Household Behaviour and Property Division upon Divorce (Joint Saving with Endogenous Separation)

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Motivation

- Two-sided limited commitment with dynamic contracts is a popular framework to study risk sharing.
  - Applications: marriage, partnerships, economic unions

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- Two-sided limited commitment with dynamic contracts is a popular framework to study risk sharing.
  - Applications: marriage, partnerships, economic unions
- Most of the relevant theoretical literature does not feature neither separations along the optimal path, nor asset accumulation by the 'couple.'
  - For empirical applications these features are relevant.

Motivation – household saving

Modern macroeconomics considers optimising households who decide on: consumption and saving, labour supply, human capital investment, fertility

Representative household or some heterogeneity, e.g. in age (life-cycle models), income and wealth (Aiyagari), but still one decision-maker

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We aim to build **a theory of household saving** (and other aggregate variables later) **taking into account the possibility of separation/divorce** (and in light of other observed changes to the family)

What we do in this paper

- (For now:) We characterise some key properties of the allocation.
- First paper to study formally jointly two-sided lack of commitment, efficient separations, and asset accumulation.
- Based on Ábrahám and Laczó (2016).

  Two key differences:
  - Constrained-efficient separations through shocks to individual match value.
  - Study asset division rules upon default.

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  Two key differences:
  - Constrained-efficient separations through shocks to individual match value.
  - Study asset division rules upon default.
- Questions:
  - How separations and asset division rules affect risk sharing?
  - How asset accumulation incentives change with these new features?
  - Should agents who file for divorce be penalised in the property division rule? Should divorce be costly?

What we do in this paper – key trade-off

The possibility of separations makes the role of asset division rules non-trivial.

- More assets upon default make default more attractive. ⇒ Less assets should be provided for agents with high default incentives.
- More assets upon default may help consumption smoothing through separations. ⇒ Assets should be provided for those agents who would have low consumption after separation.

Literature

- Two-sided lack of commitment models with or without storage: Thomas and Worrall (1988), Kocherlakota (1996), Ábrahám and Laczó (2016)

- Applications of limited-commitment models to couples: Mazzocco (2007), Mazzocco, Ruiz, and Yamaguchi (2013), Voena (2015), Lise and Yamada (2015), Bayot and Voena (2015), Low, Meghir, Pistaferri, and Voena (2016); survey by Chiappori and Mazzocco (2015)

- Other relevant applications of limited commitment models: risk sharing in economic unions/between countries (Kehoe, and Perri, 2012), employer and employee (Thomas and Worrall, 1988), partnerships

Setup

Our starting point is the lack of commitment framework of Kocherlakota (REStud, 1996) extended for storage by Ábrahám and Laczó (2016).

- Two infinitely lived, risk-averse, ex-ante identical agents. Utility is unbounded below.
- Income y has discrete support and is perfectly negatively correlated across the two agents, i.e., there is no aggregate risk in the sense that the aggregate endowment Y is constant.
- **Agents also face match-specific individual 'love' shocks.** Love shocks are perfectly negatively correlated and have mean zero. $\phi()$ and $\Psi()$ are the continuous density and CDF, respectively.
- We consider a storage/saving technology with exogenous return $-1 \leq r \leq 1/\beta - 1$. Borrowing is not allowed.
- Storage B' is decided (constrained-)efficiently by the couple.

‘Divorce’

- Agents can leave the relationship at any time if they find it optimal.
- They live in autarky permanently after divorce, where they can use the storage technology.
- We assume that the property division rule can condition on income, but not on the value of the love shock, at most its sign. More precisely, upon divorce the accumulated assets of the couple are distributed as follows:
    - $\gamma\delta$ share is given to the agent who files for divorce/is at fault (the one with $\alpha < 0$ when divorce happens), $0 \leq \gamma \leq 0.5$, and $0 \leq \delta \leq 1$ ($1 - \delta$ is the cost of divorce).
    - $(1 - \gamma)\delta$ share is given to the other party.
- These assumptions on love shocks prevent consensual divorce.

The first best

- Assume equal Pareto weights of the two partners.
- The first-best solution of this model implies perfect risk sharing (equal and constant consumptions), no separations, and no storage.
- This is because storage is inefficient and love shocks are additive and perfectly negatively correlated.

Limited commitment

- Large enough negative love shocks imply that participation constraints are violated for any income distribution.
- Therefore, limited commitment can lead to constrained-efficient separations.

