The dynamics of US inflation: Can monetary policy explain the changes?

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\textbf{ABSTRACT}

We investigate the relationship between monetary policy and inflation dynamics in the US using a medium scale structural model. The specification is estimated with Bayesian techniques and fits the data reasonably well. Policy shocks account for a part of the decline in inflation volatility; they have been less effective in triggering inflation responses over time and qualitatively account for the rise and fall in the level of inflation. A number of structural parameter variations contribute to these patterns.

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1. Introduction

The US economy has gone through a number of important structural changes over the last forty years. For example, the level of inflation and of nominal interest rates shows an inverted U-shaped pattern, rising at the end of the 1970s and falling at the beginning of the 1980s; while the persistence and the volatility of inflation have dramatically declined since the mid-1980s; see e.g. Stock and Watson (2002). These patterns are well documented in the literature. What is still to be determined is the cause of these changes.

The prevailing view suggests that the run-up of inflation occurred because monetary authorities believed that there was an exploitable trade-off between inflation and output. Since output was low following the oil shocks of the 1970s, the temptation to inflate was strong. However, the option of keeping inflation temporarily high was unfeasible: in the medium run, inflation reached a higher level with output settling at its potential. Since the 1980s, central banks learned that the output–inflation trade-off was not exploitable and concentrated on the objective of fighting inflation. A low inflation regime ensued, and the larger predictability of monetary policy made the macroeconomic environment less volatile (see e.g. Sargent (1999), Clarida et al. (2000), and Lubik and Schorfheide (2004)). There are two alternative views as regards this prevailing wisdom: one focuses on “real” causes (see e.g. McConnell and Perez Quiroz (2000), Campbell and Herkovitz (2006)) and the other hinges on “good luck” (see e.g. Bernanke and Mihov (1998), Leeper and Zha (2003), Sims and Zha (2006)) to explain the changes in the level and in the autocovariance function of inflation.

One reason for this heterogeneity of explanations is that the empirical strategy used to study the issue matters. In general, VAR based evidence tends to support the good luck hypothesis; calibration exercises point to real reasons for the changes; and structural econometric analyses favor the idea that monetary policy is responsible for the observed variations (see, e.g., Ireland (2001), Lubik and Schorfheide (2004) and Boivin and Giannoni (2006)). However, while structural VAR exercises allow for time varying coefficients and variances, the evidence produced by more structural calibration or econometric analyses is mostly restricted to arbitrarily chosen subsamples. Because inflation and the nominal rate displayed an inverted U-shaped pattern, subsample evidence may depend on the selected break point.
and Primiceri (2008) have estimated evolving structural models but their conclusions are only suggestive, because computational complexities force them to consider variations only in a subset of the parameters. Given that one expects important covariations in the evolution of structural parameters, allowing only a subset of the parameters to change may bias inference. Hence, it is of interest to know whether less computationally intensive and yet intuitively appealing structural methods can tell us more about the nature of the changes experienced by US inflation. This paper provides a step in that direction by estimating a structural model over rolling samples of fixed length with Bayesian techniques. Bayesian methods, which have become popular tools for bringing DSGE models to the data thanks to the work of Smets and Wouters (2003) and Del Negro and Schorfheide (2004) among others, have inferential and computational advantages over traditional limited and full information classical techniques when dealing with models which are known to be a misspecified description of the data. In these situations, unrestricted classical estimates are often unreasonable or on the boundary of the parameter space and tricks must be employed to produce economically sensible estimates. Furthermore, asymptotic standard errors attached to classical estimates – which are constructed assuming that the model is “true” – are meaningless. Rolling samples allow us to use relatively standard techniques to study the nature of the time variations present in interesting parameters while maintaining some form of rationality in the economy and keeping computational costs manageable. For example, in contrast to that of Fernandez Villaverde and Rubio Ramírez (2008), our setup allows the use of Kalman filtering techniques in building the likelihood function and permits time covariances in all the parameters.

The specification that we consider deviates somewhat from what is standard in the literature by allowing money to play a role. The stock of money has been neglected in all recent monetary policy discussions (see e.g. Woodford (2003)) and Ireland (2004) provided some empirical evidence supporting this approach. In our setup real balances can potentially affect the Euler equation and the growth rate of real balances is allowed to enter the monetary policy rule. Since we will use loosely specified but proper priors in the estimation, the data will decide whether these features are important in characterizing the experience. Overall, the statistical fit of the model looks satisfactory, in particular, in comparison with other structural specifications. We estimate the preferred specification a number of times over rolling samples, analyze the time evolution of interesting inflation statistics, measure the contribution of monetary policy to the observed changes and study the evolution of the structural parameters.

Our model captures the fall in inflation volatility over time and attributes part of the changes to monetary policy shocks. We detect level but not shape differences in the transmission of policy shocks which tend to make inflation less reactive to policy disturbances as time goes by. Finally, variations in the level of inflation are qualitatively related to policy shocks: had those been absent, the rise of the 1970s and the fall of 1980s would have been much more modest.

A number of structural changes drive these results. We find support for the conjecture that the Fed had a much stronger dislike for inflation but also notice that in the latest samples the coefficient resembles the one obtained at the beginning of the sample. Moreover, the estimate of the long run coefficient on monetary aggregates has been steadily declining over time. We detect, in agreement with the good luck hypothesis, variations in the posterior mean estimate of the variance of the policy shocks. Nevertheless, as in Sims and Zha (2006), the variations that we discover are typically reversed over time. Finally we also find, in consistency with non-monetary explanations of the facts, that important private sector parameters such as the slope of the Phillips curve and the variability of real demand shocks have significantly changed in the later samples.

In sum, we find, in consistency with the conclusions of Gambetti et al. (2008), that a combination of causes appears to be responsible for the changes in the level and the autocovariance function of US inflation over the last forty years: changes in the variance of the shocks, in the parameters regulating private sector behavior and in the policy rule all more or less contributed to explain why inflation rose and fell, and why inflation volatility subsided.

The rest of the paper is organized as follows. Section 2 describes the model, the estimation technique and the diagnostics used to evaluate the quality of the model’s approximation to the data. Section 3 presents estimation results for the full sample. Section 4 reports the time profile of inflation statistics over the rolling samples. Section 5 interprets these time profiles in terms of rolling structural parameter estimates. Section 6 concludes.

2. The framework of the analysis

2.1. The theoretical model

We consider a medium scale model featuring several shocks and frictions. Households maximize a utility function which depends on three arguments (money, consumption and leisure), and money and consumption are potentially non-separable. Labor is differentiated over households, so there is some monopoly over wages. Households allocate wealth between cash and a riskless bond, and bond demand is perturbed by a preference disturbance (as in Smets and Wouters (2007)). Households also rent capital services to firms and decide how much capital to accumulate. As the rental price of capital goes up, the capital stock can be used more intensively according to some schedule cost. There are two kinds of firms: a final good representative firm that aggregates intermediate goods, and a continuum of intermediate producing firms that combine labor and capital in a monopolistic competitive market where price decisions are subject to a Calvo lottery. Prices that cannot be optimally adjusted are assumed to be partially indexed to past inflation. Similarly, in the labor market unions sell differentiated units of labor in a monopolistic competitive environment with a Calvo type scheme. When unions receive positive signals, they are allowed to re-optimize wages; otherwise they adjust wages, indexing them to past inflation. Finally, profits generated from the imperfectly competitive intermediate goods and the labor markets are redistributed to households. The nominal interest rate is controlled by a monetary authority who set it in reaction to inflation, output gap and real balances.

