THE EFFICIENT ALLOCATION OF CONSUMPTION UNDER MORAL HAZARD AND HIDDEN ACCESS TO THE CREDIT MARKET

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Abstract
In this paper, we describe the properties of the optimal allocation of consumption in a world with moral hazard and hidden borrowing and lending. We discuss how and under what conditions the efficient allocation can be distinguished from that of the permanent income (self-insurance) model. We also compare our allocation with the complete markets (full information) case, and with the standard moral hazard model with monitorable and fully contractible asset holdings. (JEL: D82, E21)

1. Introduction

There is a large literature that studies optimal long-term contracts under moral hazard assuming that agents cannot buy or sell assets (or equivalently assuming that their financial positions are perfectly observable and it is possible to contractually restrict the acquisition of additional assets and liabilities by the agents). In particular, Rogerson (1985) shows that preventing the agent from entering the credit market is critical—even in the presence of borrowing constraints—since in the optimal contract, the agent is actually willing to save. On the other hand, the observability and contractability of the agent's wealth and consumption cannot be guaranteed in many situations. For example, in transitional and developing countries agents often use foreign currency, gold, or some other forms of storage of value for self-insurance. These forms of asset accumulation are typically not observable by other agents or institutions. In other cases, agents can have access to domestic or foreign accounts and credit lines where they can save and borrow secretly.

Therefore, relaxing the assumption of perfect observability and contractability on agents' asset holdings and analyzing the resulting optimal allocation is potentially a very valuable exercise from both the theoretical and applied point

Acknowledgments: Nicola Pavoni thanks the Spanish Ministry of Science and Technology. Grant BEC2001-1653.

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Journal of the European Economic Association April–May 2005 3(2–3):370–381

of view. However, previous literature suggests that this study could be uninteresting. In particular, Allen (1985) and Cole and Kocherlakota (2001) (ACK) show that in a pure endowment economy with adverse selection (hidden information moral hazard) the efficient allocation does not differ from that in a pure bond economy, i.e., where the agents can insure themselves through borrowing and lending, without the opportunity of further risk sharing. On the other hand, Ábrahám and Pavoni (2004a) (APa) show that within the class of moral hazard models this is generically not the case. There, it is shown that, as long as the agent has imperfect control over publicly observable (income) histories, the efficient allocation differs from the self-insurance allocation.

This paper analyzes the implications of the efficient contracting model for households' consumption behavior in a stochastic environment with moral hazard, where individuals can secretly borrow and lend at a given risk-free interest rate. We compare the optimal allocation of consumption in this case with the self-insurance allocation and with other benchmark models, such as the complete market (full information) framework, and the standard moral hazard model with fully observable and contractible assets.

2. Model

Although business cycle theory provides an important motivation for the study of households' consumption decision, we simplify our analysis by abstracting from aggregate economic disturbances and focus on how consumption reacts to idiosyncratic risk.

Consider a small open economy consisting of a large number of agents that are exante identical, and who each live for $T \leq \infty$ periods. At period t, each agent can take an unobservable effort level $e_t \geq 0$ which affects the probability distribution of the publicly observable individual income level $y_{t+1} \in Y = (y^1, \dots, y^N)$, with $y^i < y^{i+1}$. The conditional probabilities over Y are defined by $p_i(e_t) = \Pr\{y_{t+1} = y^i | e_t\} > 0$. The history of public outcomes up to period t will be denoted by $h^t \equiv (y_t, \dots, y_t)$.

Agents are allowed to buy and short-sell a one period risk-free bond that pays a constant interest rate $r \geq 0$. Their asset holdings are private information, and we assume that each agent is born with no wealth.[1] However, since income processes of distinct agents are not perfectly correlated, agents might enhance welfare by trading additional potentially state-contingent claims (risk sharing). These net trades can be represented by a transfer $\tau_t = \tau_t(h^t)$, that the agent receives (or pays) at each date t, contingent on the realized individual history h^t. An *allocation*

1. As long as agents are ex ante identical, the initial level of assets does not affect the qualitative characteristics of the efficient allocation.