Preliminary characterisation

- For each income y and asset level B, there exists a unique threshold $\alpha^*(B, y) > 0$ such that if an agent's love shock is $\alpha < -\alpha^*(B, y)$, then he files for divorce.
- For the same agent, if $\alpha > \alpha^*(B, Y - y)$, then the other agent files for divorce.
- $\frac{\partial \alpha^*(B,y)}{\partial y} < 0$, i.e., separations thresholds get tighter with income.

Below we define these thresholds formally.

Intuition

- For negative love shocks agents need to be compensated by current consumption and future utility.
- Future utility is limited by participation constraints and increasing current consumption reduces the consumption of the partner.
- For α high enough in absolute value, both participation constraints cannot be satisfied simultaneously.
- Higher income increases the autarky value, hence the separation threshold becomes tighter.
- The capacity to compensate agents for adverse love shocks depends on aggregate assets.

Recursive formulation

$$V(U,B,y,\alpha) = \max_{w(y',\alpha'),B',c} u(c) + \alpha$$

$$+ \beta \left[\sum_{y'} \pi(y') \left(\int_{-\alpha^*(B',y')}^{\alpha^*(B',Y-y')} \phi(\alpha') V\left(w\left(y',\alpha'\right), B', y', \alpha'\right) d\alpha' \right.\right.$$

$$\left.\left. + \int_{-\infty}^{-\alpha^*(B',y')} \phi(\alpha') V^D\left(\gamma\delta B', y'\right) d\alpha' + \int_{\alpha^*(B',Y-y')}^{\infty} \phi(\alpha') V^D\left((1-\gamma)\delta B', y'\right) d\alpha' \right) \right].$$

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Promise-keeping constraint:

$$U \le u(Y - c + (1+r)B - B') - \alpha + \beta \left[\sum_{y'} \pi(y') \left(\int_{-\alpha^*(B',y')}^{\alpha^*(B',Y-y')} \phi(\alpha') w(y',\alpha') \, d\alpha' \right.\right.$$
$$\left.\left. + \int_{-\infty}^{-\alpha^*(B',y')} \phi(\alpha') V^D((1-\gamma)\delta B', Y-y') \, d\alpha' + \int_{\alpha^*(B',Y-y')}^{\infty} \phi(\alpha') V^D(\gamma\delta B', Y-y') \, d\alpha' \right) \right]$$

[1] U is the current and $w(y',\alpha')$ is tomorrow's state-contingent life-time utility promised to the partner.

Recursive formulation (cont.)

Participation constraints:

$$
\begin{aligned}
& w\left(y', \alpha'\right) \geq V^D\left(\gamma\delta B', Y - y'\right) \\
\text{and} \quad & V\left(w\left(y', \alpha'\right), B', y', \alpha'\right) \geq V^D\left(\gamma\delta B', y'\right) \\
& \quad \forall\left(y', \alpha'\right) \text{ such that } -\alpha^*\left(B', y'\right) \leq \alpha' \leq \alpha^*\left(B', Y - y'\right)
\end{aligned}
$$

The outside option is

$$
V^D(b, y) = \max_{b'} u\left(y + (1+r)\, b - b'\right) + \beta \sum_{y'} \Pr\left(y'\right) V^D\left(b', y'\right).
$$

The threshold $\alpha^*(B, y)$ is implicitly defined as:

$$
V\left(V^D\left(\gamma\delta B, Y - y\right), B, y, -\alpha^*(B, y)\right) = V^D\left(\gamma\delta B, y\right)
$$

The effect of storage

- Provides consumption smoothing benefits while the couple stays together.
- Increases default incentives as long as $\gamma\delta > 0$.
- Provides consumption smoothing benefits along separation for both agents if $\gamma\delta > 0$, and only one agent if $\gamma = 0$ and $\delta > 0$.
- Affects the separation margin. Higher assets make the separation thresholds looser if $\gamma\delta$ is low.
  - Intuition: The agent who files for divorce loses access to a large chunk of the assets after divorce, hence his/her divorce incentives are reduced.

No love shocks ($\alpha = 0$ always), and $\gamma\delta = 0$

- Ábrahám and Laczó (2016).
- For sufficiently high return on storage ($\beta(1+r) \leq 1$), storage takes positive values in equilibrium.
- Similar results for risk sharing as above, but as long as assets change over time, persistence and amnesia do not hold.

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- For sufficiently high return on storage ($\beta(1+r) \leq 1$), storage takes positive values in equilibrium.
- Similar results for risk sharing as above, but as long as assets change over time, persistence and amnesia do not hold.
- Euler inequality for asset accumulation:

$$u'(c) \geq \beta(1+r) \sum_{y'} \pi(y') (1 + \mu_1(B', y')) u'(c'(B', y')),$$

where $\mu_1(B', y')$ is the Lagrange multiplier on agent 1's participation constraint tomorrow in state y'.

