Many authors have examined the “Great Moderation” episode in the US (see Olivier Blanchard and John Simon 2001; Richard Clarida, Jordi Gali, and Marc Gertler 2000; Timothy Cogley and Thomas J. Sargent 2002, 2005; Robert J. Gordon 2005; James H. Stock and Marc W. Watson 2003; Giorgio E. Primiceri 2005; Andres Arias, Gary D. Hansen, and Lee E. Ohanian 2007; Christopher A. Sims and Tao Zha 2006; Luca Gambetti, Evi Pappa, and Fabio Canova 2008, among others) and its international features are currently investigated (see Stock and Watson 2005; Canova, Gambetti, and Pappa 2007, or Luca Benati 2008). Most analyses agree on the observation that the volatility and the persistence of output and inflation declined since the late 1970s but explanations differ. The literature is mainly divided into two fronts—those who support the “bad policy” hypothesis (failure of the Fed to appropriately respond to inflation) and those who lean toward the “bad luck” hypothesis (shocks are drawn from a time varying distribution)—with a few authors claiming that changes in the private sector (see e.g., Margaret M. McConnell and Gabriel Perez-Quiros 2000; Jeffrey R. Campbell and Zvi Hercowitz 2005; Urban Jermann and Vincenzo Quadrini 2006; Canova 2009; Gali and Gambetti 2009) or reduced activism combined with decreased misperceptions (Athanasios Orphanides 2004; Orphanides and John C. Williams 2005b) may be responsible for the phenomenon.

We examine the role of expectations in the Great Moderation episode. We derive theoretical restrictions in a New-Keynesian model and test them using measures of expectations obtained from survey data, the Greenbook and bond markets. Expectations explain the dynamics of inflation and interest rates but their importance is roughly unchanged over time. Systems with and without expectations display similar reduced form characteristics. Results are robust to changes in the structure of the empirical model. (JEL E23, E24, E31, E32)
The division appears to be linked, in part, to the type of data used (real time versus historical) and, in part, to the type of empirical analysis conducted. While narrative and reduced form approaches consistently point to “bad policy” as key to explain the facts, structural VARs favor the “bad luck” conclusion. Given the strong prior of many commentators, some have questioned the ability of structural VARs to detect true sources of variations in the data (see Benati and Paolo Surico 2008).

The most convincing formalization of the “bad policy” hypothesis appears in Thomas A. Lubik and Frank Schorfheide (LS) (2004) who, building on the work of Clarida, Gali, and Gertler (2000), estimate a three-equations New-Keynesian model with Bayesian methods over subsamples and find an indeterminate equilibrium in the first subsample (up to the end of the 1970s) but not in the second one (from the beginning of 1980s up today). Jean Boivin and Marc P. Giannoni (2006) confirm this conclusion with an alternative estimation technique. One important consequence of this finding is that expectations were driven by non-fundamental forces in the 1970s, and became function of fundamental factors when the Fed strengthened the reaction of the nominal rate to inflation. Despite the fact that the dynamics of expectations are crucial to understand the facts and to assess the credibility of the explanation, no one has formally examined whether expectations fit the role that the indeterminacy-determinacy story of the Great Moderation has given to them. Leduc, Sylvain, Keith Sill, and Tom Stark (2007) studied how much the nominal rate moves in response to expected inflation shocks and whether there has been a change in the magnitude and the persistence of expected inflation shocks, but they do not directly examine the importance of inflation expectations in the two regimes.

In this paper, we study the role of expectations in the Great Moderation episode using reduced form techniques. To start with we take a simple New-Keynesian model, parameterized so as to replicate the most salient aspects of LS estimates, and show that there is a state variable entering the solution in the indeterminate regime which fails to appear when the equilibrium is determinate. If expectations play the role of this additional state variable, they should help to predict other endogenous variables in the indeterminate sample and there should be a break in the significance of predictive tests, as we move from the indeterminate to the determinate regime. Moreover, omitting expectations from the empirical model causes the variance of the shocks to be overestimated in the indeterminate regime but not in the determinate one.

We show that these two implications are the only testable ones the theory imposes, and that existing approaches may be unable to detect regime switches. For example, the standard counterfactuals conducted in the literature are uninformative because variations in the policy rule imply changes in both the impact coefficients, and the lagged responses to shocks, regardless of whether policy changes occur within or across regimes. Moreover, we show that certain structural methods are unlikely to be more informative than reduced form ones about the type of regime in place.

In our analysis, we proceed as follows. We collect alternative measures of one year ahead expectations using survey data (Michigan, Professional, Livingstone), the Greenbook, and the term structure of nominal interest rates. Then, we run several VARs which include output growth, inflation, the nominal interest rate, and a proxy measure of expectations and examine: whether the coefficients on lagged expectations are significant, and whether their significance changes over time; and whether
omitting expectations from the estimated system causes time-varying biases in the variance of reduced-form shocks.

Our results suggest that the role of expectations differs from that postulated by the indeterminacy-determinacy story. In particular, regardless of the specification of the empirical model and the statistics used, we find that lags of expectations are either always significant or always insignificant, and there is no clear switch over time in their importance in any equation of the system; and that reduced-form variances, estimated in systems with and without expectations, display similar features and little evidence of time-varying biases.

The rest of the paper is organized as follows. The next section examines the implications of the theory. Section II describes our expectation measures. Section III presents the empirical evidence. Section IV discusses the robustness of the results. Section V concludes.

I. What Does the Theory Tell Us?

A. A Simple Example

To set up ideas, it is useful to consider a simple univariate example. Suppose

\[ y_t = \frac{1}{\theta} y_{t+1}^e + e_t \]

where \( e_t = \phi e_{t-1} + \eta_t \), \( 0 < \phi \leq 1 \), \( \eta_t \) is iid \((0, \sigma^2)\) and \( y_{t+1}^e \) is the time \( t \) expectations of \( y_{t+1} \). Suppose expectations are rational, i.e \( y_t^e = E_t y_{t+1} \). If \( |\theta| > 1 \) (the determinate regime), the solution for \( y_t \) is \( y_t = (\theta/(\theta + \phi)) e_t = \phi y_{t-1} + (\theta/(\theta + \phi)) \eta_t \). Since \( E_{t-1} y_t = \phi y_{t-1} \) time \( t-1 \) expectations of \( y_t \) are irrelevant in predicting \( y_t \) when \( y_{t-1} \) is available. In other words \( E_{t-1} y_t \) does not Granger-cause \( y_t \) in this regime.