(or social contract) $\mathcal{W} = (\tau, \sigma)$ in this economy is hence a contingent plan of transfer $\tau = \{\tau_t(h^t)\}_{t=1}^T$ and agent's decisions $\sigma = \{\mathbf{e}_t(h^t), \mathbf{b}_t(h^t), \mathbf{c}_t(h^t)\}_{t=1}^T$ of effort, assets and consumption, as a function of the realized history h^t. An admissible allocation must satisfy at any date and for each agent the following individual budget constraint:

$$c_t + b_t = y_t + \tau_t + (1+r)b_{t-1}, \tag{1}$$

where y_t is today's income level, τ_t is the received transfer, b_{t-1} is the asset level accumulated previous period, and $c_t \geq 0$ and b_t represent period t consumption and bond holding decisions respectively. In addition, $\mathbf{b}$ must satisfy the standard No Ponzi Game condition (or solvency constraint): $\lim_{\tau \to T} \left(\frac{1}{1+r}\right)^{\tau-1} \mathbf{b}_\tau(h^\tau) \geq 0$ almost surely.

2.1. *Constrained Efficiency*

A (constrained) efficient allocation can be computed by solving the problem of a benevolent planner whose aim is to reallocate resources optimally in order to insure agents, subject to the feasibility and incentive constraints. For any given initial distribution $\mathbf{p}^0$ over Y, the problem of the planner can be formulated as follows

$$\sup_{\mathcal{W} \in \Omega} \sum_{i=1}^N p_i^0 \mathbf{U}_1^T(\mathcal{W}; y_1^i); \quad \text{s.t.} \quad \sum_{i=1}^N p_i^0 \mathbf{\Pi}_1^T(\mathcal{W}; y_1^i) \geq 0, \tag{2}$$

where Ω is the set of incentive feasible allocations and, given history $h^1 = y_1^i$,

$$\mathbf{U}_1^T(\mathcal{W}; h^1) = \mathbf{E}_1 \left[\sum_{t=1}^T \beta^{t-1} u(\mathbf{c}_t(h^t), \mathbf{e}_t(h^t)) | \mathbf{e} \right]$$

is the agent's expected discounted lifetime utility induced by the social contract $\mathcal{W}$ after node h^1. It is important to notice that the expectation operator depends on the effort plan $\mathbf{e} = \{\mathbf{e}_t(h^t)\}_{t=1}^T$, as income histories do. The per-period utility u is strictly increasing, concave, and smooth in both arguments, with $u_e'(c, 0) \equiv 0$. The parameter $\beta \in (0, 1)$ represents the discount factor.

An allocation $\mathcal{W}$ satisfying the sequence of individual budget constraints at each node is (sequentially) incentive compatible, that is, $\mathcal{W} \in \Omega$, if for any history h^t we have

$$\mathbf{U}_t^T(\tau, \sigma; h^t) \geq \mathbf{U}_t^T(\tau, \tilde{\sigma}; h^t), \tag{3}$$

where $\tilde{\sigma}$ is any other possible continuation plan of actions after node h^t that satisfies the budget constraint (1) and $\lim_{\tau \to T} \left(\frac{1}{1+r}\right)^{\tau-1} \tilde{\mathbf{b}}_\tau(h^\tau) \geq 0$ almost surely, given the transfer plan τ. Finally, the constraint

$$\Pi_1^T(\mathcal{W}; h^1) = \mathbf{E}_1 \left[\sum_{t=1}^{T} \left(\frac{1}{1+r} \right)^{t-1} (-\tau_t(h^t)) | \mathbf{e} \right] \geq 0$$

in equation (2) represents the planner's intertemporal budget restriction.