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- The couple accumulates assets because
  - it allows for better consumption smoothing, and
  - it reduces future default incentives.

No love shocks ($\alpha = 0$ always), $\gamma\delta \geq 0$

- Now default incentives are higher as $V^D(\gamma\delta B, Y)$ increases with $\gamma\delta$.
- Less risk sharing is implementable.

The Euler equation for asset accumulation is

$$u'(c) \geq \beta(1+r)\sum_{y'}\pi(y')\left[(1+\mu_1(B',y'))u'(c'(B',y'))\right.$$
$$\left. -\delta\underbrace{\{\mu_1(B',y')\gamma u'(c^{au}(\gamma\delta B',y')) + x'\mu_2(B',y')\gamma u'(c^{au}(\gamma\delta B',Y-y'))\}}_{>0}\right],$$

x' is the Lagrange multiplier on the promise-keeping constraint, and $x'\mu_2(B',y')$ is the Lagrange multiplier on agent 2's participation constraint tomorrow in state y'.

- In this environment $\gamma\delta = 0$ maximises ex-ante welfare.

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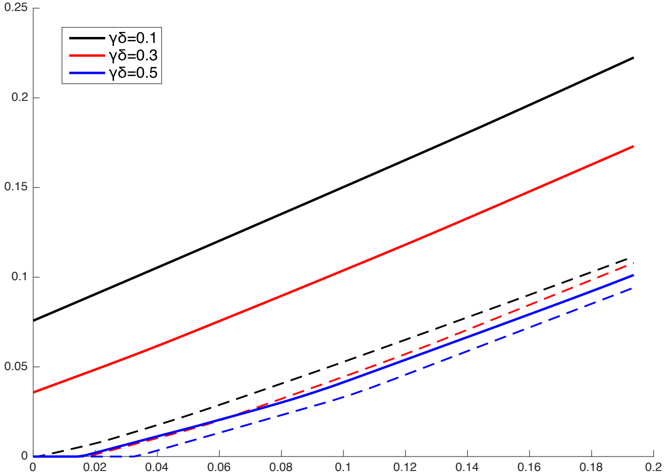
- In this environment $\gamma\delta = 0$ maximises ex-ante welfare.
- Asset accumulation incentives are reduced because of the autarky effects.
- However, reduced risk sharing increases the ‘precautionary motive.’

No love shocks – effect of $\gamma\delta$ on storage

Coef. of RRA=1.5, incomes: 0.329, 0.443, 0.557, 0.671 i.i.d. with equal probabilities, $r = 0.06$, $\beta = 0.89$; B' as a function of B in the more unequal state:



Storage, separations, any γ and δ

$$
\begin{aligned}
u'(c) = {} & \beta(1+r)\sum_{y'}\pi(y')\Bigg[\int_{-\alpha^*(B',y')}^{\alpha^*(B',Y-y')}\phi(\alpha')\{(1+\mu_1(y',B',\alpha'))u'(c(w(y',\alpha'),B',y',\alpha')) \\
& -\delta\underbrace{[\mu_1(B',y')\gamma u'(c^{au}(\gamma\delta B',y'))+x'\mu_2(B',y')\gamma u'(c^{au}(\gamma\delta B',Y-y'))]}_{>0}\}d\alpha' \\
& +\underbrace{\Psi(-\alpha^*(B',y'))}_{\text{prob. agent 1 files}}\underbrace{[\gamma\delta u'(c^{au}(\gamma\delta B',y'))+x'(1-\gamma)\delta u'(c^{au}((1-\gamma)\delta B',Y-y'))]}_{>0} \\
& +\underbrace{(1-\Psi(\alpha^*(B',Y-y')))}_{\text{prob. agent 2 files}}\times\underbrace{[(1-\gamma)\delta u'(c^{au}((1-\gamma)\delta B',y'))+x'\gamma\delta u'(c^{au}(\gamma\delta B',Y-y'))]}_{>0} \\
& -\frac{1}{1+r}\phi(\alpha^*(B',Y-y'))\frac{\partial\alpha^*(B',Y-y')}{\partial B'}\left[V^D((1-\gamma)\delta B',y')-V^D(\gamma\delta B',y')\right] \\
& -\frac{x'}{1+r}\phi(-\alpha^*(B',y'))\frac{\partial(\alpha^*(B',y'))}{\partial B'}\left[V^D((1-\gamma)\delta B',Y-y')-V^D(\gamma\delta B',Y-y')\right]\Bigg]
\end{aligned}
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Storage, separations, any γ and δ

