), \( \hat{r}_L > r_B^L \), then at the optimum borrowers choose \( r_L = \hat{r}_L \), and \( q < 1 \), \( \Pi > 0 \). If, however, for a given \( k \), \( \hat{r}_L < r_B^L \), then \( \hat{r}_L \) is no longer a feasible solution since \( \Pi (\hat{r}_L) < \Pi (r_B^L) = 0 \). In this case, the optimal loan rate is the lowest rate for which \( \Pi = 0 \), which is \( r_B^L \). This is because no lower rate is feasible since \( \Pi (r_L) < 0 \) for any \( r_L < r_B^L \). A higher \( r_L \) is feasible but not optimal since it follows from the concavity of \( BS \) that \( BS \) must be decreasing for \( r_L > \hat{r}_L \).

The analysis above demonstrates that we have two candidate solutions: either \( r_L = r_B^L \) with \( \Pi (r_B^L) = 0 \), or \( r_L = \hat{r}_L \) with \( \Pi (\hat{r}_L) \geq 0 \). The level of \( k \) chosen by the regulator remains to be determined for the two solutions. Start with the case where \( r_L = r_B^L \), so that
$\Pi \left( r_L^* \right) = 0$. Here, the maximization of $SW$ is equivalent to the maximization of $BS$, for which we know from Proposition 1 that the solution involves $q = 1$ and $r_L = 2 - k$. This implies $\Pi = \frac{1}{2} - kr_E$, and since by assumption we have $\Pi = 0$, this implies that $k = \frac{1}{2r_E}$ at the optimum. Under this solution social welfare equals

$$SW_1 = qR - (1 - k) - kr_E - \frac{1}{2}q^2 = R - \frac{3}{2} + \frac{1}{2r_E} (1 - r_E) = R - 2 + \frac{1}{2r_E}.$$

We note that $SW_1 \geq 0$ for $R \geq 2 - \frac{1}{2r_E}$.

Next, consider the candidate loan rate $r_L = \hat{r}_L = \frac{R}{2} + \frac{2(1-k)}{R}$, with $q < 1$. For this solution to be feasible, it must satisfy $\Pi \geq 0$. Substituting the equilibrium loan rate into the bank’s monitoring effort as in (13), we have $q = \frac{R}{2}$.

Consider now social welfare, and note that $\frac{\partial SW}{\partial k} = 1 - r_E < 0$, so that the regulator prefers the lowest possible $k$. From above, this lowest value is given by $k = 1 - \frac{R^2}{4}$. Now $\hat{r}_L = R$ when evaluated at $k = 1 - \frac{R^2}{4}$. For this level of $k$ and $r_L$, $BS = 0$. However, $q = R/2$, which implies

$$SW_2 = \Pi = \frac{R^2}{8} - (1 - \frac{R^2}{4})r_E.$$

We compare the two candidate solutions. This amounts to finding the minimum value of $R$ such that $SW_2 \geq SW_1$. This value is given by

$$R_{AB} = \frac{4r_E + 2\sqrt{r_E + 2r_E^2 - 6r_E^3 + 4r_E^4}}{r_E + 2r_E^2},$$

so that, for $R > R_{AB}$, $SW$ is maximized by setting $k = 1 - \frac{R^2}{4}$, with $q = \frac{R}{2}$, $r_L = R$, and $\Pi = SW > 0$. This is part A.2 in the proposition. For $R < R_{AB}$, $SW$ is maximized by setting $k = \frac{1}{2r_E}$, with $q = 1$, $r_L = 2 - k$, $\Pi = 0$ and $SW = R - 2 + \frac{1}{2r_E}$. This is part B of the proposition.

Finally, if $R - 2 + \frac{1}{2r_E} < 0$ no intermediation occurs and this is part C of the proposition. □

**Proof of Proposition 4:** With deposit insurance $r_D = 1$ and (2) simplifies to $q = \min \{r_L - (1 - k), 1\}$.

We then proceed in two steps. The first is to show that at the optimum $k < 1$, $r_L < R$ and $\Pi = 0$. The second is to characterize parts A and B of the proposition (see Figure 3).