The equations that we employ can be derived from first principles—optimizing and forward looking consumers and firms and general equilibrium considerations. Since derivations of this type exist in the literature (see e.g. Smets and Wouters (2003) and Smets and Wouters (2007)), we simply present the optimality conditions and highlight how they link to the objective functions and the constraints of the agents. The system in log-linear form is

\[
\omega_1 l_t = -c_t + h_{c_{t-1}} - \omega_2 m_t + \omega_2 u_t
\]

\[
\omega_1 \ell_t = \theta_t + w_{zt}^{RH}
\]

\[
\theta_t = -\omega_3 c_t + h_{os_{t-1}} - \omega_4 m_t + \omega_4 u_t - \frac{1}{R-1}(\epsilon_t^b + r_t)
\]

\[
z_t = d_t/\alpha_t^{k_t}
\]

\[
k_t = z_t + k_{t-1}
\]

\[
\ell_t = \omega_t + \ell_t - r_t^k
\]

\[
mc_t = (1-\alpha)\ell_t + \alpha r_t^k - \ell_t^a
\]
\[
\begin{align*}
y_t &= \frac{\psi}{\psi - 1} e_t^r + \frac{\psi}{\psi - 1} \alpha(w_t - r_t^1) + \frac{\psi}{\psi - 1} k_t \\
y_t &= C_t = K_t - \delta k_{t-1} + \epsilon_t e_t^r + \alpha z_t \\
\kappa_t &= (1 - \delta) k_{t-1} + \delta i_t + \delta e_t^r \\
r_t &= \lambda_r r_{t-1} + \lambda_y y_t + (\lambda_p + \lambda_m) \pi_t + \lambda_m (m_t - m_{t-1}) + \zeta_t \\
\theta_t &= E_t [k_{t+1} + \epsilon_t^2 + r_t - \pi_{t+1}] \\
q_t &= E_t [\beta r_t^e + \beta (1 - \delta) q_{t+1} - \theta_t + \theta_{t+1}] \\
(1 + \beta) S^t i_t &= E_t [q_t + \epsilon_t^2 + S^t i_{t-1} + \beta S^t i_{t+1}] \\
(k_t + \beta w_t) u_t &= E_t \left[ \left( k_t w_{t+1} + \frac{\epsilon_t^m}{\epsilon_t^m + 1} e_t^r \right) + u_{t-1} \right] + \beta w_{t-1} + \left( 1 + i_w \beta \right) \pi_t + i_w \pi_{t-1} - \beta \pi_{t+1} \\
\pi_t &= E_t \left[ \left( \frac{\rho}{1 + \beta_p} \pi_{t-1} + \frac{\beta}{1 + \beta_p} \pi_{t+1} \right) + k_p \left( m c_t + e_t^p \right) \right] \\
\end{align*}
\]

where variables with the time subscript are log deviations from the steady state, and variables without are variables at the steady state. The \(\omega\)'s are convolutions of steady state values and parameters that capture the curvature of the utility function. In particular, \(\omega_1 = -\frac{\xi_2}{\omega_{10}}, \omega_2 = -\frac{\xi_{10}}{\omega_{02}}, \omega_3 = -\frac{\xi_{11}}{\omega_{03}}, \omega_4 = -\frac{\xi_{02}}{\omega_{04}}\) and \(\omega_5 = -\frac{\xi_{00}}{\omega_{05}}\). Eq. (1) relates the marginal utility of consumption, \(\theta_t\), to current consumption, \(c_t\), past consumption (controlled by a degree of habit, \(h\)), real balances, \(m_t\), and a money demand shock, \(\epsilon_t\). When preferences are separable in real balances and consumption, \(\alpha \sigma i_{cm} = 0\), so \(\omega_2 = 0\) and \(\omega_5 = 0\) and real balances do not enter into the Euler equation. Unions pay a wage, \(w_t^{HI}\), constant across households (HH), which reflects the capital returns of stratification between working, \(l_t\), and consumption; see Eq. (2). Eq. (3) is a money demand equation which depends on current and past consumption, the nominal interest rate, \(r_t\), and a preference disturbance, \(\epsilon_t^p\). Current capital services, \(k_t\), are a function of the capital installed in the previous period, \(k_{t-1}\), and the degree of capital utilization, \(z_t\); see Eq. (5). Household cost minimization implies that the degree of capital utilization is a positive function of the rental rate of capital, as in Eq. (4). Eq. (6) relates the capital per worker to the cost of capital, \(r_t^1\), and the cost of labor, \(u_t\). In Eq. (7) the marginal cost, \(m c_t\), is determined by the real cost of the two factors in production, \(r_t^1\) and \(w_t\), with weights given by their respective share in production, net of the total factor productivity disturbance, \(\epsilon_t^r\). Final output, \(y_t\), is produced using a Cobb–Douglas production function (see Eq. (8)), where \(\Psi\) captures the fixed cost in production, and \(\alpha\) is the capital share. Total output is absorbed by exogenous government spending, \(\epsilon_t^g\), by investment, \(i_t\), by consumption, \(c_t\), and by the capital utilization cost, \(z_t\); see Eq. (9)]. New installed capital is formed by the flows of investment and the old capital net of depreciation, \((1 - \delta) k_{t-1}\); see Eq. (10). The capital accumulation equation is hit by the investment-specific technology disturbance \(\epsilon_t^i\); Eq. (10) represents a policy rule. The specification is standard in the first three terms, reflecting an interest rate smoothing desire and the wish to respond to fluctuations in the output and inflation. We allow the policy rule to depend on the growth rate of nominal balances in order to mimic concerns that Central Banks had over monetary aggregates for part of the sample and let \(\lambda_m \geq 0\). The disturbance to this equation represents a monetary policy shock which is, by construction, orthogonal to the other structural disturbances. Eq. (11) describes the dynamics of consumption and real balances, where current consumption and money balances depend on their expected values and the ex ante real interest rate, \(r_t - E_t \pi_{t+1}\). Eq. (12) is the Q equation that gives the value of capital stock, \(q_t\), which depends positively on the expected future value of the capital stock and on the real rental rate and negatively on the marginal utility of consumption. Eq. (13) is an investment equation: the current value of the investment depends on the past and expected future values of the investment and on the current value of the stock of capital. Eq. (14) gives the dynamics of the real wage, which moves sluggishly because of the wage stickiness and the partial indexation assumption; the wage responds to past and future expected real wages, and to the (current, past and expected) movements in inflation. The real wage depends also on the wage paid by the union and on the wage markup, \(\epsilon_t^w\), with slope \(k_w = \frac{\xi_w - \xi_{1w}}{\xi_{1w}}\), where \(1 - \xi_w\) is the probability of re-optimizing wages. Eq. (15) is the New Keynesian Phillips curve: current inflation depends positively on past and expected inflation, and on marginal costs. This equation is perturbed by a price markup disturbance, \(\epsilon_t^p\), and the slope is \(k_p = \frac{(\xi_{1p} - \xi_p)}{(1 + \rho_{p})\xi_{1p}}\), where \(\rho\) is the time discount factor, and \(\xi_p\) is the probability of keeping the prices fixed. The system is driven by eight exogenous processes: \(\epsilon_t = \rho \epsilon_{t-1} + \zeta_t^\epsilon\) (money demand), \(\epsilon_t^b = \rho b \epsilon_{t-1} + \zeta_t^b\) (technological), \(\epsilon_t^i = \rho \epsilon_{t-1} + \zeta_t^i\) (investment specific), \(\pi_t - \ln \epsilon_t = \zeta_t^\pi = \rho \pi_{t-1} + \zeta_t^\pi\) (government), \(\ln \epsilon_t - \ln \epsilon_t^p = \zeta_t^\epsilon = \rho \epsilon_{t-1} + \zeta_t^\epsilon\) (price markup), \(\ln \epsilon_t^p - \ln \epsilon_t = \zeta_t^\epsilon^p = \rho \epsilon_{t-1} + \zeta_t^\epsilon^p\) (wage markup) and \(\epsilon_t^m = \zeta_t^m\) (monetary policy shock), where the \(\zeta_t^\cdot\)'s are normal i.i.d. shocks. We assume that the investigator observes output, consumption, investment, hours worked, real wages, real balances, the inflation rate, and the nominal interest rate. The available sample goes from 1959:1 to 2006:1 and the data set is obtained from the FRED database at the Federal Reserve Bank of St. Louis. We stop at 2006 to maintain comparability with the existing literature and this avoids headaches as regards how to deal with the recent financial crisis episode. For real balances we use real M2 deflated by the GDP deflator. The inflation rate is measured by the growth rate of the GDP deflator, and the nominal interest rate by the federal funds rate, and real variables are scaled by the civilian noninstitutional population (CNP16GV) to transform them to per capita terms. Despite these transformations, some series still display upward trends. We therefore separately eliminate this from their log using a simple quadratic specification. While the resulting fluctuations display long periods of oscillation, they are overall stationary as the model assumes (see Fig. 1). As a referee has pointed out, one feasible alternative to the strategy that we use to match the data to the model’s counterparts is to allow the technology or the investment specific shock to be non-stationary and remove the upward trend in output and real balances using a model consistent method. We do not follow this approach for three reasons. First, when these shocks have a unit root, output and real balances share the same trend, which is not the case with the available data. Second, the data set transformed with a model based trend still displays a clear non-stationary behavior and this makes parameters estimates unreliable and the quality of the fit quite poor. Finally, it is unclear whether all non-cyclical fluctuations can be safely attributed to non-stationary technology shocks. For example, Chang et al. (2006) have recently fit a model with non-stationary preference shocks to US data with good results.