2.2. *First-Order Conditions and Ex Post Verification Approach*

Note that the incentive constraint (3) is a very complicated object. In particular, notice that a deviation on bond holdings b_t at period t typically generates wealth effects that most likely will induce further future deviations of effort and consumption. Thus, a "brute force" approach to solving (2) requires keeping track of all these off the equilibrium deviations that makes this kind of direct procedures typically infeasible in practice (see APa for details).

On the other hand, notice that any interior contract $\mathcal{W}$ that is incentive feasible according to equation (3) must also satisfy the following first-order conditions for each $h^t \neq h^T$:

$$e: \qquad -u'_e(\mathbf{c}_t(h^t), \mathbf{e}_t(h^t)) = \beta \sum_i p'_i(\mathbf{e}_t(h^t)) \mathbf{U}^T_{t+1}(\mathcal{W}; (h^t, y^i)) \tag{4}$$

and

$$b: \; u'_c(\mathbf{c}_t(h^t), \mathbf{e}_t(h^t)) \geq \beta(1+r) \sum_i p_i(\mathbf{e}_t(h^t)) u'_c(\mathbf{c}_{t+1}(h^t, y^i), \mathbf{e}_{t+1}(h^t, y^i)), \tag{5}$$

where we used the budget constraint (1) to eliminate the planner transfers τ_t, and assumed interiority with respect to e.[2]

It turns out that a much more viable procedure is hence to first solve the model by restricting the planner to only satisfy the first-order conditions of the agent, and then verify ex post that the obtained allocation is truly optimal. More specifically, let Ω_{FOC} be the set of social contracts satisfying these first order conditions, and $\mathcal{W}^*_{FOC}$ be the solution to the relaxed planner's problem (2) where Ω_{FOC} is replaced for Ω. Since $\Omega \subset \Omega_{FOC}$, if we can show that $\mathcal{W}^*_{FOC} \in \Omega$, then we have actually derived the efficient contract. In Section 3.4, we will see that the first-order conditions approach facilitates the characterization of the optimal allocation to a great extent.

In practical terms, the ex post verification procedure can be implemented as follows (see APa for details). After computing the optimal contract according

2. Notice that when $T < \infty$, there is an obvious corner solution $e_T(h^T) \equiv 0$ for effort. Moreover, condition (5) at period T becomes $u'_c(c_T(h^T), e_T(h^T)) \geq 0$, which implies $b_T(h^T) \equiv 0$.

to the relaxed problem, we allow the agent to remaximize his lifetime utility by choosing effort, consumption, and bond holdings taking the relaxed-optimal transfer scheme implied by $\mathcal{W}^*_{FOC}$ as given. If the optimal decisions of this remaximization problem do not improve the agent's welfare, it must be that the agent does not have any profitable deviation from the proposed plan. Hence $\mathcal{W}^*_{FOC}$ is an incentive compatible allocation, i.e., it is an optimal contract. In APa, we show that the use of the first-order approach also permits the problem to be written in a tractable recursive form. The recursive formulation is useful not only for numerical computations, but it also simplifies considerable the ex post verification procedure.

3. Characteristics of the Efficient Allocation

In this section, we discuss the distinctive characteristics of the efficient allocation by comparing it to three other different model scenarios: the basic permanent-income model, the model with complete markets and the standard moral hazard model with observable assets. To make our analysis more applicable for empirical work, we will focus on the properties of consumption and income processes alone. We will also restrict to the additive separable preferences case ($u(c, e) = u(c) - v(e)$) and, for expositional simplicity, we set $\beta(1 + r) = 1$.

3.1. Permanent Income (Self-Insurance): The Efficiency Gains Result

The permanent income (self-insurance) model is probably the most commonly used workhorse in consumption theory. Various formulations of this model have been tested in a wide variety of environments (see Attanasio 1999 for a comprehensive survey).