When \( |\theta| < 1 \) (the indeterminate regime), equation (1) can be rewritten, shifting the time index by one period, as

\[ y_t = \theta y_{t-1} - \theta e_{t-1} + v_t \]

where \( v_t \equiv y_t - E_{t-1} y_t \). Clearly, if \( v_t = \eta_t \), the solution for \( y_t \) still is \( y_t = (\theta/(\theta + \phi)) e_t \) and, conditional on \( y_{t-1} \), expectations play no role also in this regime. Suppose instead that \( v_t \) is a independently and identically distributed (pure sunspot) shock orthogonal to \( e_{t-1} \). Since \( E_{t-1} y_t = \theta y_{t-1} - \theta e_{t-1} \), time \( t-1 \) expectations of \( y_t \) may help to forecast \( y_t \), given \( y_{t-1} \), because they contain information about \( e_{t-1} \) that is not included in \( y_{t-1} \).

This discussion indicates that, under rational expectations, two basic features distinguish indeterminate from determinate regimes: (i) conditional on \( y_{t-1} \) past expectations should help to predict \( y_t \) in the former but not in the latter regime; (ii) excluding expectations from an empirical model should make prediction errors larger in the indeterminate regime, but not in the determinate one. These two implications of the theory constitute the null hypotheses of the reduced form tests we conduct below.
As the editor has pointed out to us, it is unclear whether rational expectation is a reasonable working assumption when the economy drifts into an indeterminate regime. Since our empirical analysis may have stronger appeal if the tests we propose have power when the rational expectation assumption fails hold in this regime, we next examine whether the implications we emphasize holds under an alternative expectation formation mechanism. Suppose that expectations are formed using a constant gain learning scheme:

\[
y_{e_{t+1}} = y_{e_t} + \gamma (y_{t-1} - y_{e_t}^e).
\]

Using equation (3) into (1) we obtain

\[
y_t = \frac{1 - \gamma}{\theta} y_{e_t}^e + \frac{\gamma}{\theta} y_{t-1} + e_t.
\]

Hence, given \( y_{t-1} \), past expectations help forecasting \( y_t \), so long as \( \gamma \neq 1 \). Intuitively, expectations matter because they proxy for longer lags of \( y_t \), which are important to characterize current values of \( y_t \).

It is relatively easy to show that the above result holds also if agents use more complicated learning schemes, or a Kalman filtering scheme

\[
y_{e_{t+1}} = y_{e_t} + \kappa_{t-1} \varepsilon_{t-2},
\]

where \( \kappa_{t-1} \) is the time varying gain, \( \varepsilon_{t-1} = y_{t-1} - y_{t-1|t-2} \) is the time \( t - 1 \) forecast error and the notation \( y_{t|t-1} \) indicates the best predictor of \( y_t \) using information available at \( t - 1 \). Nevertheless, as the above derivation clearly indicates, under learning \( y_{e_t} \) will help to predict \( y_t \) in both regimes, unless \( \gamma \) is time varying and changes in a particular way. Hence, the basic tests we perform in Section IV are meaningful if rational expectations hold at least in the determinate regime—the expectation formation in the indeterminate regime could be any of the ones we have considered.

If one it is not willing to assume that expectations are rational, even in the determinate regime, a weaker version of our tests would still be meaningful, provided \( \theta \) is sufficiently away from one. In fact, when \( \gamma \neq 1 \), and again conditional of \( y_{t-1} \), \( y_{e_t} \) will have a (much) larger coefficient under indeterminacy than under determinacy, and the difference will be statistically significant in large samples. Therefore, even though the distinction across regimes is not as sharp as under rational expectations, there is a sense in which, under learning, expectations are more important in an indeterminate regime than a determinate one. In Section IV, we report a time-varying coefficient model that can detect these differences if they are present in the data.

### B. The Basic Model

To show that the two basic implications we care about carry over to more interesting setups, consider a standard three-equation New-Keynesian model which includes a log-linearized Euler condition, a log-linearized Phillips curve, and a log-linearized policy rule. In deviation from a non-stochastic steady state, the equations are:

\[
R_t = \phi_r R_{t-1} + (1 - \phi_r) (\phi_\pi \pi_t + \phi_\lambda (\lambda_t - z_t)) + e_{R,t}
\]
\begin{align}
\pi_t &= \beta \pi_{t+1|t} + \kappa(x_t - z_t) \\
xt &= xt_{t+1|t} - \tau(R_t - \pi_{t+1|t}) + g_t
\end{align}

where \( g_t = \rho_g g_{t-1} + eR_t, z_t = \rho_z z_{t-1} + e_z, x_t \) is the output gap, \( \pi_t \) the inflation rate, \( R_t \) the nominal rate, and the notation \( t + 1 | t \) denotes conditional expectations. Here, \( g_t \) is a demand shifter, \( z_t \) exogenously shifts the marginal cost of production while \( \beta, \kappa, \tau, \phi_\pi, \phi_x, \phi_R, \sigma_g, \sigma_z, \rho_{gz} \), the contemporaneous correlation between \( g_t \) and \( z_t \), are structural parameters.

To describe the population features of this model in different regimes we use a parameterization similar in spirit to the estimates of Lubik and Schorfheide (2004) (see table 1, columns 1 and 2), which they obtained with US data and Bayesian methods over the subsamples (1960:Q1–1979:QII, 1982:QIV–1997:QIV). None of the points we make, however, depends on the exact parameter selection. Note that these two columns differ only in the coefficients of the policy rule, \( (\phi_\pi, \phi_x, \phi_R) \).