2.2. The prior and the estimation technique

The model (1)–(10) and (10)–(15) contains up to 38 parameters: 18 structural ones \(\eta_1 = (h, i, \beta, \alpha, d'/a', S^i, \xi_{1w}, \epsilon_{1w}, \delta_p, \epsilon_{1p}, \lambda_r, \lambda_y, \lambda_m, \lambda_n, \epsilon^f, \delta, \alpha)\), 5 semi-structural ones \(\eta_2 = (\alpha_1, \alpha_2, \alpha_3, \ldots)\).
ο4, ο5), and 15 auxiliary ones, η1 = (ρe, ρb, ρu, ρl, ρg, ρw, ρp, σe, σb, σg, σl, σw, σp, σw, σp). In our exercises we fix a few of them (the time discount factor β = 0.995, the capital depreciation rate δ = 0.025, the steady state government spending over GDP εδ = 0.18, the capital share α = 0.3, and the steady state inflation π = 1.006) since they are not identifiable from the data that we use, and we obtain posterior distributions for the remaining elements of η = (η1, η2, η3) using loosely specified but proper priors.

We assume univariate prior densities for each of the parameters even if, in principle, there should be some correlation structure among the priors (see e.g. Del Negro and Schorfheide (2008)). Since all the priors are proper, posterior distributions are proper and various specifications can be compared with simple odd ratios. The prior shapes that we use in our benchmark specification are relatively standard (see Table 1), and very similar to the ones used in Smets and Wouters (2007) but with much larger prior standard deviations.

The model can be solved with standard methods. Its solution has a state space format:

\begin{align}
\begin{align}
X_{t+1} &= A_1(\eta)X_t + B_1(\eta)z_{t+1} \\
X_t &= A_2(\eta)X_t
\end{align}
\end{align}

where \( X_t = [m, h, y, c, l, w, \pi_t, r_t] \), \( z_t = [\xi^e, \xi^b, \xi^u, \xi^l, \xi^g, \xi^p, \xi^w] \) and the matrices \( A_i(\eta), i = 1, 2, 3, \) are complicated nonlinear functions of the \( \eta \)'s.

Bayesian estimation of (16) and (17) is simple: given some \( \eta \) and a sample \( t, \ldots, T \), we compute the likelihood, denoted by \( f(y_{[t,T]}|\eta) \), by means of the Kalman filter and the prediction error decomposition. Then, for any specification of the prior distribution, denoted by \( g(\eta) \), the posterior distribution for the parameters is \( g(y_{[t,T]}|\eta) = \frac{g(\eta) f(y_{[t,T]}|\eta)}{\int g(\eta) f(y_{[t,T]}|\eta) d\eta} \). The analytical computation of the posterior is impossible in our setup since the denominator of the expression, \( f(y) \), requires the integration of \( g(\eta)f(y_{[t,T]}|\eta) \) with respect to \( \eta \), which is a high dimensional vector. In order to obtain draws from the unknown posterior distribution we use the following (Metropolis) algorithm:

1. Choose an \( \eta_0 \). Evaluate \( g(\eta_0) \); use the Kalman filter to evaluate the likelihood \( f(y_{[t,T]}|\eta_0) \).

2. For each \( i = 1, \ldots, k \), where \( k \) is the dimension of the estimated parameter vector, set \( \eta_i = \eta_{i-1} \) with probability \( 1 - p \) and \( \eta_i = \eta^*_i \) with probability \( p \), where \( \eta^*_i = \eta_{i-1} + v_i \) and \( v = [v_1, \ldots, v_k] \) follows a multivariate normal distribution and \( p = \min\{1, \frac{\mathcal{L}(y_{[t,T]}|\eta^*_i)^{\kappa}}{\mathcal{L}(y_{[t,T]}|\eta_{i-1})^{\kappa}}\} \).

3. Repeat steps 1 and 2 \( \hat{L} + L \) times and keep the last \( L \) draws.

At the end of the routine one has \( L \) draws with which to conduct the structural analysis.

Two important issues concern the convergence of simulated draws, that is, what the size of \( \hat{L} \) is, and the acceptance rate. We set the number of iterations to 500,000, checked for convergence using the cumulative sum of the draws (CUMSUM) statistics and found that convergence is achieved typically after 200,000 draws. For inference we discard the first half of the chain and keep one from every 250 draws, so that we remove the correlation among draws and have 1000 draws from the posterior distribution to work with. To get reasonable acceptance rates, it is important to properly select the variance of \( v_i \). If the acceptance rate is “too small” the chain will not visit the parameter space in a reasonable number of iterations. If it is too high, the chain will not stay long enough in the high probability regions. We set the variance of \( v_i \) to target an acceptance rate in the range 20%-40%.