Notice that the self-insurance allocation of consumption, effort, and bond holdings constitutes an incentive compatible social contract since it corresponds to the solution of problem (2) when transfers are identically set to zero, i.e., when $\tau_t(h^t) \equiv 0$. As we mentioned above, ACK find that the self-insurance allocation is the only incentive compatible one in the hidden information model. Proposition 2 of APa shows that a sufficient condition for the optimal contract to be welfare improving with respect to self insurance is that the agent has imperfect control over income histories. In order to provide risk sharing, the planner must be able to impose taxes in good states and pay transfers in bad states. Since his transfers are based on publicly observable outcomes (income histories) alone, when each agent has perfect and costless control over them, the planner is not able to collect tax revenues, as the agent would never make the taxable incomes to appear. In APa, it is also shown, that in its general formulation, the model presented in Section 2 nests that of ACK. Further, in terms of our framework, the hidden information model of ACK is specific precisely in the sense, that the agents have perfect and costless control over public outcomes.

In terms of observable characteristics, notice first that the standard Euler equation is a characterizing feature of both self insurance and of our efficient allocation (see condition(5)). Hence, the properties of the univariate time series of consumption alone are not particularly informative in distinguishing these two allocations. Notice however that, in the self-insurance framework, individual budget feasibility requires that the discounted present value of income flows must be equal to the discounted present value of consumption flows for each agent. In other terms, the budget constraint (1) together with $\tau_t(h^t) \equiv 0$ implies that each agent must have a zero net present value (NPV), i.e.,

$$\sum_{t=1}^{T}\left(\frac{1}{1+r}\right)^{t-1}(y_t - c_t) = 0 \text{ almost surely for all } h^T. \tag{6}$$

In contrast with this, since the planner typically provides additional insurance by making history dependent transfers, the zero NPV condition (6) is violated in an optimal allocation, at least for some history h^T.[3] Attanasio and Pavoni (2004) exploit this property and argue that, in an efficient allocation, consumption does not exhibit "excess sensitivity" (à la Flavin 1981) but, because of the additional insurance provided to consumers relative to self-insurance, it exhibits "excess smoothness" in the spirit of Campbell and Deaton (1989). Using a closed form specification of the model, which incorporates the ACK and the complete market model as special extreme cases, they also show how one can directly relate the magnitude of the excess smoothness of consumption to the degree of control the agent has over publicly observable income levels.

Using a modification of the VAR techniques adopted for aggregate data by West (1988), Hansen, Roberds, and Sargent (1991), and Galí (1991), Attanasio and Pavoni show a rejection of the intertemporal budget constraint with a single asset for U.K. micro data, indicating excess smoothness of consumption. At the same time, they fail to reject the standard overidentifying restrictions implied by the Euler equation for consumption. Both results are consistent with the implications of our model.

3.2. Complete Markets (Full Insurance)

The interest in models with private information has often been motivated by the failure of another important workhorse in consumption theory: the complete markets model.

3. Notice that the planner's intertemporal budget constraint guarantees that an optimal allocation satisfies the zero NPV condition in expected terms, that is, at the aggregate level.

The complete markets allocation can be derived by solving a modification of the planner's problem (2) where both effort and consumption decisions are fully observable, and the set of feasible contracts is hence restricted by the sequence of the individual budget restrictions (1) alone. Combining the first order conditions gives us

$$u'(c_t(h^t)) = u'\left(c_{t+k}\left(h^{t+k}\right)\right), \quad \text{for all } t \geq 1 \text{ and } k \geq 0. \tag{7}$$

That is, when there are no aggregate shocks, marginal utility (hence consumption) remains fixed both intertemporally and cross-sectionally. The fact that (7) involves no expectations operator is the defining property of full insurance. There is no individual variation in consumption because any fluctuation in idiosyncratic income is perfectly shared across agents. In contrast, it is easy to see that the incentive compatibility constraint (3) implies that, as long as the efficient level of effort is positive, consumption cannot be independent of individual income. These simple observations are at the core of many tests of complete markets proposed in the literature. Virtually all studies soundly reject the null hypothesis of full consumption insurance (e.g., Attanasio and Davis 1996).[4]