As in the univariate example, when the reaction of the nominal rate to inflation is weak \( (\phi_\pi < 1) \) an indeterminate equilibrium is obtained; when the reaction is strong \( (\phi_\pi > 1) \), a determinate equilibrium emerges. Since in the indeterminate regime there is a continuum of solutions, we examine two special situations: one where the forecast error is a function of the structural errors—the “continuity” solution—and another where it is a pure sunspot shock—the “orthogonality” solution. Letting \( \eta_{1t+1} = \pi_{t+1} - \pi_{t+1|t}, \eta_{2t+1} = xt_{t+1} - xt_{t+1|t} \) and \( \eta_t = [\eta_{1t}, \eta_{2t}]' \), then the continuity solution makes \( \eta_t = G\epsilon_t \), where \( \epsilon_t = (eR_t, z_t, g_t)' \) and \( G \) is a function of the structural parameters of the model (see Lubik and Schorfheide (2004,* 199, equation (5))). The orthogonality solution instead makes \( \eta_t = F\nu_t \), where \( \nu_t \) are sunspot shocks and \( F \) is a function of the structural parameters of the model. Since no analytical expression for the solution for the system (5)–(7) is available, we present the log-linearized decision rules for the nominal rate, the inflation rate and the output gap the model delivers. The continuity solution of the indeterminate regime produces:
c
elements of parameterization and the choice of solution. For example, two of the three diagonal
the magnitude of the variance of the shocks in the two regimes depends on the
these results have little to say about the implications we care about. Furthermore,
determinate one. Moreover, omitting \( \hat{\zeta}_{t-1} \) disturbances in the indeterminate regime.
the structural model differs across regimes only in the coefficients of the policy
obtained in more complex models with additional shocks or frictions. Second, while
the variance of the reduced form shocks to be larger than the true ones in the inde-
Thus, regardless of the solution one considers, there is an additional state variable
under indeterminacy when sunspots are present.1 Hence, \( \hat{\zeta}_{t-1} \) should help predicting
In the determinate regime, instead we have:
Thus, we are not saying that the importance of \( \hat{\zeta}_{t-1} \) lose their predi-
unconditionally across regimes, and that the variance of the shocks in the indeterminate regime is larger than in the determinate one. Unconditionally,
the magnitude of the variance of the shocks in the two regimes depends on the
parameterization and the choice of solution. For example, two of the three diagonal
elements of \( \Sigma_u \) are larger in the determinate than in the indeterminate regime under
orthogonality. Rather than comparing unconditional variances across regimes, we
emphasize that omission of \( \hat{\zeta}_{t-1} \) induces biases in the variance of the reduced form
disturbances in the indeterminate regime.
We would like to emphasize three additional points. First, the model we consider
is stark but the conclusions it delivers about regime switches are the same as those
obtained in more complex models with additional shocks or frictions. Second, while
the structural model differs across regimes only in the coefficients of the policy

\[
\begin{bmatrix}
\hat{R}_t \\
\hat{\pi}_t \\
\hat{x}_t
\end{bmatrix} = \begin{bmatrix}
0.61 & -0.06 & 0.05 & 0.36 \\
0.001 & -0.001 & -0.001 & 0.99 \\
0.28 & -0.99 & 0.84 & 0.81
\end{bmatrix} \begin{bmatrix}
\hat{R}_{t-1} \\
\hat{\pi}_{t-1} \\
\hat{x}_{t-1} \\
\hat{\zeta}_{t-1}
\end{bmatrix} + \begin{bmatrix}
\hat{u}_{1t} \\
\hat{u}_{2t} \\
\hat{u}_{3t}
\end{bmatrix}, \quad \Sigma_u = \begin{bmatrix}
3.57 & 8.18 & 17.52 \\
8.18 & 24.86 & 5.30 \\
17.52 & 5.30 & 20.57
\end{bmatrix},
\]

where \( \hat{\zeta}_{t-1} \) represents \( t - 1 \) expectations of inflation or of output or a combination
of the two, while the orthogonality solution delivers:

\[
\begin{bmatrix}
\hat{R}_t \\
\hat{\pi}_t \\
\hat{x}_t
\end{bmatrix} = \begin{bmatrix}
0.61 & -0.06 & 0.05 & 0.36 \\
0.001 & -0.001 & -0.001 & 0.99 \\
0.28 & -0.99 & 0.84 & 0.81
\end{bmatrix} \begin{bmatrix}
\hat{R}_{t-1} \\
\hat{\pi}_{t-1} \\
\hat{x}_{t-1} \\
\hat{\zeta}_{t-1}
\end{bmatrix} + \begin{bmatrix}
\hat{u}_{1t} \\
\hat{u}_{2t} \\
\hat{u}_{3t}
\end{bmatrix}, \quad \Sigma_u = \begin{bmatrix}
1.31 & 0.92 & -1.13 \\
0.92 & 2.04 & -2.50 \\
-1.13 & -2.50 & 3.07
\end{bmatrix}.
\]

In the determinate regime, instead we have:

\[
\begin{bmatrix}
\hat{R}_t \\
\hat{\pi}_t \\
\hat{x}_t
\end{bmatrix} = \begin{bmatrix}
0.86 & 0.17 & 0.02 \\
0.06 & 0.57 & -0.04 \\
0.11 & -0.59 & 0.81
\end{bmatrix} \begin{bmatrix}
\hat{R}_{t-1} \\
\hat{\pi}_{t-1} \\
\hat{x}_{t-1}
\end{bmatrix} + \begin{bmatrix}
\hat{u}_{1t} \\
\hat{u}_{2t} \\
\hat{u}_{3t}
\end{bmatrix}, \quad \Sigma_u = \begin{bmatrix}
0.80 & 0.80 & -0.44 \\
0.80 & 4.89 & 2.61 \\
-0.44 & 2.61 & 10.94
\end{bmatrix}.
\]

1 We are not the first ones to point out this fact, see Lubik and Schorfheide (2004) or Benati and Surico (2008),
but neither use it to derive testable reduced form restrictions.
equation, the solution is such that lagged dynamics as well as the variance of the reduced-form shocks change. Hence, standard reduced-form counterfactuals conducted in the literature, switching coefficients and variances across subsamples, are not useful to check what regime is in place. Third, changes in the structural parameters, within or across regimes, produce changes in the lagged dynamics and in the variance parameters, and the magnitude of the changes is roughly similar. Thus, the size of the relative changes in the lagged coefficients and the variances is uninformative about regime switches.