The chosen prior shapes reflect the restrictions on the support of the parameter space. Various parameterizations of the Beta distributions were tried and results are, by and large, insensitive to settings of the parameters controlling the location of these distributions. Finally, the choice of prior distributions for the policy parameters implies that multiple equilibria are unlikely to characterize the data.

2.3. A measure of the fit and model comparisons

To assess the quality of our model’s approximation to the data we have estimated a number of interesting structural benchmarks. We present the marginal likelihood (ML) of our model and of four alternative specifications where e.g. real balances play no role in the Euler equation, or real balances do not enter the policy rule, or both. For each model \( \mathcal{M}_j \), we approximate \( \mathcal{L}(y_{[t,T]}|\mathcal{M}_j) \), the marginal likelihood of \( \mathcal{M}_j \), using \( \left[ \frac{1}{L} \sum_{l=1}^{L} \frac{f(y_{[t,T]}|\eta_l, \mathcal{M}_j) g(\eta_l|\mathcal{M}_j)}{\int f(y_{[t,T]}|\eta, \mathcal{M}_j) g(\eta|\mathcal{M}_j) d\eta} \right]^{-1} \).
of the interest rate to real balances, \( \lambda_m = 0 \) (R1), (iii) a specification which assumes no reaction of the interest rate to real balances, \( \lambda_p = 0 \) (R2), and (iv) a specification which employs a separable utility function, \( \omega_2 = \omega_5 = 0 \) (R3).

A few aspects of the table deserve comments. First, priors and posteriors tend to have different locations, spreads and, in several cases, shapes. Therefore, the sample appears to be informative about the properties of many of the parameters. Second, the mean estimates of \( \omega_2 \) and \( \omega_5 \) are positive and the posterior distribution is tight, both in an absolute sense and relative to the prior. The elasticities of the three standard arguments in the utility function, i.e. \( \omega_1 \), \( \omega_3 \) and \( \omega_4 \), have the expected signs.

Third, the parameters controlling the backward looking components of the Euler equation and the Phillips curve are small (h and \( L_p \))—smaller than those obtained, for example, by Smets and Wouters (2003) and Justiniano and Primiceri (2008). One reason for this is that the preliminary data transformation that we use is different.

Fourth, the coefficients in the policy rule imply a relative mild smoothing desire and a somewhat more aggressive response to inflation and real balances. We experimented with several versions of the monetary policy rule, where for instance the interest rate was allowed to react to real balances (instead of real balances growth) or where the reaction is lagged rather than contemporaneous. The fits of these specifications were poor and, in particular, the estimate of the smoothing coefficient of the Taylor rule was close to zero, making monetary policy residuals highly

3. Full sample estimation and specification searches

We start estimating the model for the full sample, 1959:1–2006:1. We are interested in verifying that it fits the data reasonably well and, therefore, can be used to undertake the type of analysis that we care about; and in examining whether the model’s parameterizations can be simplified and certain specification choices matter for the results. Table 1 presents the posterior medians and standard deviations for (i) a specification which employs the full set of parameters (UR), (ii) a specification which employs a separable utility function, \( \omega_2 = \omega_5 = 0 \), and no reaction
serially autocorrelated. The rule that we use has two important features, which turns out to be important in the estimation: it makes the relationship between interest rate and the right hand side variables dynamic; money growth enters the rule.

To gain insights into the plausibility of the estimates, it is interesting to analyze the transmission mechanism of shocks and the decomposition of the variance implied by the unrestricted and the restricted models. Fig. 2 reports the responses of the interest rate, inflation, real balances and GDP to impulses in supply (technology, investment) and demand (preference, money demand and government) shocks and monetary policy in the unrestricted and the \( R_1 \) specifications. The dynamics induced by supply shocks are in line with those of Smets and Wouters (2007), where for example following a positive TFP shock, output increases while inflation and the interest rate drop. An investment shock produces similar co-movements of the interest rate, the inflation and GDP and induces a drop in money demand. These dynamics are roughly similar in models where money matters and where money does not matter. The transmission of a monetary policy shock is however different. In the baseline model, a monetary policy tightening (positive monetary policy shock) increases the interest rate and makes the M2 supply drop, generating a liquidity effect. As a result, the economy contracts, and output and inflation decrease. All the responses display some inertia and the response of the GDP is quite sluggish.

When money does not matter, the correlation between interest rate and money conditional on monetary policy shocks is positive, implying that contractionary monetary policy generates a significant increase of the interest rate and of the money supply. Moreover, while money demand shocks have a non-negligible impact on nominal and real variables in the unrestricted model, in the restricted specification all these effects are forced to zero and are captured by other shocks.

Table 2 presents the \( k \)-step-ahead forecast error for GDP and inflation in terms of structural shocks. The table contains several results. First, while in the unrestricted specification the money demand shock contributes significantly to the volatility of GDP and inflation at horizons from 2 to 10 years, in the restricted specifications money demand shocks play a negligible role. In particular, they do not contribute to explaining the GDP forecast error at any horizons, and they explain a small portion of the volatility of inflation as long as \( \lambda_m \neq 0 \). Second, while markup shocks have some role in explaining GDP volatility at short horizons, their impact tends to disappear in the long run, and supply shocks, i.e. technology shocks, appear to be the predominant source of GDP fluctuations. In line with Smets and Wouters (2007), in the most restricted specification \( R_1 \) wage markup shocks counterintuitively explain a large portion of the volatility of GDP at business cycles horizons. In sum, the dynamics described by the unrestricted and the restricted specifications differ substantially, in particular as far as the transmission of monetary policy shocks and the share of the variance of output and inflation attributed to shocks are concerned. Given the focus of the investigation, models where money is not allowed to matter seem unable to capture important features of the data.

It is also instructive to inspect the time path of the estimated residuals to check for an interesting pattern left unexplained in the estimation. Fig. 3 plots the smoothed residuals obtained with the unrestricted specification. It is clear that during the 1970s, the US economy experienced a series of negative monetary policy shocks, while at the beginning of the 1980s, monetary policy is characterized by a series of positive shocks.

Thus, the path of policy shock implies that monetary policy was accommodative for most of the 1970s and became much

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2 In the appendix, we report the impulse response for specifications \( R_2 \) and \( R_3 \), see Fig. A.1.
Table 2

$k$-step-ahead forecast error variance decomposition for GDP and inflation.