3.3. Standard Moral Hazard (with Observable and Fully Contractible Assets)

Consider now the case where each agent has private information on the effort level e, but the planner can fully monitor consumption and asset decisions. In such an environment, the optimal allocation of consumption can be computed by solving a variation of the planner's problem (2) where instead of (3), the set of feasible contracts is derived by imposing an incentive constraint of the form

$$\mathbf{U}_t^T(\tau, \sigma; h^t) \geq \mathbf{U}_t^T(\tau, \tilde{\mathbf{e}}, \mathbf{b}, \mathbf{c}; h^t), \quad \text{for all } \tilde{\mathbf{e}} \tag{8}$$

i.e., allowing the agent to deviate only in his effort decisions.

In order to derive one key distinguishing property of this model, imagine that the principal manipulates the transfer scheme so that he takes away some utility Δ from the agent in one period, but then he returns Δ/β utils with certainty in the following period. A quick check of condition (8) (or of condition (4)) should convince the reader that this reallocation does not affect the incentive compatibility constraint. The key now is to notice that the cost (in terms of the consumption good) of providing z utils to the agent is just the inverse of the agent's utility function evaluated at z, or $u^{-1}(z)$. Thus after imposing the budget

4. Notable exceptions are Altug and Miller (1990) and Mace (1991). In these papers, however, failure to reject the null hypothesis of full consumption insurance is likely to be due to econometric and sample selection issues. See, e.g., Nelson (1994).

constraint, the first-order condition with respect to Δ (evaluated at $\Delta = 0$) yields (e.g., Rogerson 1985)

$$\frac{1}{u'(c_t(h^t))} = \mathbf{E}_t\left[\frac{1}{u'(c_{t+1}(h^{t+1}))}\right]. \tag{9}$$

Roughly speaking, it says that in a standard moral hazard model where agents do not have access to the credit market the inverse of the marginal utility $1/u'(c_t)$ follows a martingale. Notice that, since $1/x$ is a strictly convex transformation, Jensen's inequality and (9) imply that $u'(c_t) < E_t[u'(c^i_{t+1})]$, which is in sharp contrast with the Euler equation (5) characterizing our efficient allocation. An important consequence of this difference is that the standard moral hazard model leads to a more frontloaded consumption profile than our allocation. A may be more important implication is that, under some conditions, by using this discrepancy in the intertemporal properties of consumption, one can distinguish empirically our efficient allocation from the standard moral hazard model with monitorable borrowing and lending. Notice that condition (5) implies that the marginal utility $u'(c_t)$ *itself* follows a martingale. Hence, in the case of CRRA utility (i.e., $u(c) = c^{1-\sigma}/(1-\sigma)$), both equations (5) and (9) can be written in a general form as follows

$$c_t^{\gamma}(h^t) = \mathbf{E}_t[c_{t+1}^{\gamma}(h^{t+1})]. \tag{10}$$

Since risk aversion implies that $\sigma > 0$, if the data suggests that $\gamma > 0$ we might interpret this as evidence in favor of the standard moral hazard model (with observable assets) as the "true" model explaining the data. On the other hand, when $\gamma < 0$ the data supports that the Euler equation is satisfied suggesting that our model does better instead.

Ligon (1998) used condition (10) to test the standard moral hazard model versus self insurance with rural South Indian data. In two of the three villages he studies, he finds a violation of the Euler equation $\gamma > 0$. On the other hand, for developed countries, the common view seems to that—once changes in labor supply decisions and demographics are appropriately accounted for—the Euler equation cannot be rejected in micro data, at least in its weak form of condition (5) (Attanasio 1999).