**Step 1:** To demonstrate that $k = 1$ is not optimal, assume that $k = 1$. Then it is possible
to increase $BS$ both for $q = 1$ and $q < 1$ by appropriately changing $k$ and $r_L$. Similarly, it is possible to show that $r_L = R$ cannot be optimal because it is always possible to lower $r_L$ and set $k$ so that $q > 0$ and $BS$ is increased. Finally, it can be shown that if $\Pi > 0$ it is always possible to alter $k$ and $r_L$ so that $BS$ is increased.

**Step 2:** We now turn to the expressions for $k$, $r_L$ and $q$ knowing that $k < 1$, $r_L < R$ and $\Pi = 0$. There are two possibilities for the monitoring effort, $q = 1$ and $q < 1$, and these correspond to parts A and B in the proposition.

Consider $q < 1$ first. This implies $q = r_L - (1 - k) < 1$. Using this and the constraint $\Pi = 0$, we have

$$r_L = 1 - k + \sqrt{2r_E k} \quad \text{and} \quad q = \sqrt{2r_E k} < 1. \quad (16)$$

The last inequality implies $k < \overline{k} = \frac{1}{2r_E}$ for $q < 1$. Given (16), it follows that

$$BS(k) = \sqrt{2r_E k}[R - 1 + k - \sqrt{2r_E k}]. \quad (17)$$

Differentiating with respect to $k$, putting $\frac{\partial BS}{\partial k} = 0$, multiplying through by $k^{1/2}$, and squaring, we obtain $k = \left(\frac{\sqrt{2r_E} \pm \sqrt{2r_E - 3(R - 1)}}{3}\right)^2$. This gives two distinct roots for $2r_E - 3(R - 1) > 0$ or, equivalently, $R < \frac{2r_E}{3} + 1$. Since $BS|_{k=0} = 0$ and $\frac{\partial BS}{\partial k}|_{k=0} > 0$, the root for $k$ with a minus, $k^{INT}$, is a local maximum while the root with a plus, $k^{MIN}$, is a local minimum. To see then whether $k^{INT}$ is a global maximum, we first note that $\overline{k} = \frac{1}{2r_E}$ is the maximum possible optimal value of $k$ since for $k > \frac{1}{2r_E}$, $q = 1$, $BS = R - r_L$ with $r_L = 3/2 + k(r_E - 1)$ satisfying the constraint $\Pi = 0$, and $\partial BS/\partial k = -(r_E - 1) < 0$, so that $k > \overline{k}$ is never optimal for borrowers. Then, we compare $k^{INT}$ and $k^{MIN}$ with $\overline{k}$. To do this, we distinguish between two cases given by $r_E > \frac{3}{2}$ and $r_E \leq \frac{3}{2}$.

(i) Consider $r_E > \frac{3}{2}$. Setting $k^{INT} = \overline{k}$ and solving for $R$ yields $R = R_{AB} = 3 - \frac{3}{2r_E}$. Since $k^{INT}$ is increasing in $R$, this implies that $k^{INT} < \overline{k}$ for $R < 3 - \frac{3}{2r_E}$. Now notice that for $r_E > \frac{3}{2}$ we have

$$\overline{k}^{1/2} = \frac{1}{\sqrt{2r_E}} < \frac{1}{\sqrt{3}} < \frac{1}{\sqrt{3}} \frac{\sqrt{2r_E}}{\sqrt{3}} = \frac{\sqrt{2r_E}}{3}.$$

This inequality together with $\left(\frac{\sqrt{2r_E}}{3}\right)^2 < k^{MIN}$ implies $\overline{k} < k^{MIN}$. Thus, if $r_E > \frac{3}{2}$, we have $k^{INT} < \overline{k} < k^{MIN}$ for $R < R_{AB}$. This, together with the fact that at $q = 1$, $\partial BS/\partial k < 0$ and
that $\frac{\partial BS}{\partial k} \big|_{k=k_R} = r_L \left( R - 3 + \frac{3}{2r_L} \right) < 0$, implies that $BS(k^{INT}) > BS(\bar{k})$ and therefore that $k^{INT}$ is the global maximum for $R < R_{AB}$ and $r_L > \frac{3}{2}$. By contrast, for $R_{AB} < R < \frac{3}{2r_L} + 1$, $k^{INT} > \bar{k}$ and $\bar{k} = \frac{1}{2r_L}$ is the global optimum since $q = 1$, $\frac{\partial BS}{\partial k} \big|_{k=\bar{k}} > 0$ and $\partial BS/\partial k = -(r_L - 1) < 0$ for $k > \frac{1}{2r_L}$. Finally, for $R > \frac{3}{2r_L} + 1$, no real value for $k^{INT}$ exists. It follows that for $0 \leq k < \frac{1}{2r_L}$, $\partial BS/\partial k > 0$, while $\partial BS/\partial k < 0$ for $k > \frac{1}{2r_L}$. Thus $k = \frac{1}{2r_L}$ is the global maximum and $q = 1$.