<table>
<thead>
<tr>
<th>Steps ahead</th>
<th>UR</th>
<th>$R_1$</th>
<th>$R_2$</th>
<th>$R_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Forecast error variance of GDP</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Money shock</td>
<td>24.7</td>
<td>14.9</td>
<td>11.1</td>
<td>8.0</td>
</tr>
<tr>
<td>Preference</td>
<td>0.6</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>Technology</td>
<td>21.9</td>
<td>61.2</td>
<td>82.2</td>
<td>59.4</td>
</tr>
<tr>
<td>Investment</td>
<td>17.1</td>
<td>1.9</td>
<td>1.5</td>
<td>0.3</td>
</tr>
<tr>
<td>Government</td>
<td>6.3</td>
<td>3.0</td>
<td>1.8</td>
<td>0.4</td>
</tr>
<tr>
<td>Monetary policy</td>
<td>0.6</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>Wage markup</td>
<td>5.1</td>
<td>11.2</td>
<td>0.4</td>
<td>14.3</td>
</tr>
<tr>
<td>Price markup</td>
<td>23.6</td>
<td>7.8</td>
<td>3.0</td>
<td>25.6</td>
</tr>
</tbody>
</table>

Forecast error variance of inflation

| Money shock | 2.3  | 15.2 | 11.0 | 0.0  |
| Preference  | 2.8  | 0.4  | 0.2  | 0.7  |
| Technology  | 22.2 | 0.5  | 7.8  | 59.7 |
| Investment  | 51.7 | 66.4 | 16.6 | 0.3  |
| Government  | 1.3  | 6.0  | 3.3  | 0.3  |
| Monetary policy | 6.2 | 0.4  | 0.5  | 0.0  |
| Wage markup | 0.2  | 10.1 | 32.4 | 14.4 |
| Price markup | 13.3 | 1.1  | 28.2 | 24.6 |

Fig. 3. Residuals; unrestricted specification.

tighter later on. Interestingly, the model also predicts a sequence of negative interest shocks just after 2001. The monetary policy residuals in the restricted specifications are similar; see Fig. 4. However, the implied paths display a more pronounced autoregressive component and residuals tend to be asymmetric and skewed towards negative values.

Table 1, which reports the marginal likelihood of each model, confirms previous conclusions. For a wide range of $p$-values, the unrestricted specification is superior to all other structural alternatives that we considered. Thus, allowing real balances to play a role and specifying a policy rule which depends on, among other things, the growth rate of nominal balances provide important advantages in terms of the model’s fit and properties of the residuals. We hope that this superiority will also help in providing a better economic interpretation of the evidence.

4. Inflation dynamics

There is substantial controversy in the literature regarding the role that monetary policy had in shaping the dynamics of US inflation over the last forty years. We can shed some light on the issue, conditionally on the model, by analyzing the time profile of the estimated autocovariance function of inflation, by measuring the contribution of policy shocks to the changes, and by interpreting the evolution of these statistics in terms of changes in the parameters of the model.

Roberts (2006) and Cecchetti et al. (2007) have studied, by way of stochastic simulations and/or standard sensitivity analysis, how changes in the parameters of a small scale model where money plays no role affect inflation variability and persistence. Both consider a somewhat hybrid model, where the Euler equation is ad hoc, a time varying inflation target enters the policy rule and the policy shock is persistent. Roberts finds that changes in the policy parameters can account for the fall in the variance of inflation but not its persistence. Cecchetti et al. find that two parameters are crucial: the degree of indexation in the Phillips curve and the variance of the policy shocks. In particular, only a considerable fall in the indexation parameter is capable of explaining the absolute fall in persistence that researchers have documented. In addition, when indexation is low, a fall in the variance of the policy shocks may also decrease inflation persistence.

The analysis that we conduct has two main differences with these papers. First, unlike these authors, we examine the contribution of monetary policy shocks to the evolution of the variability and persistence of inflation (this is done in much the
same spirit as the analysis of Gambetti et al. (2008) but using a structural model rather than a VAR and examine how the evolution of certain parameter estimates changed the contribution of monetary policy shocks. To do this, let \( x_t = [x_{t1}, x_{t2}]' \) and represent the solution of the model as \( x_{t+1} = A_1(\eta)x_t + A_2(\eta)\zeta_{t+1} \). Then \( \text{var}(x_t) = \Sigma_x = A_1(\eta)\Sigma_x A_1(\eta)' + A_2(\eta)\Sigma_x A_2(\eta)' \) and \( \text{ACF}(1)(x_t) = A_1(\eta)\Sigma_x \), where \( \Sigma_x \) is the covariance matrix of the shocks and \( \text{ACF}(1) \) the first autocovariance coefficient. Since monetary shocks are uncorrelated with the other disturbances, the contribution of monetary shocks to these statistics can be computed by replacing \( \Sigma_x \) with a matrix which is zero everywhere except in the diagonal position corresponding to the monetary shocks. As estimates of \( \eta \) change over time, we can assess how the contribution of monetary shocks to these statistics has changed and attribute the variations to particular parameters.

Second, while one would be tempted to present a graph showing how inflation dynamics change when \( \eta \) varies within a reasonable range, keeping all the other parameters fixed at some value (for example, those estimated in the first column of Table 1), one should be aware that such a graph would be meaningless in our context, since the correlation structure of estimates implies that the effect of changes in some parameters cannot be measured independently of the others. To address the questions of interest we have instead estimated a version of the model on many overlapping samples. We started from the sample [1959.1, 1976.1] and repeated estimation moving the starting date by two years, while keeping the size of the sample constant at 17 years. Keeping a fixed window size is necessary to eliminate differences produced by the different precisions of the estimates. The last subsample is [1989.1—2006.1], so we produce 16 posterior distributions for the parameters. Since the final sample roughly corresponds to Greenspan’s tenure, we can compare the estimated stance of monetary policy in the 1990s, where inflation was low, with that of the 1970s, where inflation was high.

Given the large number of parameters involved and the sample length (69), we have decided to reduce the number of parameters to estimate and focus on those which are of interest. Thus, in each estimation window, in addition to keeping fixed the time discount factor \( \beta = 0.995 \), the capital depreciation rate \( \delta = 0.025 \), the steady state government spending over GDP \( \kappa^g = 0.18 \), the capital share \( \alpha = 0.3 \), and the steady state inflation \( \pi = 1.006 \), we set the two elasticities in the labor and intermediate good markets to 13.

### 4.1. The volatility and the persistence of inflation

It is well documented in the literature that the times series properties of US inflation have changed over time (see e.g. Stock and Watson (2002)). Is the model able to capture these facts? What is the contribution of policy shocks to these changes? Fig. 5 presents statistics recursively computed from the data, the estimates of these statistics obtained in the model and the statistics produced if shocks of only one type — technology shocks (TFP and investment), monetary policy shocks or real demand shocks (government spending and preference) — were present. Money demand shocks are not reported here since their role is minimal.

We measure volatility in the model using the population unconditional variance and persistence using the population first-order autocorrelation coefficient obtained using the solution of the model and posterior mean estimates of the parameters in each sample.

The statistics in the data show important time variations: for example, there is a large drop in volatility when the samples exclude the early 1970s, while the fall persistence is evident only if windows including the 1980 and the 1990s are considered. Interestingly, after that, inflation persistence starts rising again. While the model is able to track quite well the volatility dynamics, it fails to capture the drop in persistence experienced in the sample including the 1980s and 1990s. Notice, however, that the fall in inflation persistence is neither smooth nor long-lived—the same pattern is maintained if the windows change in size of if the sample is split into less overlapping windows. Thus, changes in volatility and persistence could be due to different causes.