It is often presumed that structural estimation methods have an edge relative to less structural ones in detecting regimes, because they take expectation formation into account. To illustrate the fallacy of such a presumption in our specific case, we take the population dynamics generated by the model under indeterminacy (the continuity solution) as given and ask: are there parameter values which make the dynamics under determinacy “close” to those produced under indeterminacy?

Figure 1, which uses a formal minimum distance estimator to try to replicate the dynamic responses of output, inflation and the nominal rate generated by the structural shocks, shows that this is indeed possible. If rather than taking one parameterization, we take estimation uncertainty seriously and construct response bands for the indeterminate regime using Monte Carlo simulations, these bands would always include the point estimate of the responses under determinacy. Thus, even in the unlikely case that a very large number of observations were available, structural methods focusing on the dynamics induced by structural shocks will find it hard to detect regime switches.

The parameters generating Figure 1 are in the third column of Table 1. Note that it is impossible to simply change the variance of the shocks to make the dynamics of the indeterminate and of the determinate solutions close; that is, the “bad luck” hypothesis is not local to the indeterminacy/determinacy story. However, alternative
explanations in which private sector parameters change together with the structural variances, or in which the parameters of the policy rule change together with the structural variance, keeping private sector parameters fixed (see fourth column of Table 1) have this feature. Thus, the near observational equivalence of various hypotheses makes certain structural estimation exercises incredible.

It is important to know whether the type of reduced-form tests we suggest have reasonable power to detect regimes in the typical samples used in macroeconomics. As it will be clear below, we have only about 80 data points on each side of the potential break date, making small sample problems an issue. To check whether our approach can detect regime breaks in this situation, we have simulated data from each of the two regimes by using the parameter values reported in the first two columns of Table 1, employing either the continuity or the orthogonality solution when generating data from the indeterminate regime. We then constructed two samples of 160 data points (one with 80 data from the continuity regime and 80 from the determinate regime, the other with 80 data from the orthogonality regime and 80 from the determinate regime); ran a VAR(2) including experimental data for output, inflation, and the nominal rate and one of the expectational variables; tested the hypothesis that lags of the expectational variables significantly enter the first three equations; and measured the differences in the covariance matrix of the reduced form shocks when the expectational variable is included or excluded from the VAR. Tables 2 and 3 show that our tests do have power to detect regime changes, even in these relatively small samples. In particular, i) one of the expectational variables is significant in some equations when up to the first 80 data points are used, but not if more data is included or if estimation starts at a later date. And ii) the variance of the reduced-form shocks in a system without inflation expectations is larger than in a system which includes them only if the first 80 data points are used. Benati and Surico (2008) have argued that VARs may be unable to correctly capture regime switches with this DGP. Tables 2 and 3 show that such a claim is generally invalid.

In sum, regime changes may be hard to detect with standard methods. However, if the indeterminacy/determinacy story is correct, expected inflation, expected output, or a combination of the two must behave as a state variable up to the end of the 1970s but not afterward. That is to say, lags of these variables must help in predicting output, inflation, and the interest rates given their lags, up to the end of the 1970s but not afterward, and the change should be a permanent one. Furthermore, omitting expectations from the system should change the variance of reduced-form shocks only for samples up to the end of the 1970s.

In the next sections, we focus attention on the role of inflation expectations as a state variable. Later, we examine how our conclusions change if a measure of output expectations is used in place or in addition to an inflation expectation measure, or if the first principal component of all the available measures of inflation and output expectations is used in the empirical model.

II. Measures of Expectations

Expectations are not observable, but there are proxies one could use. Since they differ in the time coverage and in their reliability as predictors of future variables,
we dedicate this section to describe their properties and motivate our selection of expectation measures.

The Michigan survey reports average expected changes in consumer prices for the incoming year and is available quarterly since 1960:Q1. This survey has 100 respondents each period, covers primarily households, and is conducted before the inflation figure of the middle month of the quarter are available. We assign the forecast to the end of the quarter, giving the survey a bit more information than it
actually has. We use the mean forecast as our measure, since median estimates are available only since 1978, despite the fact that Lutz Kilian and Atsushi Inoue (2005) have raised doubts about its reliability.

The Survey of Professional Forecasters, constructed by the Federal Reserve Bank of Philadelphia, has data on the implicit price deflator and real GDP expected yearly changes since 1970:Q1 (1968:Q1 for real GDP growth) while CPI forecasts are available only since 1981. The number of respondents changes somewhat with the quarter and the year in which the survey is run, and respondents are primarily members of the business community. As with the Michigan survey, it is conducted in the middle of each quarter, but we assign the reported value to the end of the quarter. In this case, we use the median forecast as our measure.

The Livingstone survey is biannual—it is conducted in April and October since 1955:Q1—and reports eight months ahead, level of the non-seasonally adjusted CPI. The number of respondents is smaller than the other two surveys (it covers about 50 economists from industry, government and academia per time period), and this may produce larger or more persistent biases. To make it comparable to the other survey measures, the 8 months expected rate of change is annualized. The median value is used as our estimate.

The Greenbook contains projections of inflation and real GDP growth produced by the staff at the Federal Reserve Board for FOMC meetings. The projections measure the annualized quarter-on-quarter changes of the implicit price deflation and real GDP up to 1996, and of the chain-weighted indices after that date. Forecasts providing data one year ahead are only available since 1975:Q1. Irregularly sparsed, forecasts annualized two and three quarters ahead are available since 1968:Q1 and forecasts annualized one quarter ahead since 1965:QIV. We fill in missing data using regression methods and use projections annualized three quarters ahead as our basic measure. Also, since FOMC meetings are irregularly spaced, quarterly data are constructed using the projections produced by the report which is closest to the middle of each quarter. As with survey measures, we assign this value to the end of the quarter.