(ii) Consider now $r_L < \frac{3}{2}$. Here it can be shown similarly to above that $\bar{k}^{1/2} > \sqrt{2r_L}/3$ so that $k^{INT} < \bar{k}$. Now $k^{MIN} = \bar{k}$ for $R = 3 - \frac{3}{2r_L}$, and $k^{MIN} > \bar{k}$ for $R < 3 - \frac{3}{2r_L}$ since $k^{MIN}$ is decreasing in $R$. This implies that $k^{INT}$ is the global optimum for $R \leq 3 - \frac{3}{2r_L}$ using a similar argument to the one above for $r_L > \frac{3}{2}$. On the other hand, for $R > 3 - \frac{3}{2r_L}$, $k^{MIN} < \bar{k}$ and therefore $BS(k^{INT})$ could be higher or lower than $BS(\bar{k})$. To see when $BS(k^{INT}) > BS(\bar{k})$, set them equal to each other and solve for $R$. Denoting this value by $R_{AB}$, we have $R_{AB} = \frac{3}{2} - \frac{3}{8r_L} + \frac{r_L}{2}$. Then the global optimum is at $k = k^{INT}$ for $R < R_{AB}$ and at $k = \bar{k} = \frac{1}{2r_L}$ for $R \geq R_{AB}$.

Together (i) and (ii) give the boundary for parts A and B of the proposition and the values of $k^{BS}$, $r_L$, and $q$. In part A, $BS = SW = q(R - r_L) = R - (2 - \frac{1}{2r_L})$, and in part B, $BS = q(R - 1 + k^{BS} - q)$, and $SW = BS - (1 - q)(1 - k^{BS}) = qR - q^2 - (1 - k^{BS})$.

Finally, consider the boundary where $SW = 0$ illustrated in Figure 5. In Region A $SW = R - (2 - \frac{1}{2r_L})$. Evaluating this at the boundary for Region A for $r_L < \frac{3}{2}$, given by $R_{AB} = \frac{3}{2} - \frac{3}{8r_L} + \frac{r_L}{2}$, gives $SW|_{R_{AB}} = \frac{(1-2r_L)^2}{8r_L} > 0$. This implies that social welfare is positive at the boundary as well as above it. The same holds for $r_L \geq \frac{3}{2}$, since evaluating social welfare at $R_{AB} = 3 - \frac{3}{8r_L}$ we obtain $SW|_{R_{AB}} = 1 - \frac{1}{r_L} > 0$.

Consider now social welfare in Region B as given by $SW = qR - q^2 - (1 - k^{BS})$. Evaluating this at $R_{AB} = \frac{3}{2} - \frac{3}{8r_L} + \frac{r_L}{2}$ for $r_L < \frac{3}{2}$ gives $SW|_{R_{AB}} = [5 - 46r_L + 44r_L^2 - 8r_L^3]/16r_L$. This equals zero at $r_L = 1.226$, is negative for $r_L < 1.226$ and positive for $1.226 \leq r_L < 1.5$. Consider now the case $r_L \geq \frac{3}{2}$. It can be checked that for $r_L \geq 1.226$ there exists a boundary $R$ as defined implicitly by $SW = qR - q^2 - (1 - k^{BS}) = 0$ such that $SW \geq 0$ for $R \geq R$ and $SW < 0$ otherwise. □

**Proof of Proposition 5:** We proceed in two steps. We first describe how the optimal
amount of capital $k$ is determined depending on which constraints bind. Then we find the global optimum $k^{REG}$ as a function of the parameters $R$ and $r_E$.

**Step 1.** We start by determining the optimal amount of capital $k$ depending on the constraints $\Pi \geq 0$ and $q \leq 1$ in the maximization problem.