Interestingly, monetary policy shocks seem to have a role in shaping the dynamics of inflation volatility and appear to be responsible in part for the decrease observed in the latest samples. Moreover, they track pretty well the rise and fall of volatility over time. However, other shocks appear also to be responsible for the evolution of the autocovariance function of inflation. Technology shocks, for example, affect inflation volatility and persistence mainly at the beginning of the sample and their importance is falling over time. Oil shocks which materialized at the end of the 1970s are captured here as negative supply shocks, and contribute to the inflation volatility at the beginning of the sample. Real demand shocks matter also for the dynamics of inflation volatility and persistence. However, their role is quite modest.
4.2. The transmission of monetary policy shocks

It is interesting to examine how the variations observed in Fig. 5 decompose into variations in the transmission of policy shocks over time (for a given variance of the shocks) and variations in the volatility of policy shocks. Gambetti et al. (2008) have shown, in the context of a SVAR with time varying coefficients, that there are variations in the size and the shape of the responses to monetary shocks and in the variances of these shocks. What does our structural analysis tell us about this issue? Fig. 6 presents the responses to a normalized monetary policy shock in the 16 samples that we have considered. Hence, the variations that it displays only reflect changes in the structural parameters of the model and not changes in the variance of the policy shocks (these will be analyzed later on).

Overall, the shape of the responses to policy shock has not changed much over time: when monetary policy is tight, inflation and output fall. Quantitatively, the impact effect of monetary shocks is somewhat reduced over time.

One other feature of the responses deserves some commentary: the largest inflation response is always instantaneous. This may appear surprising in view of the conventional VAR wisdom. One should stress that much of the conventional wisdom is derived using restrictions which are inconsistent with the theory embedded in models like the one that we use and that when the restrictions that the theory imposes are used, no strong delayed response typically appears (see Canova and De Nicolò (2002)).

4.3. The rise and fall in the level of inflation

One crucial event that we would like our model to explain is the rise in the level of inflation in the 1970s and the subsequent fall in the 1980s. The conventional wisdom attributes these ups and downs to a changing monetary stance, which was lax in the 1970s and became tight in the 1980s. The analysis of the previous subsections is inconclusive since the statistics that we report are not designed to shed light on this issue. Here we present the results of a historical decomposition exercise where we take mean posterior estimates of the parameters in four samples (1961–1978, 1965–1982, 1967–1984, 1969–1986), project the implied path of inflation out-of-sample, and ask whether and by how much policy shocks account for the deviation between the actual and
the projected path. Formally, we decompose inflation in terms of realized monetary policy shocks,

$$\hat{\pi}_T = SA_3(\bar{\eta}_T) \left( \sum_{m=1}^{\bar{T}} [A_1(\bar{\eta}_T)]^{\bar{T}-m} A_2(\bar{\eta}_T)\xi^*_m \right)$$

(18)

where $S$ is a selection matrix that picks inflation out of the vector of observable variables, and $\bar{\eta}_T$ is the mean posterior estimate using the information up to $T$. $[\xi^*_m]_{m=1}^{\bar{T}}$ is the set of realized monetary policy shocks in the sample considered and $\xi^*_m = [0, 0, 0, 0, 0, 0, 0, 0]$. Then, we project inflation out-of-sample, conditionally on time $T$ information,

$$\hat{\pi}_{T+k} = E_T(\hat{\pi}_{T+k}) = SA_3(\bar{\eta}_T) [A_1(\bar{\eta}_T)]^k \times \left( \sum_{m=1}^{\bar{T}} [A_1(\bar{\eta}_T)]^{\bar{T}-m} A_2(\bar{\eta}_T)\xi^*_m \right).$$

(19)

Fig. 7, which presents the actual (de-meaned) and the counterfactual inflation paths, displays interesting aspects. The sequence of policy shocks that materialized after 1978 accommodative in these periods. This pattern seems to change as we conduct, apart from being informative about the good luck–good policy debate, is also less open to the Lucas critique since we are manipulating only the shocks and not the structural coefficients.

We compute the realized innovations of the sample that goes from 1967q1 to 1984q1 using $\xi^{70} = A_2^{-1}(\eta^{70})(x_{t+1} - A_1(\eta^{70})x_t)$, where $\eta^{70}$ is the mean of the posterior distribution of the sample [1967q1–1984q1]. We then simulate inflation assuming that the economy evolves according to the estimated structure during the 1990s but is fed with shocks different than those observed in the 1990s. For example, when the shocks of the 1970s are fed in,

$$x'_{t+1} = A_1(\eta^{90})x^*_t + A_2(\eta^{90})\xi^{70}_{t+1}$$

where $\eta^{90}$ is the mean of the posterior distribution of the sample [1987q1–2006q1].

Fig. 8 plots the counterfactual paths that we generate and the actual path of inflation during the ‘Great Moderation’ period. The figure indicates that the counterfactual path for inflation would have displayed more volatile and slightly more persistent fluctuations than the actual ones of the 1990s. Thus, the intensity of the shocks matters quite a lot.

4.5. Summary

The analysis of this section has shown that the model captures the time profile of inflation volatility quite well and attributes parts of the changes to monetary policy shocks. Nevertheless, policy shocks do not have a predominant role in the decline: in fact, the transmission of policy disturbances is broadly unchanged over time. Hence, changes in the structural parameters somewhat average out and do not amplify changes in the variance of the monetary shocks. On the other hand, the rise and fall in the level of inflation appear to be linked to the sign and, to some extent, the magnitude of realized policy shocks. Without these shocks, the ups and downs of inflation would have been much smaller. Finally, the ‘Great Moderation’ episode seems to be more likely linked with the intensity of the shocks rather than changes in the monetary policy behavior. Overall, the evidence suggests that changes in the transmission of shocks other than policy disturbances could be as or more important in explaining the changes in the autocovariance function of inflation. To understand what factors may have given these shocks an important role, we next turn to examining the parameter estimates that we obtain in different samples.

4.4. Bringing the shocks of the 1970s into the Greenspan era

What would have happened to inflation if we could bring the shocks prevailing in the 1970s into the ‘Great Moderation’ era? Would we still observe low inflation with small volatility? Typical counterfactuals are performed by moving the monetary policy rule of Alan Greenspan back to the 1970s, which is equivalent to asking whether monetary policy would have been able to stabilize the economy given the shocks and the economic structure of the 1970s. Here, we take a different stand. We would like to know whether the private sector and monetary policy would be able to absorb and mitigate the realized shocks of the 1970s. The exercise that we conduct, apart from being informative about the good luck–good policy debate, is also less open to the Lucas critique since we are manipulating only the shocks and not the structural coefficients.

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5. Rolling parameter estimates

Fig. 9 presents the evolution of the posterior mean of interesting (functions of the) parameters over different samples. For the sake of readability, we omit posterior credible sets—since they are relatively tight, the variations that we present are typically a posteriori significant.