The term structure of nominal interest rates also provides an implicit measure of inflation. To construct it, let $f_{t,p,k-p} = R_{t,p}/R_{t,k}$ be the forward rate quoted at $t$, for $p$ holding periods, on a bond with maturity $k$, where $R_{t,p}$ and $R_{t,k}$ are the time $t$ returns on nominal bonds of $p$ and $k$ maturities. Thus, for example, the (quarterly) forward rate quoted at $t$, on a 10-year bond held for one year, is denoted by $f_{t,4,116}$. The one year forward rate can be decomposed as:

$$f_{t,4,k-4} = r_{t,4,k-4} + \pi_{t,4,k-4} + [f_{t,4,k-4} - E_t \ln R_{4,t+k-4}]$$

$$+ [E_t \ln R_{4,t+k-4} - r_{t,4,k-4} - \pi_{t,4,k-4}]$$

where the first term represents the expected one year real rate, the second the one year expected inflation, the third the nominal term premium (the difference between the forward rate and the expected future nominal rate), and the last the real excess return of the expected nominal rate over the expected real rate. While it is typical to assume that the first, the third, and the fourth terms of the expression are time invariant—this would allow us to identify the dynamics of expected inflation with those
of the forward rate. Such an assumption, however, is too heroic for the sample we consider to be credible. As an alternative, we use the rational expectation assumption, regress realized inflation on a constant and the forward rate, and take the predicted value as a measure of inflation expectations. This procedure is relatively common in the literature (e.g., Lars Svensson 1994, or Paul Söderlind 1995), and makes the resulting expectations close to actual inflation. To take into account potential breaks in the path of inflation, the regression is actually run on two separate subsamples (up to 1980:QII, after 1980:QII). An alternative signal extraction approach, where expected inflation is treated as unobservable random walk, while the other components in (4) have stationary AR(1) dynamics, produces similar results.

Data on the term structure of the nominal interest rates is available at the FRED databank of the Federal Reserve Bank of St. Louis. However, the data reports rates for non-zero coupon bonds. We have managed to recover a comparable data set for zero coupon bonds, but only for the period 1974:QI–2001:QIV, which makes it too short for our purposes. It turns out that the forward rates implied by the two-term structures are very similar in the overlapping sample (contemporaneous correlation 0.98), and the measures of expectations we obtain from the two different series are practically indistinguishable. To maximize the length of the sample, we therefore
work with inflation expectations obtained from non-zero coupon bonds, even though the decomposition in (8) is only approximately valid.

While inflation expectations backed out from financial market data are probably more reliable, survey data are publicly available and do not require any statistical model, or controversial assumption, to back them out. To compare their properties, we plot in Figure 2 the time path of the five expected inflation series, together with actual inflation computed using either the implicit price deflator (IPD) or the CPI (measured here by the seasonally adjusted index for all items). Confirming Yash P. Mehra (2002), Michigan expectations are a good predictor of actual inflation up to 1980. The tracking performance deteriorates somewhat over the 1980s, and over the 1990s the reported mean systematically overestimates actual inflation. Professional expectations are better over the whole sample, but in particular episodes (for example, the beginning of the 1980s), they are less reliable than Michigan expectations. Livingstone expectations appear to be free of large or persistent biases, except perhaps in the latest part of the sample. Greenbook projections closely track IPD dynamics, are highly correlated with Professional and term structure expectations, and replicate actual inflation well, except for the early 1980s.

Table 4 shows that Michigan and Term structure expectations are those most highly correlated with actual inflation (regardless of whether it is measured by IPD or CPI) and with each other. In terms of moments of the empirical distribution, Term structure expectations closely replicate those of actual inflation. Finally, Michigan expectations have the smallest in-sample MSE, both relative to IPD and CPI inflation. Hence, we initially focus on Michigan and Term structure expectations in our exercises and use other measures for robustness checks.2

III. The Evidence

We estimate a number of reduced form VAR models and examine whether lags of inflation expectations matter in a system including real output growth ($\Delta GDP$), the inflation rate ($\pi$), and a short-term nominal rate ($R$). Data is from the FRED data bank. Output growth is measured by the year-to-year change in GDP. Inflation is measured by the year-to-year change in CPI. All items and the interest rate are measured by the Federal funds rate. While the implications we have derived in Section II hold for a system where real activity is proxied by the output gap, it can easily be shown that they also hold when output growth is used. We use output growth to sidestep the thorny issue of how to compute an output gap measure that is consistent with the theory, and at the same time has reasonable statistical properties.

To start with, we use the traditional device of breaking the sample in two, even if such an approach is problematic for two reasons: since inflation and the nominal

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2 When comparing survey measures to actual inflation data one should be aware that they are not measuring the same thing. First, the reported expected rate is an average over quarters rather than an end of the period measure. Second, apart from Professional forecasts, it is not clear if agents forecast CPI levels/changes or headline CPI level/changes. Third, it is not clear if simple or compound rates are used to construct yearly measures. Fourth, forecasts are typically for non-seasonally adjusted data, while seasonally adjusted data will be used in the exercise. Ang et al. (2006) have shown that these measurement biases are small and account for none of their forecasting comparison results.
interest rate display an inverted U-shaped pattern, it is not clear which break date should be used, and whether a subset of the data (the 1979–1982 period) should be omitted or not. Furthermore, using subsamples forces a simultaneous break in all the relationships, while the moments of these variables display breaks at different dates.

To partially avoid this problem, Table 5 reports the p-value of an F-test for the exclusion of lags of inflation expectations for a number of subsamples in a VAR with four lags. When Michigan expectations are employed, lags of inflation expectations are never important in the output growth equation, always important in the inflation equation, and usually important in the nominal rate equation (the exceptions are the samples 1960:QI–1981:QII and 1960:QI–1982:QI). When term structure expectations are used, lags of inflation expectations are always significant in the nominal rate equation; significant in the output growth equation in the samples 1979–2005 and 1980–2005; and significant in the inflation equation, if the years 1979–1980–1981 are jointly included.

Table 6, which reports the estimated variance of the VAR residuals in a number of subsamples when the two proxies for expectations are used and when inflation expectations are excluded from the system, confirms the outcomes of Table 5. For appropriately selected samples, the variances of reduced-form shocks in a system where inflation expectations are included decreases over time, and a system which excludes inflation expectations has reduced-form shocks with marginally higher variability. More importantly, a system where inflation expectations are excluded displays the same qualitative features as systems which include them: for appropriately chosen samples, the variance of all shocks declines.