Case 1: Unconstrained case ($\Pi > 0$) for $q < 1$. If $q = r_L - (1 - k) < 1$, then from the first order condition $\partial BS/\partial r_L = 0$ we have $r_L = [R + (1 - k)]/2$ so that $q = [R - (1 - k)]/2 < 1$. Substituting these expressions for $q$ and $r_L$ into (10) gives

$$SW_U(k) = R \left( \frac{R - (1 - k)}{2} \right) - (1 - k) - kr_E - \frac{1}{2} \left( \frac{R - (1 - k)}{2} \right)^2. \quad (18)$$

It can be easily checked that $SW_U$ is a concave function of $k$. Given this, there are three possibilities for the optimal value of $k$ when $\Pi > 0$:

(i) $\partial SW_U/\partial k < 0$, in which case $k = 0$ is optimal.

(ii) $\partial SW_U/\partial k = 0$, in which case there is an interior optimum given by $k^{INT}_U = R + 1 - 4(r_E - 1)$.

(iii) $\partial SW_U/\partial k > 0$, in which case the optimum is at the value $k_0 = 1 + 4r_E - R - 4\sqrt{r_E} \sqrt{1 + r_E - \frac{1}{2}R}$ at which the constraint $\Pi \geq 0$ is satisfied with equality or at the value $\bar{k}_U = 3 - R$ at which the constraint $q \leq 1$ is satisfied with equality, depending on which is the smallest.

Case 2: Constrained case ($\Pi = 0$) for $q < 1$. When $\Pi = 0$, as in (16) we have $r_L = 1 - k + \sqrt{2r_E k}$, and $q = \sqrt{2r_E k}$. Substituting these into (10), the expression for $SW$, gives

$$SW_C(k) = \sqrt{2r_E k} R - (1 - k) - 2r_E k. \quad (19)$$

Again $SW_C$ is a concave function of $k$ but in this case $\frac{\partial SW_C}{\partial k}|_{k=0} > 0$. This implies that there are two possibilities for the optimal value of $k$ when $\Pi = 0$ and $q < 1$:

(i) $\partial SW_C/\partial k = 0$, so that there is an interior optimum given by $k^{INT}_C = r_0 R^2/[2(2r_E - 1)^2]$.

(ii) $\partial SW_C/\partial k > 0$, so that the optimum $k$ is where the constraint $q \leq 1$ starts to be binding. From $q = 1$, this happens when $\bar{k}_C = \frac{1}{2r_E} < 1$ since $r_E \geq 1$.

The optimal value of $k$ is at the smaller of $\bar{k}_C$ or $k^{INT}_C$. 

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Case 3: Unconstrained case ($\Pi > 0$) for $q = 1$. From $q = 1$, it follows that $r_L = 2 - k$. Substituting $q = 1$ into (10), we then have

$$SW_{U1}(k) = R - (1 - k) - kr_E - \frac{1}{2}$$

from which $\partial SW_{U1}/\partial k = 1 - r_E < 0$ as $r_E > 1$. Thus, the only possible optimal value for $k$ when $\Pi > 0$ and $q = 1$ is at the value where the constraint $q = 1$ starts to be binding, which occurs at $k = \bar{k}_U = 3 - R$.

Case 4: Constrained case ($\Pi = 0$) for $q = 1$. Substituting $q = 1$ in the expression for $SW$ we obtain $SW_{C1} = SW_{U1} = R - (1 - k) - kr_E - \frac{1}{2}$ and thus $\partial SW_{C1}/\partial k = 1 - r_E < 0$. Then the only possible optimum in this case is the lowest value of $k$ such that $\Pi = 0$ and $q = 1$ as given by $\bar{k}_C = 1/2r_E$.

In determining the global optimum the potential values of $k$ are $0, \bar{k}_U, k_0, k^{INT}_{U}, \bar{k}_C$ or $k^{INT}_{C}$. In fact it is possible to show that in all the regions where $SW \geq 0, \bar{k}_C < k^{INT}_{C}$. This will be done after considering all the regions and the other constraints.

Step 2. Now that we have derived the possible cases depending on the constraints $\Pi \geq 0$ and $q \leq 1$ and the optimal values of $k$ in each of them, we analyze how the two constraints move as a function of the parameters $r_E$ and $R$, and determine the global optimal value for $k$ in each scenario. The regions refer to those in Figure 4.