Consistent with the analysis of Clarida et al. (2000), the figure shows that the short run policy coefficient for inflation increases in the samples which mainly include the 1980s. Interestingly, and consistently with the structural VAR evidence of Canova and Gambetti (2009), the posterior mean estimate falls somewhat in the latest samples. Hence, while Greenspan was tougher than Burns in fighting inflation, the posterior distributions which include only the 1990s and the beginning of the 2000s show a reduced short run concern for inflation. This contrasts with the dynamics displayed by the long run coefficient for nominal variables \((1 - \lambda_r)^{-1}(\lambda_\pi + \lambda_m)\). Here a clear decreasing pattern emerges with a peak located at the beginning of the sample. Note that the coefficient for the output (not shown here) is stable over time and shows no sign of increase in correspondence to the drop in the long run coefficient of nominal aggregates.

The posterior mean of the standard deviation of the policy shock \(\sigma_r\) shows some variations over time. As conventional wisdom suggests, there is an increase (to 0.035 from 0.015) in the samples from (1959–1976) to (1967–1984) but this decrease is reversed in the next few samples. This evidence is a bit surprising, but it is in line with the evidence produced by the Markov switching approach of Sims and Zha (2006), and the recursive analysis of Gambetti et al. (2008). It is worth mentioning that decreasing the length of the windows does not change this pattern: the increase present in the last few years of the sample is consistent with the more active role that the Fed has taken since 2001.

There are changes also in the posterior mean of other important parameters. For example, there are shifts in responsiveness of the output to real balances (see the real balance trade-off, \(\omega_2/\omega_1\)) and in the responsiveness of inflation to the marginal cost (Phillips curve trade-off, \(\kappa_p\)). The fact that the real balance trade-off is increasing, coupled with a volatile but overall constant Phillips curve trade-off, indicates that real balances may have played different roles at different points in time. For example, it is possible that real balances behaved as close proxies of consumer purchasing power in the early samples while they proxy for segmented asset markets in later samples (see e.g. Alvarez and Lippi (2009)). Note that the price indexation mechanism \(\iota\) does not show any significant falls as we move through the sample.

Interestingly, many other standard deviations display significant time variations. For instance, the standard deviations of government spending, preference and money demand shocks increased at the beginning of the samples, and then steadily decreased, giving support to the idea that many structural shocks have been less intense in the latter part of the sample; see e.g. Gali and Gambetti (2009).

In sum, the parameter changes that we describe can explain why shocks other than monetary policy disturbances have an important role in explaining the variations in the volatility of inflation documented in Fig. 5. It is because output changes in response to these shocks are larger, and because changes in output are proportionally more important for explaining inflation changes (and because the size of these shocks fell) that we see a large decline in the volatility of inflation over time.

6. Conclusions

This paper examines the contribution of policy shocks to the dynamics of inflation using a medium scale structural model estimated with US post-WWII data and Bayesian techniques over rolling samples. The model belongs to the class of New Keynesian structures that have been extensively used in the literature but explicitly allows money to play a role. Bayesian techniques are preferable to standard likelihood methods or to indirect inference (impulse response matching) exercises, because the model that we consider is a false description of the DGP of the data and misspecification may be important. We show that our approach delivers interesting estimates of the structural parameters when priors are broadly non-informative and the policy reaction function appropriately chosen. We also demonstrate that the model fits the data reasonably well and that alternative structural specifications produce lower marginal likelihoods and fail to capture important aspects of the data.

Our model captures the fall in inflation volatility and attributes parts of the changes to monetary policy shocks. We detect level but not shape differences in the transmission of policy shocks which tend to make inflation and output somewhat less reactive to policy disturbances as time goes by. Finally, variations in the level of
inflation are qualitatively related to policy shocks: had those been absent, the rise of the 1970s and the fall of the 1980s would have been much more modest.

A number of structural changes drive these results. We find support for the conjecture that the Fed had a much stronger short run “dislike” for inflation in later samples but also notice that the long run coefficient on nominal variables has been steadily decreasing. We detect, in agreement with the good luck hypothesis, variations in the posterior mean estimate of the variance of the policy shocks. Perhaps surprisingly, we find that these variations do not change the way in which monetary shocks are transmitted to the economy.

In sum, several causes in combination are responsible for the changes in the level and the autocovariance function of US inflation over the last forty years: changes in the variance of the shocks, in the parameters regulating the private sector and in the policy rule all contributed to a greater or lesser extent to explaining why inflation rose and fell and why inflation variability subsided. These conclusions are rather different than those present in the literature, with the exception of Gambetti et al. (2008).

There are a number of ways in which our analysis can be extended. For example, the estimation approach that we employ treats expectations as latent variables. However, measures of output and inflation expectations do exist in the literature. While these proxies are probably contaminated with measurement error, it would be interesting to see whether they provide additional or contrasting information about the issues at stake. Similarly, it is important to consider additional statistics to the evaluation process: while the model seems by and large well specified, it may not capture the time profile of the dynamics of a particular variable well. Finally, the use of alternative rolling estimation techniques, such as those employed by Kapetanios and Yates (2008), can help us to understand whether the conclusions are also robust along this dimension. We leave all these extensions to future research.

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Appendix

The Smets and Wouter model with a non-separable utility function.

In this section, we derive the equations of the model that differ from those of the model of Smets and Wouters (2007), henceforth SW. Households have a (non-separable in money) utility function of the form

\[ U \left( C_t - hC_{t-1}, \frac{M_t}{e_t P_t}, L_t \right) = U \left( C_t - hC_{t-1}, \frac{M_t}{e_t P_t} \right) - V(L_t). \]

Let \( m_t = M_t / P_t \). The budget constraint in real terms is

\[ m_t + C_t + L_t \cdot \frac{B_t}{P_t} + \frac{B_t}{P_t} \frac{1}{\sigma_t} + T_t = m_{t-1} \frac{1}{\sigma_t} + T_{t-1} \frac{1}{\sigma_t} + \frac{B_{t-1}}{P_{t-1}} \frac{1}{\sigma_{t-1}} + \frac{K_{t-1}}{P_{t-1}} - a(Z_t)K_{t-1} + \frac{D^{\phi u}_{t}}{P_{t}} + \frac{D^{\psi u}_{t}}{P_{t}}, \]

where \( D^{\phi u}_{t} \) and \( D^{\psi u}_{t} \) are dividends paid by the intermediate sector firms and labor unions. As in SW, labor unions pay a fixed wage, \( w^{HH}_{t} \), and differentiate HH labor and sell it to labor packers in a monopolistic competitive environment (profits are redistributed to HH in terms of dividends). Labor packers then sell “packed” labor to intermediate sector firms in a perfectly competitive market. The capital accumulation law of motion is

\[ K_{t} = (1 - \delta)K_{t-1} + \epsilon_t \left( 1 - S \left( \frac{h}{k_{t-1}} \right) \right) L_t. \]

The production side and the wage setting are identical to those of SW. Also, the government budget constraint is similar with the only exception being money, i.e.

\[ G_t + \frac{B_t}{P_{t-1}} \frac{1}{\sigma_t} + m_{t-1} \frac{1}{\sigma_t} = T_t + \frac{B_t}{e_t R_t P_t} + m_t. \]

The feasibility constraint is obtained simply by integrating across households and adding the government budget constraint, i.e.

\[ G_t + C_t + L_t + a(Z_t)K_{t-1} = Y_t. \]