3.4. Further Characterization of the Efficient Allocation

In order to characterize further the efficient allocation of consumption we will restrict ourselves to the two-period version of the model ($T = 2$). This setup is studied in detail in Ábrahám and Pavoni (2004b). Moreover, since the recursive formulation is essentially an "extended" two-period model, this case exhibits most key properties of the general multiperiod case as well (see APa for details).

Using the first-order conditions (4) and (5), we can characterize the solution of the relaxed version of problem (2) by looking at the stationary points of the following Lagrangian:[5]

$$\mathcal{L} = u(c_1) - v(e_1) + \beta \sum_i p_i(e_1)u(c_2^i)$$
$$+ \lambda \left[y_1 - c_1 + \frac{1}{1+r} \sum_i p_i(e_1)(y_2^i - c_2^i) \right]$$
$$+ \mu \left[\beta \sum_i p_i'(e_1)u(c_2^i) - v'(e_1) \right] + \xi \left[u'(c_1) - \beta(1+r) \sum_i p_i(e_1)u'(c_2^i) \right],$$

where it can be shown that the multipliers γ, μ, and ξ are all positive. For later use, notice that, since the problem with observable assets does not include the incentive condition (5), the properties of the standard moral hazard model can be obtained from the above Lagrangian by imposing that $\xi = 0$.

The expression determining the stationary point with respect to second-period consumption is

$$\frac{\lambda}{u'(c_2^i)} = 1+\mu\frac{p_i'(e_1)}{p_i(e_1)} - \xi(1+r)\frac{u''(c_2^i)}{u'(c_2^i)} = 1+\mu\frac{p_i'(e_1)}{p_i(e_1)} + \xi(1+r)\rho_a(c_2^i), \quad (11)$$

where $\rho_a(c) = -u''(c)/u'(c)$ is the coefficient of absolute risk aversion at c. Condition (11) implies that consumption varies with income only to the extent income provides information about the agent's hidden action. The optimal trade-off between insurance and incentives requires that consumption dispersion is provided only if the agent's action actually affects the likelihood ratio $p_i'(e)/p_i(e)$. For all other income levels, and regardless of their dispersion, a constant level of consumption is provided. This cross sectional characteristic of the efficient allocation is typical in models with moral hazard (e.g., Grossman and Hart 1983). However, this contrasts sharply with the self-insurance allocation, where consumption is always monotone in income, but is in principle uncorrelated with the likelihood ratio.[6]

Both in the numerical exercises in APa and in the specification proposed by Attanasio and Pavoni (2004) consumption typically varies less across states in our model than in the permanent income model. This also has implications for the time series properties of consumption. Since the agent faces less future idiosyncratic

5. For simplicity, we normalize the effort cost so that $v(0) = 0$, and assume that p_i^0 is degenerate at y_1.

6. In a two-period model this property is particularly evident since $c_2^i = y_2^i + (1+r)b_1$. When $T > 2$, monotonicity in income is a well-known consequence of intertemporal consumption smoothing.

uncertainty, his precautionary motives for savings are reduced. The precautionary motive is further reduced by the fact that the agent's effective borrowing limit in Period 1 is relaxed because the transfers increase his minimum disposable second period income.[7] Both these effects imply that, for any given initial level of wealth, the agent enjoys a much smoother intertemporal path of consumption in our case than in the permanent income model (see APa, Figure 5).

We saw in the previous section, that from an intertemporal point of view, in the standard moral hazard model, consumption tends to be more frontloaded compared to the efficient allocation with hidden assets. In order to understand how the introduction of hidden asset accumulation alters the cross-sectional distribution of consumption we focus on the curvature of the optimal consumption scheme as function of income. Taking the difference in equation (11) between two adjacent states, we obtain the following expression

$$\begin{aligned}\left[\frac{\lambda}{u'(c_2^{i+1})} - \frac{\lambda}{u'(c_2^i)} - \xi(1+r)(\rho_a(c_2^{i+1}) - \rho_a(c_2^i))\right] \\ = \mu\left[\frac{p'_{i+1}(e_1)}{p_{i+1}(e_1)} - \frac{p'_i(e_1)}{p_i(e_1)}\right].\end{aligned} \qquad (12)$$