Region A: When $R \geq 3$ the optimal solution is $k = 0$ in the unconstrained case ($\Pi > 0$) with $q = 1$. From the expressions in Step 1 for the unconstrained region it can be seen that with $k = 0$ and $q = 1, r_L = \frac{R+1}{2}, BS = \frac{1}{2}(R - 1) > 0, \Pi = \frac{R}{2} - 1 > 0$ and $SW = R - \frac{3}{2} > 0$.

Region B: In this region the global optimum is at $k = \bar{k}_U = 3 - R$ in the unconstrained case ($\Pi > 0$) with $q = 1$. This requires:

$$\bar{k}_U \leq k_0, \quad \partial SW_U/\partial k|_{k=0} > 0, \quad k^{INT}_{U} \geq \bar{k}_U, \quad \bar{k}_U \geq 0.$$
between regions $i$ and $j$, and $R_{BE}$ is defined by $R_{BE} = 3 - \frac{1}{2r_E}$. The next two conditions ensure that $\bar{k}_U$ is optimal in the unconstrained region and thus also globally optimal; it can be seen from $\partial SW_U/\partial k$, and the expressions for $k^{INT}_U$ and $\bar{k}_U$ that they are both satisfied for $R \geq R_{BC}$, where $R_{BC}$ is defined by $R_{BC} = 2r_E - 1$. The last condition just requires $\bar{k}_U$ to be non-negative and is satisfied for $R \leq 3$. This implies that the boundary with region $A$ is at $R = 3$ as shown above.

Finally, using $k = \tilde{k}_U = 3 - R$ and $q = 1$ in the expressions for the unconstrained case we obtain $r_L = R - 1$, $BS = 1$, $\Pi = \frac{1}{2} - (3 - R)r_E > 0$, and $SW = \frac{1}{2} - (3 - R)r_E > 0$.

Region $C$: In this region the global optimum value is at $k^{INT}_U$ in the unconstrained case ($\Pi > 0$) for $q < 1$. This requires

$$k^{INT}_U \leq \tilde{k}_U, \quad k^{INT}_U \leq k_0, \quad \partial SW_U/\partial k|_{k=0} \geq 0, \quad \tilde{k}_C < k^{INT}_C, \quad SW_U(k^{INT}_U) \geq SW_C(\bar{k}_C).$$

Similarly to Region B, it can be shown these conditions imply $R \leq R_{BC}$, where $R_{BC}$ is given above, $R \geq R_{CD} = 4r_E - 5$, and $R \geq R_{CE} = r_E + (\sqrt{r_E - 8r_E^2 + 10r_E^3 - 3r_E^4})/r_E$.

It can be seen that the intersection of boundaries $R_{BC}$ and $R_{CD}$ is at $r_E = 2$ and $R = 3$. It can also be checked that $R_{BE}, R_{BC}$ and $R_{CE}$ intersect at $r_E = 1.866$ and $R = 2.732$. Also $R_{CD}$ and $R_{CE}$ intersect at $r_E = 1.933$ and $R = 2.732$.

To conclude, the optimal value of $k$ is $k^{INT}_U = R + 1 - 4(r_E - 1)$, and using the expressions for the unconstrained region $r_L = 2(r_E - 1)$, $q = R - 2(r_E - 1) < 1$, $BS = (2 - 2r_E + R)^2 > 0$, $\Pi = \frac{1}{2}(R + 2)^2 - 3(R + 3)r_E + 6r_E^2 > 0$ and $SW = \frac{R^2}{2} + R - (R + 5)r_E + 2r_E^2 + 2 > 0$.

Region $D$: In this region the global optimum is at the value $k = 0$ in the unconstrained case ($\Pi > 0$) with $q < 1$. Sufficient conditions for this to hold are:

$$\frac{\partial SW_U}{\partial k} \bigg|_{k=0} < 0, \quad \tilde{k}_C < k^{INT}_C, \quad SW_U(0) \geq SW_C(\bar{k}_C).$$

These conditions imply the boundaries $R \geq R_{DE} = (5\sqrt{r_E} + 2\sqrt{3 + r_E})/3\sqrt{r_E}, R \leq R_{CD}$, and $R \leq 3$. It can be checked that $R_{DE}$ intersects with $R_{CD}$ and $R_{DE}$ at $r_E = 1.933$ and $R = 2.732$.