For the steady state, SW assume that \( S(1) = 0, S'(1) = 0, Z = 1 \) and \( a(Z) = a(1) = 0 \). The steady state values are \( R = 1/\beta \sigma, r^G = 1/\beta - 1 + \delta, r^c = a'(1), mc = \frac{a'(1-a')\omega_1}{\sigma(\epsilon^c + \epsilon_1)} \frac{1}{\beta \sigma}, \]

\[ L/Y = \frac{1-a}{w}, K/Y = \frac{\alpha}{1-\beta-1+\delta}, \Psi = (K/Y)^\alpha (L/Y)^{1-\alpha}. \]

From the first-order condition for consumption optimization we obtain that

\[ U_t \left( C_t - hC_{t-1}, \frac{M_t}{e_t P_t} \right) = \theta_t. \]

We rewrite the right hand side of this equation as

\[ U_t \left( \tilde{C}_t, \tilde{m}_t \right) = U_t(t) \]

where \( \tilde{C}_t = C_t - hC_{t-1} \) and \( \tilde{m}_t = \frac{m_t}{\tilde{e}_t} \) and \( m_t = \frac{M_t}{\tilde{e}_t P_t} \). The partial derivatives are

\[ \frac{\partial U_t(t)}{\partial \tilde{C}_t} = \frac{\partial U_t(t)}{\partial \tilde{C}_t} \frac{\partial \tilde{C}_t}{\partial \tilde{m}_t} = \frac{\partial U_t(t)}{\partial \tilde{C}_t} \]

\[ \frac{\partial U_t(t)}{\partial \tilde{C}_{t-1}} = \frac{\partial U_t(t)}{\partial \tilde{C}_{t-1}} \frac{\partial \tilde{C}_{t-1}}{\partial \tilde{m}_{t-1}} = \frac{\partial U_t(t)}{\partial \tilde{C}_t} \]

\[ \frac{\partial U_t(t)}{\partial \tilde{m}_t} = \frac{\partial U_t(t)}{\partial \tilde{m}_t} \frac{\partial \tilde{m}_t}{\partial m_t} = \frac{\partial U_t(t)}{\partial \tilde{m}_t} \]

\[ \frac{\partial U_t(t)}{\partial \tilde{e}_t} = \frac{\partial U_t(t)}{\partial \tilde{e}_t} \frac{\partial \tilde{e}_t}{\partial \tilde{e}_t} = \frac{\partial U_t(t)}{\partial \tilde{e}_t}, \]

At the steady state (\( e = 1 \)), the partial derivatives are

\[ \frac{\partial U_t}{\partial \tilde{C}_t} = \frac{\partial U_t}{\partial \tilde{C}_t} = U_{\tilde{C}_t} \]

\[ \frac{\partial U_t}{\partial \tilde{C}_{t-1}} = \frac{\partial U_t}{\partial \tilde{C}_{t-1}} = hU_{\tilde{C}_t} \]

\[ \frac{\partial U_t}{\partial \tilde{m}_t} = \frac{\partial U_t}{\partial \tilde{m}_t} = U_{\tilde{m}_t} \]

\[ \frac{\partial U_t}{\partial \tilde{e}_t} = \frac{\partial U_t}{\partial \tilde{e}_t} = U_{\tilde{e}_t}. \]

The log-linearized version of the condition is

\[ U_{\tilde{C}_t} \tilde{C}_t - hU_{\tilde{C}_t} \tilde{C}_{t-1} + U_{\tilde{m}_t} \tilde{m}_t - U_{\tilde{e}_t} \tilde{e}_t = \theta_t. \]

Since from the steady state equation we have that \( \theta = U_c \), the latter can be rewritten as

\[ \omega_1 \tilde{e}_t = -\tilde{c}_t + h\tilde{c}_{t-1} - \omega_2 \tilde{m}_t + \omega_3 \tilde{e}_t \]

where \( \omega_1 = -\frac{U_{\tilde{C}_t}}{U_c} \) and \( \omega_3 = \frac{U_{\tilde{e}_t}}{U_c} \).

From the optimality condition with respect to \( L_t \), we obtain

\[ V_t(L_t) = -\theta_t W_t^{HH}/P_t = -\theta_t w_t^{HH}. \]
where $W_t^{HH}$ is the wage paid by the unions. In the steady state,

$$V_t = -\theta W_t^{HH}/P.$$ 

Thus, the log-linear version of this optimality condition is

$$V_t \tilde{U}_t = -\theta U_t^{HH} \tilde{U}_t - \theta U_t^{HH} \tilde{V}_t^{HH}$$

$\omega_3 \tilde{U}_t = \tilde{h}_t + \tilde{w}_t^{HH}$

where $\omega_3 = -\frac{V_3}{V_0}$.

Finally, maximization with respect to $m_t = M_t/P_t$ and $b_t = B_t/P_t$ leads to

$$U_m \left( C_t - hC_{t-1}, \frac{M_t}{\varepsilon P_t} \right) = \theta_t - \beta \varepsilon_t \left( \theta_{t+1} \frac{1}{\pi_{t+1}} \right)$$

$$\theta_t - \frac{1}{\varepsilon_t R_t} = \beta \varepsilon_t \left( \theta_{t+1} \frac{1}{\pi_{t+1}} \right)$$

and combining the two we obtain

$$U_m \left( C_t - hC_{t-1}, \frac{M_t}{\varepsilon P_t} \right) = \theta_t \left[ 1 - \frac{1}{R_t \varepsilon_t} \right].$$

In the steady state,

$$U_m \left( C(1-h), \frac{M}{P} \right) = \theta \left[ 1 - \frac{1}{R} \right].$$

Log-linearizing we have

$U_m C \tilde{C}_t - hU_m C \tilde{C}_{t-1} + U_{mm} \tilde{m}_t - U_{mm} \tilde{m}_t$

$= \theta \left[ 1 - \frac{1}{R} \right] \tilde{\theta}_t + \theta \tilde{r}_t + \theta \tilde{\epsilon}_t$

$U_{mc} C \tilde{C}_t - hU_{mc} C \tilde{C}_{t-1} + U_{mm} \tilde{m}_t - U_{mm} \tilde{m}_t$

$= \theta \left[ 1 - \frac{1}{R} \right] \tilde{\theta}_t + \theta \tilde{r}_t + \theta \tilde{\epsilon}_t$

and dividing by $U_m$ we get

$$U_{mc} C \tilde{C}_t - hU_{mc} C \tilde{C}_{t-1} + U_{mm} \tilde{m}_t - U_{mm} \tilde{m}_t = \theta \left[ 1 - \frac{1}{R} \right] \tilde{\theta}_t + \theta \tilde{r}_t + \theta \tilde{\epsilon}_t$$

$= \theta_t + \frac{1}{R} \tilde{r}_t + \frac{1}{R} \tilde{\epsilon}_t - \omega_3 \tilde{C}_t + h \omega_3 \tilde{C}_{t-1}$

$= \omega_3 t + \omega_4 \tilde{C}_t = \tilde{\theta}_t + \frac{1}{R} \tilde{r}_t + \frac{1}{R} \tilde{\epsilon}_t$

where $\omega_3 = -\frac{\partial U_m}{U_m}$ and $\omega_4 = -\frac{mU_{mm}}{U_m}$.

The remaining equations are the same as in SW.

References