$$u'(c) = \beta(1+r)\sum_{y'}\pi\left(y'\right)\Bigg[\int_{-\alpha^*(B',y')}^{\alpha^*(B',Y-y')}\phi\left(\alpha'\right)\{(1+\mu_1\left(y',B',\alpha'\right))u'\left(c\left(w\left(y',\alpha'\right),B',y',\alpha'\right)\right)$$

$$-\delta\underbrace{\left[\mu_1(B',y')\,\gamma u'\left(c^{au}\left(\gamma\delta B',y'\right)\right)+x'\mu_2(B',y')\,\gamma u'\left(c^{au}\left(\gamma\delta B',Y-y'\right)\right)\right]}_{>0}\}d\alpha'$$

$$+\underbrace{\Psi\left(-\alpha^*\left(B',y'\right)\right)}_{\text{prob. agent 1 files}}\underbrace{\left[\gamma\delta u'\left(c^{au}\left(\gamma\delta B',y'\right)\right)+x'(1-\gamma)\delta u'\left(c^{au}\left((1-\gamma)\delta B',Y-y'\right)\right)\right]}_{>0}$$

$$+\underbrace{\left(1-\Psi\left(\alpha^*\left(B',Y-y'\right)\right)\right)}_{\text{prob. agent 2 files}}\times\underbrace{\left[(1-\gamma)\delta u'\left(c^{au}\left((1-\gamma)\delta B',y'\right)\right)+x'\gamma\delta u'\left(c^{au}\left(\gamma\delta B',Y-y'\right)\right)\right]}_{>0}$$

$$-\frac{1}{1+r}\phi\left(\alpha^*\left(B',Y-y'\right)\right)\frac{\partial\alpha^*\left(B',Y-y'\right)}{\partial B'}\underbrace{\left[V^D\left(\left(1-\gamma\right)\delta B',y'\right)-V^D\left(\gamma\delta B',y'\right)\right]}_{>0}$$

$$-\frac{x'}{1+r}\phi\left(-\alpha^*\left(B',y'\right)\right)\frac{\partial\left(\alpha^*\left(B',y'\right)\right)}{\partial B'}\underbrace{\left[V^D\left(\left(1-\gamma\right)\delta B',Y-y'\right)-V^D\left(\gamma\delta B',Y-y'\right)\right]}_{>0}\Bigg]$$

Storage, separations, $\delta = 0$, $\gamma = \frac{1}{2}$

- Remember that $1 - \delta$ can be thought of as the cost of divorce.
- This parameter may especially play a prominent role when conditioning the property-division rule on who is at fault for divorce is not possible.

The Euler simplifies to

$$u'(c) \geq \beta(1+r) \sum_{y'} \pi(y') \int_{-\alpha^*(B',y')}^{\alpha^*(B',Y-y')} \phi(\alpha') \times (1 + \mu_1(B',y',\alpha')) u'(c(w(y',\alpha'),B',y',\alpha'))\, d\alpha'.$$

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- $\delta = 0$ makes the participation constraints the tightest possible (as $\gamma = 0$).
- It minimises the consumption smoothing benefits of assets across marital statuses for both the spouse who files for divorce and the other spouse.
- No inefficient marriages.

Storage, separations, $\delta = 1$, $\gamma = \frac{1}{2}$

$$
\begin{aligned}
u'(c) = \beta(1+r)\sum_{y'}\pi\left(y'\right)\Bigg[\int_{-\alpha^*(B',y')}^{\alpha^*(B',Y-y')}\phi\left(\alpha'\right)\Big\{\left(1+\mu_1\left(y',B',\alpha'\right)\right)u'\left(c\left(w\left(y',\alpha'\right),B',y',\alpha'\right)\right) \\
-\frac{1}{2}\underbrace{\left[\mu_1\left(y',B',\alpha'\right)u'\left(c^{au}\left(\frac{1}{2}B',y'\right)\right)+x'\mu_2\left(y',B',\alpha'\right)u'\left(c^{au}\left(\frac{1}{2}B',Y-y'\right)\right)\right]}_{>0}\Big\}d\alpha' \\
+\underbrace{\Psi\left(-\alpha^*\left(B',y'\right)\right)}_{\text{prob. agent 1 files}}\frac{1}{2}\left[u'\left(c^{au}\left(\frac{1}{2}B',y'\right)\right)+x'u'\left(c^{au}\left(\frac{1}{2}B',Y-y'\right)\right)\right] \\
+\underbrace{\left(1-\Psi\left(\alpha^*\left(B',Y-y'\right)\right)\right)}_{\text{prob. agent 2 files}}\frac{1}{2}\left[u'\left(c^{au}\left(\frac{1}{2}B',y'\right)\right)+x'u'\left(c^{au}\left(\frac{1}{2}B',Y-y'\right)\right)\right]\Bigg]
\end{aligned}
$$