With $k = 0$ and $q = \frac{R - 1}{2} < 1$, we have that $r_L = \frac{R}{2}, BS = \frac{1}{4}R(R - 1) > 0, \Pi = \frac{1}{8}(R^2 - 4R + 3) > 0$ and $SW = \frac{1}{8}(3R^2 - 2R - 9) > 0$. 38
Region E: The global optimum is at the value \( k = \bar{k}_C = \frac{1}{2r_E} \) in the constrained case (\( \Pi = 0 \)) and \( q = 1 \). Sufficient conditions for this to hold are

\[
k_0 \leq \bar{k}_U, \quad \bar{k}_C \leq k^{INT}_C, \quad SW_C(\bar{k}_C) \geq SW_U(k^{INT}_U), \quad SW_C(\bar{k}_C) \geq SW_U(0), \quad SW_C(\bar{k}_C) \geq SW_U(k_0).
\]

These conditions imply the boundaries are \( R \leq R_{BE}, R \leq R_{CE}, R \leq R_{DE} \) and \( R \geq R_{EF} = 2 - \frac{1}{2r_E} \).

Finally, given \( k = \frac{1}{2r_E}, q = 1 \), and \( r_L = 2 - \frac{1}{2r_E} \), it can be shown that \( BS = SW = R - (2 - \frac{1}{2r_E}) > 0 \), and \( \Pi = 0 \).

Region F: It can be seen that for the solution that is optimal in Region E where \( k = \bar{k}_C = \frac{1}{2r_E}, \quad SW = R - (2 - \frac{1}{2r_E}) < 0 \) for \( R < R_{EF} \). However, this is not the only solution that might be optimal in Region F. So far we have assumed throughout that \( \bar{k}_C \leq k^{INT}_C \). If this inequality is reversed then \( k^{INT}_C \) is optimal. Using \( \bar{k}_C = 1/(2r_E) \) and \( k^{INT}_C = r_E R^2/[2(2r_E - 1)^2] \) it can be shown that the boundary for this constraint is \( R = 2 - \frac{1}{r_E} \). For \( R > 2 - \frac{1}{r_E} \) we have \( \bar{k}_C \leq k^{INT}_C \). Since \( 2 - \frac{1}{r_E} < 2 - \frac{1}{2r_E} \) it follows that \( \bar{k}_C \leq k^{INT}_C \) holds in Regions C, D, and E as required above.

For \( R \leq 2 - \frac{1}{r_E}, \quad k^{INT}_C \) is the optimal solution. However, it can be shown using the expressions for the constrained solution in step 1 that \( SW < 0 \) for all these values of \( R \) and \( r_E \). Thus \( SW < 0 \) in Region F and there is no intermediation. □

Proof of Proposition 6: To prove this, we overlap Figures 3 and 4 and compare \( k^{BS} \) and \( k^{reg} \) in each region to give Figure 5. We note first that the boundary between Regions A and B in Figure 3 lies above the one between Regions E and F in Figure 4 and intersects the boundary between regions D and E in Figure 4 at \( r_E = 3.52 \). We consider now each region of Figure 5 in turn. For clarity, in what follows we define the regions of Proposition 4 as 4.A and 4.B, and those of Proposition 5 as 5.A-5.F. Regions without a prefix refer to Figure 5.

Region A: \( k^{BS} > k^{reg} \). This region consists of Regions 5.A, 5.B, 5.C and 5.D.

Region 5.A: It can be seen directly that \( k^{BS} = \frac{1}{2r_E} > k^{reg} = 0 \).

Region 5.B: In this region for \( k^{BS} > k^{reg} \) to hold, it is necessary that \( k^{BS} = \frac{1}{2r_E} > k^{reg} = 3 - R \), or equivalently \( R > 3 - \frac{1}{2r_E} \). It can be seen directly that Region B satisfies this constraint since the lower boundary is \( R_{BE} = 3 - \frac{1}{2r_E} \).
Region 5.C: In this region for $k^{BS} > k^{reg}$ to hold, it is necessary that $k^{BS} = \frac{1}{2r_E} > k^{reg} = R + 1 - 4(r_E - 1)$, or equivalently $R \leq 4r_E - 5 + \frac{1}{2r_E}$. It can be seen that the boundary of this intersects with $R = 2r_E - 1$ at the corner of Region 5.C where $R_{BC} = 2r_E - 1$ intersects with $R_{BE} = 3 - 1/2r_E$. It can straightforwardly be checked that apart from this point Region 5.C lies below $R = 4r_E - 5 + \frac{1}{2r_E}$ so that $k^{BS} > k^{reg}$.