Notice that, when the access to the asset markets is observable ($\xi = 0$) the absolute risk aversion of the agent does not play a role in explaining the curvature of the consumption scheme once $1/u'$ is known. On the other hand, if absolute risk aversion is decreasing and convex (key properties of the popular CRRA preferences), whenever the agent has hidden access to credit, the consumption scheme becomes more convex than in the standard moral hazard case with monitorable assets. For example, when income levels are equally spaced and the likelihood ratio increases linearly with income, log utility implies a linear consumption scheme in the standard moral hazard model, and a convex one in the case of hidden access to the credit market.

In order to obtain a clearer intuition of this result we may examine (11) further. This expression equates the planner's costs and benefits of a marginal increase of the utility of the agent in state i normalized by $p_i(e_1)(1+r)$. As we saw before, this change costs the planner $\lambda/u'(c_2^i)$ in terms of resources to be collected in order to satisfy the intertemporal budget constraint. This cost is first of all offset by a one-to-one increase in the agent's welfare. Further, increasing the agent's utility also relaxes the effort incentive compatibility constraint by $\mu p'_i(e_1)/p_i(e_1)$. Notice that these effects are present in the standard moral hazard model as well. In the hidden asset case ($\xi > 0$), there is an additional gain though: by increasing $u(c_2^1)$, the planner also alleviates the saving motives of the agent, a gain measured

7. Notice that the agent's solvency condition requires that $b_1 \geq -\min_i \left\{\frac{y_2^i + \tau_2^i}{1+r}\right\}$.

TABLE 1. Comparisons across the different models.

	Full insurance	Permanent income (PIH)	Moral hazard obs. assets	Moral hazard hidden assets
Euler equation(5)	yes	yes	no	yes
Inverse Euler equation (9)	yes	no	yes	no
Intertemp. budget (6)	no	yes	no	no
Excess smoothness	0%	100%	partial	partial

in utility terms by $\xi(1+r)u''(c_2^i)/u'(c_2^i)$. Since consumption is increasing in i, decreasing and convex absolute risk aversion implies that the latter gain is getting smaller at a decreasing rate as income is getting higher. Eventually, this implies that in order to distribute incentives optimally, the planner has to increase consumption between state i and $i+1$ by a factor, which is increasing in i, even if $p'_{i+1}(e_1)/p_{i+1}(e_1) - p'_i(e_1)/p_i(e_1)$ is constant. Hence, the consumption plan becomes more convex than in the standard moral hazard case, where this additional factor is not present.

4. Summary and Conclusions

Table 1 summarizes the empirically testable comparisons we have made in the preceding sections. The first three rows in the table represent the different equilibrium restrictions we introduced before. Therefore, a "yes" ("no") entry in a particular cell implies that the model in the corresponding column satisfies (violates) the restriction in the corresponding row. The fourth row displays the degree of excess smoothness. That is, it describes how consumption reacts to innovations on permanent income in the different models.[8]

This table makes clear that in principle, the equilibrium allocations of all four of the models can be distinguished from one another on the basis of observable features.

In the previous section, we also described some more quantitative properties of the efficient allocation. The optimal consumption allocation under hidden assets tends to be less disperse both across states and intertemporally when compared to self-insurance, and its cross-sectional curvature (as a function of income) becomes more convex when compared to the standard moral hazard model with observable assets.

8. In our model, innovations in *permanent income* correspond to unexpected changes in the annuity value of the expected discounted sum of future incomes:

$$r(\mathbf{E}_{t+1} - \mathbf{E}_t) \sum_{n=1}^{\infty} \left(\frac{1}{1+r}\right)^n y_{t+n}.$$

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