Hence, Tables 5 and 6 do not support the main implications of the theory: the data tells us that if inflation expectations matter, they matter for the whole sample and
when they don’t, changes are temporary and primarily related to the Volker experiment of the late 1970s.

**IV. Is the Empirical Evidence Reliable?**

There could be many reasons for why the empirical evidence presented in Tables 5 and 6 fails to conform to the predictions of the theory. In this section, we examine six alternative possibilities. Tables documenting the results that we discuss are in the appendix, available as additional material to the paper.

First, we may be unable to detect a permanent break in the importance of inflation expectations because the lag length of the VAR is misspecified. Note that, given the overlapping nature of all expectations measures, a generous lag length is needed to whiten VAR residuals. However, if too many lags are included, lags of other variables could proxy for lags of inflation expectations, weakening our tests. Since the model of Section II has a VAR(2) format, and since inflation expectation measures induce an MA component of order three, a lag length of four strikes a balance between the two opposing forces. Changing the lag length from two to eight, however, has no effect on the conclusions we reach.

Second, as we have mentioned, several expectation measures forecast IPD inflation rather than CPI inflation. Therefore, we have rerun our tests using IPD inflation in the VAR. While there is weak evidence that term expectations matter in the right way for inflation, the basic conclusions we have derived hold also in this case.

Third, our tests may fail because the proxies for expected inflations we employ are plagued by measurement or estimation errors. Since Lloyd B. Thomas Jr. (1999), Mehra (2002), and Andrew Ang, Geert Bekaert, and Min Wei (2006) have shown that these proxies capture important information about future developments of inflation, it is hard to believe that this is the case. Nevertheless, Jon Faust and Jonathan H. Wright (2006) have shown that Greenbook projections are superior to other expectation measures, while Leduc, Sill, and Stark (2007) claim that Livingstone expectations contain information which is relevant to capture shocks to expectations. We have repeated the estimation using Greenbook forecasts—in this case the sample starts in 1968:QIV—and Livingstone survey data—in this case data for output

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<tbody>
<tr>
<td>Panel A. With Michigan expectations</td>
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<td></td>
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<tr>
<td>(\Delta \text{ GDP} )</td>
<td>0.73</td>
<td>0.79</td>
<td>0.81</td>
<td>0.91</td>
<td>0.91</td>
<td>0.70</td>
<td>0.55</td>
<td>0.99</td>
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<tr>
<td>(\pi )</td>
<td>0.01</td>
<td>0.01</td>
<td>0.01</td>
<td>0.00</td>
<td>0.02</td>
<td>0.00</td>
<td>0.00</td>
<td>0.04</td>
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<tr>
<td>(R )</td>
<td>0.07</td>
<td>0.00</td>
<td>0.11</td>
<td>0.24</td>
<td>0.00</td>
<td>0.01</td>
<td>0.10</td>
<td>0.05</td>
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<tr>
<td>Panel B. With term structure expectations</td>
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<tr>
<td>(\Delta \text{ GDP} )</td>
<td>0.69</td>
<td>0.82</td>
<td>0.52</td>
<td>0.29</td>
<td>0.02</td>
<td>0.03</td>
<td>0.10</td>
<td>0.67</td>
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<tr>
<td>(\pi )</td>
<td>0.58</td>
<td>0.51</td>
<td>0.10</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.59</td>
<td>0.24</td>
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<tr>
<td>(R )</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.06</td>
<td>0.02</td>
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*Note:* The table reports the \(p\)-values of the \(F\)-test that the coefficients on the expectation variable in the equation are all equal to zero in a four variables VAR(4) which includes the growth rate of output \(\Delta \text{ GDP}\), inflation \(\pi\), the nominal interest rate \(R\) and an expectation proxy, in various subsamples.
growth, inflation and the nominal rate is sampled bi-annually—but the same conclusions are reached. If anything, the evidence for a structural break is even weaker with Livingstone data, while Greenbook projections become more important for output growth and inflation after 1982.

It is also possible that our inflation expectation measures are not really forward-looking, making the tests weak. To check for this possibility we have constructed an expected inflation measure using the VAR. This measure, which is internally consistent but completely backward-looking, is correlated with survey and term structure measures, but not perfectly (roughly 0.6). Therefore, inflation expectations measures do contain an independent forward-looking component.

Fourth, as argued in Section II, the theory implies that there is an additional state variable under indeterminacy with sunspots. So far, we have associated this variable with inflation expectations, but any variable correlated with sunspot shocks may do the job. Hence, we have repeated the estimation using output growth expectations in place of, or jointly with inflation expectations, or using the first principal component of all output and inflation expectation measures in place of inflation expectation. Since measures of output growth expectations start only in the mid-late 1960s, the size of the first subsamples is now shorter. None of the results we have presented are affected by the addition of output growth expectations to the empirical model; the substitution of inflation expectations with output growth expectations; or with the first principal component of all expectations.

Campbell (2004) documented that the predictive power of the expectation measures for output growth contained in the Survey for Professional Forecasts (SPF) has declined since 1984. As mentioned, SPF cannot be used for our purposes because the data starts too late to make estimation credible. Nevertheless, it should be pointed out that our conclusions are different because the exercise we conduct is different. First, we are looking for a change in predictive power of output expectations, once lags of the endogenous variables are used. Second, we are looking for changes in

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<th>Table 6—Variances of Reduced Form Shocks</th>
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<td>Δ GDP</td>
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<td>π</td>
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<td>R</td>
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Panel A. With Michigan expectations

Panel B. With term structure expectations

Panel C. Without inflation expectations

Note: The table reports the variances of reduced form shocks in various samples for a VAR(4), which includes the growth rate of output Δ GDP, inflation π, the nominal interest rate R and, in the first two panels, an expectation proxy.
the predictive power of lagged, rather than current, expectations. Peter Tulip (2005) has found that the short-term predictability of output growth has increased using Greenbook forecasts. Our results agree with this evidence.