Region 5.D: As already described, the boundary between Regions 4.A and 4.B intersects with the boundary of Region 5.D so that we have to compare $k^{BS}$ as defined both in Regions 4.A and 4.B with $k^{reg}$ in Region 5.D. It is easy to see that $k^{BS} > k^{reg}$ always since $k^{reg} = 0$ in Region 5.D and $k^{BS} > 0$ in both Regions 4.A and 4.B.

Region B: $k^{BS} = k^{reg}$. This region consists of the overlap between Region 4.A and Region 5.E. It can be seen directly from Propositions 4 and 5 that $k^{BS} = k^{reg} = \frac{1}{2r_E}$.

Region C: $k^{BS} < k^{reg}$. This region derives from overlapping Regions 4.B and 5.E. It holds from Propositions 4 and 5 that $k^{BS} = \left(\sqrt{2r_E} - \sqrt{2r_E - 3(R - 1)}\right)/3$, and $k^{reg} = \frac{1}{2r_E}$. The boundary $k^{BS} = k^{reg}$ is equivalent to $R = 3 - \frac{3}{2r_E}$. This is the boundary for Regions 4.A and 4.B for $r_E \geq \frac{3}{2}$. Now given that $\partial k^{BS}/\partial R > 0$, and that $k^{reg} = \frac{1}{2r_E}$ is independent of $R$, it follows that as $R$ falls so does $k^{BS}/k^{REG}$. Thus, $k^{BS} < k^{reg}$ for $R < 3 - \frac{3}{2r_E}$ and $r_E \geq \frac{3}{2}$.

Consider now $r_E < \frac{3}{2}$. We know from the proof of Proposition 4 that in this case $k^{MIN} = \bar{k}_C$ at $R = 3 - \frac{3}{2r_E}$ and that at the boundary between Regions 4.A and 4.B, $k^{BS} < \frac{1}{2r_E}$. This, together with the fact that $\partial k^{BS}/\partial R > 0$, implies that $k^{BS} < k^{reg}$ is satisfied on the boundary between Regions 4.A and 4.B as well as below it. Thus, $k^{BS} < k^{reg}$.

Region D: $k^{BS} > 0$ and there is no intermediation in the regulatory case. Here the relevant areas are Regions 4.B and 5.F. □
References


The market

The bank chooses
$k, r_0$, and $r_i$

The firm decides whether to accept the loan

The bank chooses its monitoring effort $q$

The project matures; claims are settled

The regulator

The regulator chooses $k$, $r_0$, and $r_i$

The bank chooses

Figure 1: Timing of the model
Figure 2: Comparison of market and regulatory solutions with no deposit insurance. The figure compares the level of capital in the market solution ($k^{MS}$) and in the regulatory solution ($k^{reg}$) in the case of no deposit insurance as a function of the cost of equity $r_e$ and of the project return $R$. The figure distinguishes four regions:

Region A.1, as defined by $R \geq 2$, where $k^{MS} = 1/2r_e > k^{reg} = 0$; Region A.2, as defined by $2 < R < 4$, where $k^{MS} = 1/2r_e > k^{reg} = 1 - R^2/4$; Region B, as defined by $2 - 1/2r_e \leq R < R_{as}$, where $k^{MS} = k^{reg} = 1/2r_e$; and Region C, as defined by $R < 2 - 1/2r_e$, where there is no intermediation.