Storage, separations, $\gamma = \frac{1}{2}$, $\delta = 0$ vs. $\delta = 1$

Comparing the Eulers for $\delta = 1$ and $\delta = 0$, given $\gamma = \frac{1}{2}$, highlights the main trade-off:

- Providing insurance for spouses while they are married, which favours $\delta = 0$.
- Letting them better smooth in case of divorce, which favours $\delta = 1$.

Balancing these two goals requires $0 < \delta < 1$.

Storage, separations, optimal γ and δ

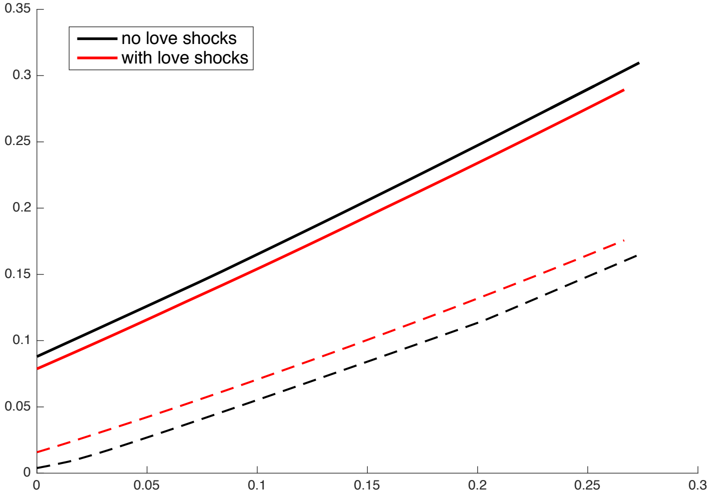
γ and δ play different roles, and both may be useful if one wanted to maximise a couple's welfare.

- A low γ, unlike a low δ improves consumption smoothing for the spouse whose partner files for divorce
- Both help enforce more risk sharing between spouses while they are married
- A γ different from $\frac{1}{2}$ implies that some marriages will be inefficient, unlike lowering δ from 1.

Therefore, at least for some parameter values, an optimal property-division rule shall involve $0 < \gamma < \frac{1}{2}$ **and** $0 < \delta < 1$.

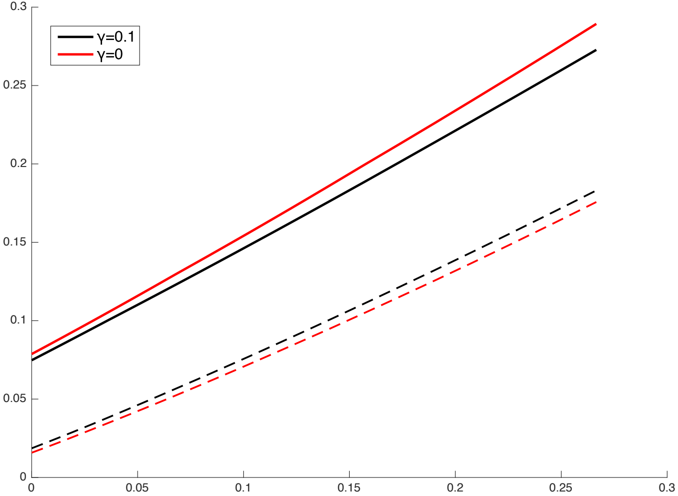
Asset-division rules and asset accumulation

B' as a function of B without and with love shocks, $\gamma = 0$, but no separations



Asset-division rules and asset accumulation

B' as a function of B with love shocks but no separations, two different γ's



Summary and Future Research

Summary:

- Characterisation of limited commitment, storage, separations, and property division rules.
- In the presence of asset accumulation, there is a trade-off between risk sharing within the match, consumption smoothing across 'marital statuses', and efficient separations.
- Optimal property division rules need to balance these effects.

Future research:

- Extend the model for empirically more plausible shock processes and outside options.
- Consider inequality within the household and alimony-type payments
- Work out a quantitative application considering changes in UK divorce law or cross-state legal differences in the US.
- Consider other household decisions such as labour supply.