Fifth, one can argue that a four-variable VAR is misspecified. If a large scale model were the true data-generating process and a four-variable system was used, many important variables would be omitted, and their presence in VAR residuals could make the detection of regime changes hard. Therefore, we repeated estimation using a VAR which, in addition to the previous four variables, includes the first principal component obtained from 16 price variables contained in the quarterly macroeconomic dataset described in Stock and Watson (2007). Two lags are sufficient to whiten the residuals of this system. With this empirical model, the results still hold, but now Michigan expectations explain output growth in some samples, but not others. However, the change in predictive performance is neither permanent nor timewise related to the event of interest. Interestingly, inflation expectations have very little predictive power for the principal component of the price vector in any of the samples we consider.

Finally, we have argued that arbitrarily splitting the sample and forcing the break to be common to all equations is less than ideal to examine the role of expectations over time. Time-varying coefficient models are particularly suited for our purpose because they avoid strong restrictions on the nature of the breaks, and because they can track the time evolution of the relationships. The specification of a time-varying coefficient also allows us to examine the weaker hypothesis that the importance of expectations has declined as we move from the 1970s to the later part of the sample. The model we consider is

\[ y_t = X_t' \theta_t + \varepsilon_t \]

where \( y_t \) is a \( 4 \times 1 \) vector, \( X_t \) is a matrix including lags of \( y_t \) and a constant, \( \theta_t \) is a \( 4(4p + 1) \times 1 \) vector, \( p \) is the number of lags and \( \varepsilon_t \sim N(0, \Sigma_t) \). We assume that

\[ \theta_t = \theta_{t-1} + u_t \]

where \( u_t \) is a normal \( 4(4p + 1) \times 1 \) white noise with zero mean, covariance \( \Omega \). Let \( \Sigma_t = FD_tF' \), with \( F \) a lower triangular matrix and \( D_t \) a diagonal matrix, and let \( \sigma_t \) be the vector of the diagonal elements of \( D_t \). We assume:

\[ \log \sigma_{it} = \log \sigma_{i(t-1)} + \xi_{it} \]

where \( \xi_{it} \sim N(0, \Xi_{i}) \) and \( \xi_{it}, u_t, \) and \( \varepsilon_t \) are mutually independent.

We estimate the model with Bayesian MCMC techniques setting \( p = 2 \) and discarding draws for \( \theta_t \) producing diverging paths for \( y_t \). The details of the implementation are described in the Appendix. Since both \( \theta_t \) and \( \Sigma_t \) are time varying rather than using classical \( F \)-tests for the significance of lags of inflation expectations at each date, we present the evolution of the median and of the 68 percent central posterior credible interval, for the statistics of interest.

Figures 3 and 4, which plot the evolution of the median and the posterior credible intervals for the lags of inflation expectations, and for their long run value in
each equation when Michigan and Term expectations are used, broadly agree with Table 5. When Michigan expectations are used, inflation expectations are practically never significant in the output growth equation, and almost always significant in the inflation equation, at least in the long run. The significance of inflation expectations in the interest rate equation depends on the sample, but changes over time in the long run effects are statistically insignificant.

When Term expectations are used, the evidence is mixed. Nevertheless, it is still true that the importance of inflation expectations in the output growth equation is small and increasing somewhat since the early 1980s. However, for the other two equations, the effect is time-varying but inconsistent with the hypothesis of interest. For example, decreases in the median value of the coefficient of the first lag in the interest rate equation are compensated by increases in the median value of the coefficient of the second lag. Overall, inflation expectations are more important after 1982.

Figure 5, which reports the posterior median of the variance of the reduced-form shocks with inflation expectations (Michigan solid line, Term dashed line) and without them (dotted line), also broadly agrees with Table 6. For instance, there is a general decline in the variability of the reduced-form shocks over time, which is similar in magnitude and timing across measures of inflation expectations; including or excluding inflation expectations from the system hardly
changes the time path of the reduced form variances. Furthermore, given the considerable uncertainty associated with point estimates, differences in systems with and without inflation expectations are a-posteriori insignificant at any date in the sample.

To conclude, regardless of the proxies employed; of the specification of the VAR and the horizon where we measure the effect; of whether we allow coefficients to be time-varying or not; and of other specification choices; the importance of expectations does not decline as we move from the 1970s to the end of the sample—neither in the sense of a structural break nor in the sense of a slow moving but unidirectional change.

V. Conclusions

This paper examines whether the restrictions imposed by a simple indeterminacy-determinacy story of the Great Moderation are satisfied. We show that there is an additional state variable in the indeterminate regime which fails to appear in the determinate one; that standard counterfactuals may have hard time to detect regime changes; and that several explanations are “locally” indistinguishable from the indeterminacy-determinacy story. Using several VAR models, we study whether the significance of lagged expectations changes over time; and whether omitting
expectations from the estimated system causes time-varying biases in the variance of reduced-form shocks. We find that there is no clear switch over time in the importance of lags of expectations in any equation of the system; and that reduced-form variances estimated in systems with and without expectations display similar paths and little evidence of time-varying biases.

We show that the empirical results we obtain are robust to a number of potential empirical problems. Therefore, if one insists on taking the “bad policy” hypothesis as a benchmark, one has to conclude that the model we have used to derive restrictions is inappropriate. While the implications we emphasize hold in larger system with additional frictions (such as habit in consumption or wage stickiness), some omitted features could matter.

First, if regimes change in a Markov chain fashion, and agents are aware of the law of motion of the switches (as in Troy Davig and Eric M. Leeper 2007), the equilibrium is either determinate or indeterminate for the whole sample, but “bad policy” can contribute to volatility and persistence bursts even in a globally determinate regime. The fact that the role of expectations is unchanged over time, and the volatility in the explanatory power of structural shocks falls over time, is consistent with an explanation of the Great Moderation where the equilibrium is always determinate, but bad policy prevailed in the 1970s.

Empirical evidence suggesting that the case for “bad policy” in the 1970s is overstated, comes from the work of Orphanides (2004), who found little evidence of violation of the Taylor principle in the 1970s once real-time data are used. John V. Duca and Tao Wu (2007) concurred, pointing out that the presence of regulation-Q made the
effective real interest rate very different from the ex-post real rate. Furthermore, with the effective rate, the Taylor principle is almost never violated in the 1970s.