$R_{as} = 4r_e^2 + 2\sqrt{r_e^2 + 2r_e^2 - 6r_e^2 + 4r_e^2}/(r_e^2 + 2r_e^2)$, where $k^{MS} = 1/2r_e > k^{reg} = 1 - R^2/4$.
Figure 3: Market solution with deposit insurance. The figure shows the level of capital in the market solution \( (k^{BS}) \) for the case of deposit insurance as a function of the cost of equity \( r_E \) and of the project return \( R \). The figure distinguishes two regions: Region A, as defined by \( R_{as} \geq R_{st} \), where \( k^{as} = \frac{1}{2r_e} \); and Region B, as defined by \( R_{st} < R_{as} \), where \( R_{st} = 3 - \frac{3}{2r_e} \). The boundary between the two regions is given by \( R_{as} = 3\sqrt{2r_e - \sqrt{2r_e - 3(R-1)/3}} \). The figure also shows the boundary for \( SW \geq 0 \). This coincides with \( R_{st} \) for \( r_e < 1.266 \) and equals \( \hat{R} \) for \( r_e \geq 1.266 \), where \( \hat{R} \) solves \( SW = qR - q^p - (1 - k^{as}) = 0 \).
Figure 4: Regulatory solution with deposit insurance. The figure shows the level of capital in the regulatory solution \( k^{\text{reg}} \) for the case of deposit insurance as a function of the cost of equity \( r_E \) and of the project return \( R \). The figure distinguishes six regions: Region A, as defined by \( R \geq 3 \), where \( k^{\text{reg}} = 0 \); Region B, as defined by \( R_{\text{ac}} \leq R < 3 \) and \( R_{\text{ac}} \leq R < 3 \), where \( k^{\text{reg}} = 3 - R \); Region C, as defined by \( R < R_{\text{ac}} < 3 \), \( R_{\text{ac}} \leq R < 3 \) and \( R_{\text{cd}} \leq R < 3 \), where \( k^{\text{reg}} = R + 1 - 4(r_E - 1) \); Region D, as defined by \( R < R_{\text{ac}} < 3 \) and \( R_{\text{ac}} \leq R < 3 \), where \( k^{\text{reg}} = 0 \); Region E, as defined by \( R_{\text{ac}} \leq R \), \( R < R_{\text{ac}} \), \( R < R_{\text{cd}} \) and \( R < R_{\text{cd}} \), where \( k^{\text{reg}} = 1/2r_E \); Region F, as defined by \( R < R_{\text{ac}} \), where there is no intermediation. The boundaries between the regions are as follow: \( R_{\text{ac}} = 2r_E - 1 \), \( R_{\text{cd}} = 3 - 1/2r_E \), \( R_{\text{cd}} = r_E + \sqrt{r_E^2 - 8r_E^2 + 10r_E^3 - 3r_E^4} / r_E^2 \), \( R_{\text{cd}} = 4r_E - 5 \), \( R_{\text{cd}} = (5\sqrt{r_E^2 + 2\sqrt{3 + r_E^2}}) / 3\sqrt{r_E^2} \), \( R_{\text{cd}} = 2 - 1/2r_E \). The proof of Proposition 5 contains the intersection points between the boundaries.
Figure 5: Comparison of market and regulatory solution with deposit insurance. The figure compares the levels of capital in the market solution ($k_{BS}$) and regulatory solution ($k_{reg}$) for the case of deposit insurance as a function of the cost of equity $r_E$ and of the project return $R$. The figure distinguishes four regions: Region A, where $k_{BS} > k_{reg}$; Region B, where $k_{BS} = k_{reg}$; Region C, where $k_{BS} < k_{reg}$; and Region D, where $k_{reg} > 0$ and there is no intermediation with regulation. Region A exists for $R \geq R_{sw} = \frac{3 - \sqrt{5}}{2} r_E$, $R \geq R_{sc} = r_E + \sqrt{r_E^2 - 8r_E^2 + 10r_E^2 - 3r_E^2 r_E} / r_E$, and $R \geq R_{sd} = \left(5\sqrt{r_E^2 + 2\sqrt{3 + r_E}}\right) / 3\sqrt{r_E}$; Region B exists between $R < R_{sw}$, $R < R_{sc}$, and $R \geq \hat{R}$ where $\hat{R} = 3/2 - 3/2 r_E$ for $r_E < 3/2$ and $\hat{R} = 3 - 3/2 r_E$ for $r_E \geq 3/2$. Region C exists for $2 - 1/2 r_E \leq R \leq \hat{R}$; and Region D exists for $R \geq 2 - 1/2 r_E$. The proof of Proposition 6 contains the intersection points between the boundaries of Regions A, B and C.