Second, we have seen that under learning, expectations become a state variable, regardless of the monetary regime in place. Therefore, our results are not necessarily inconsistent with a indeterminate-determinate story where agents learn over time about changes in the economy (see Schorfheide 2005). Furthermore, when we allow the coefficients to drift over time, we learn that the reduced-form representation of the model will be time-varying.

Third, the model assumes that there is no frictions in the flow of information. In models where information is sticky, such as Gregory N. Mankiw and Ricardo Reis (2006), the role of inflation expectations does not necessarily change with the regime. Sticky information models, however, have one counterfactual implication: inflation expectations should be almost perfectly correlated with lagged inflation. In our data, the correlation is small.

Hence, while the theoretical restrictions implied by the model of Section II are rejected, it is difficult to draw general conclusions about more sophisticated versions of the “bad policy” hypothesis, which allows for learning, misperception, or informational frictions. Future work in the area needs to examine these situations in more details.

Appendix: The Estimation of the TVC-VAR Model

I. Priors

Let $z^T$ denote the sequence of $z$’s up to time $T$. Let $\gamma$ be the vector containing the non-zero non-one elements of $F^{-1}$ stacked by rows and $\Xi$ a vector including all the $\Xi_i$. The transition density of the state is assumed to be

$$p(\theta_t | \theta_{t-1}, \Omega) \propto I(\theta_t)f(\theta_t | \theta_{t-1}, \Omega)$$

$$f(\theta_t | \theta_{t-1}, \Omega) = N(\theta_{t-1}, \Omega),$$

where $I(\theta_t)$ is an indicator function which discard draws for $\theta_t$ implying explosive paths for $y_t$. We assume that the hyperparameters and the initial states are independent so that the joint prior is simply the product of the marginal densities. Following Cogley and Sargent (2005) we assume:

$$P(\theta_0) \propto I(\theta_0)N(\bar{\theta}, \bar{P})$$

$$P(\Omega) = IW(\bar{\Omega}^{-1}, T_0)$$

$$P(\log \sigma_{i0}) = N(\log \bar{\sigma}_i, 10)$$

$$P(\gamma) = N(0, 10000 \times I_4)$$

$$P(\Xi_i) = IG\left(\frac{0.01^2}{2}, \frac{1}{2}\right)$$
where $\overline{\theta}$ and $\overline{\sigma}$ are the OLS estimates of the VAR coefficients and their variances obtained with the initial sample, $\Sigma = \lambda \overline{\sigma}$, $T_0$ is the number of observations in the initial sample (1960:QI–1971:QIV, 48 observations), $\sigma_i$ is the estimate of the variance of the residual in equation $i$ obtained using the initial sample. The hyperparameter $\lambda$ is set to 0.005 for all the parameters except for the constant terms of inflation, inflation expectations and the interest rate. For these constants it is set to 0.01.

II. Posteriors

To draw realizations from the posterior density we use the Gibbs sampler. Each iteration is composed of four steps and, under regularity conditions and after a burn-in period, iterations on these steps produce draws from the joint density.

**Step 1:** $p(\theta^T | y^T, \gamma, \sigma^T, \Xi, \Omega)$

Conditional on $y^T, \gamma, \sigma^T, \Xi, \Omega$, the unrestricted posterior of the states is normal. To draw from the conditional posterior we employ the algorithm of Carter and Kohn (1994). The conditional mean and variance of the terminal state $\theta_T$ is computed using standard Kalman filter recursions while for all the other states the following backward recursions are employed

$$\theta_{i|t+1} = \theta_{i|t} + P_{i|t} P_{t+1|t}(\theta_{t+1} - \theta_{i|t})$$

$$P_{i|t+1} = P_{i|t} - P_{i|t} P_{t+1|t} P_{t|t},$$

where $p(\theta_{i|t+1}, y^T, \gamma, \sigma^T, \Xi, \Omega) \sim N(\theta_{i|t+1}, P_{i|t+1}).$

**Step 2:** $p(\gamma^T | y^T, \theta^T, \sigma^T, \Xi, \Omega)$

Given that $\sigma^T$ and $y^T$ are known, $\varepsilon_i$ is know, and since $u_i$ is a standard Gaussian white noise, we have $D_{t+1/2}^{-1/2} \varepsilon_{t} = v_t$ or $D_{t+1/2}^{-1/2} \varepsilon_{t} = -D_{t+1/2}^{-1/2} F^* \varepsilon_t + v_t$ with $F^* = F^{-1} - I$. We can rewrite the $i$-th equation as $z_{it} = -w_{it} \gamma_i + v_{it}$, where

$$z_{it} = \varepsilon_{it}/\sqrt{\sigma_{it}}, \quad w_{it} = \left[\varepsilon_{1t}/\sqrt{\sigma_{1t}}, \ldots, \varepsilon_{it}/\sqrt{\sigma_{it}}\right],$$

and $\gamma_i$ is the column vector formed by the nonzero elements of the $i$th row of $F^*$. Given the normal prior, the posterior is $\gamma_i = N(F_{1i}, V_{1i})$, where $F_{1i} = V_{1i}(V_{0i}^{-1} \gamma_{0i} + w_{1i} z_{1i})$ and $V_{1i} = (V_{0i}^{-1} + w_{1i}^t w_{1i})^{-1}$ with $V_{0i}$ and $\gamma_{0i}$ the prior variance and mean respectively. Drawing for $i = 2, 3, 4$ we obtain a draw for $\gamma_i$.

**Step 3:** $p(\sigma^T | y^T, \theta^T, \gamma, \Xi, \Omega)$

The elements of $\sigma^T$ are drawn using the univariate algorithm described in Cogley and Sargent (2005, see appendix B.2.5 for details).

**Step 4:** $p(\Xi | y^T, \theta^T, \gamma, \sigma^T, \Omega), p(\Omega | y^T, \theta^T, \gamma, \sigma^T, \Xi)$

Conditional on $y^T, \theta^T, \gamma, \sigma^T$ and under conjugate priors, all the remaining hyperparameters, can be sampled in a standard way from Inverted Wishart and Inverted Gamma densities.
We perform 20,000 repetitions, we discard the first 5,000 draws and, for inference, we keep one every 10 of the remaining draws to break the autocorrelation of the draws.

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